


Saturation of Vortex Rings Ejected from Shock-Accelerated Interfaces

Michael J. Wadas¹,* Loc H. Khieu, Griffin S. Cearley¹, Heath J. LeFevre¹, Carolyn C. Kuranz¹, and Eric Johnsen¹
University of Michigan, Ann Arbor, Michigan 48109, USA

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Structures evoking vortex rings can be discerned in shock-accelerated flows ranging from astrophysics to inertial confinement fusion. By constructing an analogy between vortex rings produced in conventional propulsion systems and rings generated by a shock impinging upon a high-aspect-ratio protrusion along a material interface, we extend classical, constant-density vortex-ring theory to compressible multifluid flows. We further demonstrate saturation of such vortex rings as the protrusion aspect ratio is increased, thus explaining morphological differences observed in practice.

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The mixing induced by the interaction of shocks with interfaces separating different fluids has implications for problems ranging from inertial confinement fusion (ICF) to astrophysics. In ICF, laser-generated shocks pass through layers of a capsule containing fusion fuel, compressing and heating it to extreme conditions at which self-sustained nuclear fusion may occur [1–4]. The mixing of the capsule material with fuel in the hot spot due to imperfections on the capsule surface and from the gas fill tube is a primary source of decreased performance [5–11]. In core-collapse supernovae, shocks generated from rapid energy release propagate outward through the layers of the collapsing stars, causing heavy core elements to jet into outer layers [12–16]. An understanding of these turbulent, shock-driven mixing phenomena is crucial for the success of ICF and may help elucidate the mechanisms by which heavy and light elements mix in the universe. While efforts are underway to develop models describing these turbulent phenomena [17,18], their initialization is problematic given the lack of understanding of the transition to turbulence in these shock-accelerated flows.

As it traverses a perturbed material interface, a shock deposits baroclinic vorticity due to the misalignment of the density and pressure gradients, driving interfacial perturbation growth via the Richtmyer-Meshkov instability (RMI) [19,20]. Perturbations initially grow linearly in time before saturating and rolling up into a mushroomlike shape as nonlinear effects dominate [21,22]. Theoretical studies have long described early-time RMI dynamics in terms of point vortices or vortex sheets [23–26]. In complex situations (e.g., multimode, reshock, late times, etc.), the multimaterial mixing region may evolve to turbulence [27–31], which spreads as eddies entrain adjacent fluid parcels of different densities [32,33]. Although linear and, to some extent, early nonlinear theory describing perturbation growth are well established [22,34–36], turbulence transition and late-time mixing are poorly understood due to the inhomogeneity and intermittency of the flow. There is no doubt, however, that

vorticity dynamics play a role in transition and mixing [26,37,38].

Advances in diagnostics, in particular, micron-scale x-ray imaging with Fresnel zone plates and lithium fluoride detectors [39–42], and numerical methods have enabled high-resolution visualizations of shock-induced mixing regions. In recent studies [10,29,43–49], flow structures intermittently ejected from the mixing region can be discerned, in some cases to distances many times their characteristic length, as illustrated in Fig. 1. Though explicitly connected to vortex rings [30,48,50,51], these structures cannot be described by (nonlinear) RMI theory or existing models [34,35,47,52–54]. A criterion for vortex dipole ejection based on the initial interface geometry was developed [47], though this study fell short of quantitatively describing the ejection mechanism. Predicting this shock-induced ejection of vortex rings is critical to determine the growth of the mixing region because these structures transport kinetic energy and circulation from the mixing region and therefore affect its size and intensity.

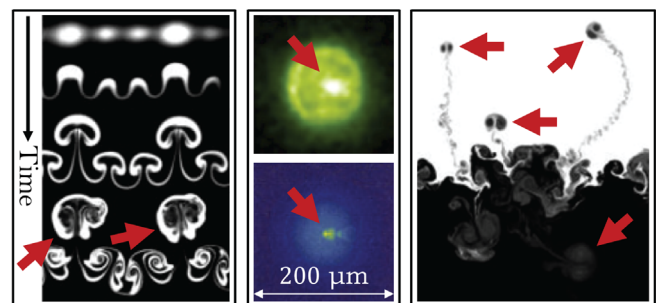


FIG. 1. Left: evolution of a multimode shocked fluid layer experiment [44]. Middle: experimental (top) and simulated (bottom) x-ray self-emission during an ICF capsule implosion [45]. Right: volume fraction from RMI simulations [43]. Red arrows indicate likely vortex rings and dipoles. All images are reproduced with permission.

Furthermore, such rings may perturb other flow structures away from the mixing region (e.g., other nearby interfaces). While vortex rings generated by drawing a piston in an open-ended cylinder have received significant attention in a completely different context [55,56], the mechanism explaining the formation and ejection of vortex rings and dipoles from shock-induced mixing regions has yet to be established.

In this Letter, we outline a theory quantitatively describing the formation, ejection, and evolution of vortex rings from shock-accelerated interfaces. Drawing from classical studies [55], we develop a setup enabling us to systematically investigate the formation and ejection of vortex rings from mixing regions generated by the interaction of a shock with an interface. Our analysis, verified by direct simulations of the Euler equations, fully describes the vortex ring dynamics, thereby extending classical theory for piston-cylinder systems to more general geometries and vorticity sources, including compressible multifluid flows. This theory allows us to calculate the kinetic energy and circulation leaving the confines of the mixing region, which are key quantities in turbulence modeling, and identify situations when vortex ring saturation occurs. Furthermore, our theory may elucidate flows dominated by shock-induced jets at interfaces, such as those observed in ICF due to the fill tube and capsule defects [10,45,46], astrophysics [16,57–59], and possibly ejecta physics [60,61].

We first revisit the basic attributes of classical vortex rings in propulsion systems, in which an impulsively started jet is generated by forcing a piston through a hollow cylinder. The vortex sheet produced along the inner edge of the orifice separates and rolls up to form a ring [62–64]. As long as the ring is attached to the jet, it accumulates circulation as it convects downstream. However, beyond a critical stroke-to-diameter ratio, the ring detaches from the jet, and no additional circulation can be imparted to it. Over a wide range of experiments and simulations [65–71], this saturation occurs at a stroke-to-diameter ratio of approximately 4, deemed the *formation number*, which equivalently represents the timescale over which vortex rings form by expressing the stroke length as the time-integrated piston velocity [55,56].

Given morphological similarities between classical vortex rings and rings ejected from shock-accelerated interfaces (see Supplemental Material [72]), we postulate that the vorticity deposited by a shock interacting with a high-aspect-ratio protrusion along a material interface serves the same purpose as that produced along the cylinder wall. If this hypothesis is correct, vortex rings ejected from shock-accelerated interfaces would exhibit behavior similar to classical rings, e.g., in terms of their circulation and formation number (including saturation), as well as their energy and impulse. We therefore expect that an initial interfacial geometry analogous to the cylinder may lead to

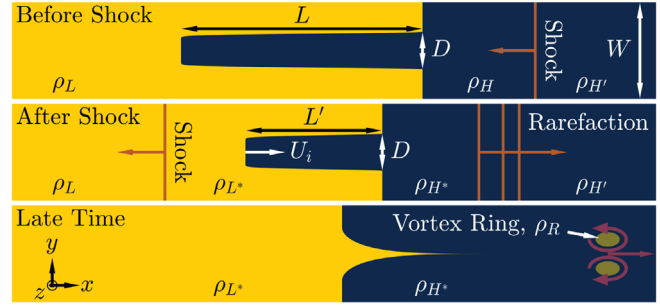


FIG. 2. Schematic showing the problem setup. Top: a shock approaches an interface separating heavy and light fluids with a heavy-fluid protrusion. Middle: the shock is transmitted through the interface, compressing the protrusion and setting the interface into motion, while a rarefaction is reflected. Bottom: the protrusion inverts and ejects a vortex ring into the heavy fluid.

similar vorticity dynamics. As illustrated in Fig. 2, we set up our problem with heavy fluid of density ρ_H adjacent to light fluid of density ρ_L , with a heavy-fluid protrusion of diameter D and depth L in the light fluid. The shock propagates from right to left. The compression ratio across the shock $\rho_{H'}/\rho_H$ characterizes the shock strength (equivalently, a Mach number, M , could be defined), where $\rho_{H'}$ is the density of the shocked heavy fluid. The shock is transmitted through the interface, compressing the light fluid to density ρ_{L^*} , and a rarefaction is reflected from the interface, decompressing the shocked heavy fluid to density ρ_{H^*} . These densities are obtained by solving the Riemann problem initialized between the unshocked light fluid and the shocked heavy fluid. The protrusion, modeled as decompressed shocked heavy fluid of depth $L' = (\rho_H/\rho_{H^*})L$, starts inverting due to the sign of the vorticity. During inversion, the fluid in the protrusion is ejected into the heavy fluid at speed U_i relative to the interface, analogous to the piston speed in the classical system, entraining the light fluid and forming a vortex ring of density ρ_R . As in past studies [56], we model the ring as belonging to the Norbury [73] family, but must account for compressibility and multiple fluids. For an unsaturated ring, the circulation, impulse, and energy of the fluid slug ejected from the protrusion equal those corresponding to the ring. Extending the analysis to the present multi-material system yields

$$\Gamma_{\text{slug}} = \frac{1}{2}L'U_i = \frac{1}{2}\frac{\rho_H}{\rho_{H^*}}LU_i = (\Omega\alpha l)\Gamma_N = \Gamma_{VR}, \quad (1a)$$

$$I_{\text{slug}} = \frac{1}{4}\rho_H\pi D^2LU_i = \rho_R(\Omega\alpha l)^3I_N = I_{VR}, \quad (1b)$$

$$E_{\text{slug}} = \frac{1}{8}\rho_H\pi D^2LU_i^2 = \rho_R(\Omega\alpha l)^2l^3E_N = E_{VR}, \quad (1c)$$

where Ω is the vorticity density (i.e., vorticity divided by distance from the ring centerline [73]), α is the nondimensional mean vortex core radius, l is the ring

radius, and the subscript N denotes nondimensional quantities that are functions only of α . Equations (1) can be rearranged to yield the formation number

$$\frac{L}{D} = \sigma \sqrt{\frac{\pi I_N^{1/2} \Gamma_N^{3/2}}{E_N}} = \sigma \sqrt{\frac{\pi \Gamma_N^{3/2}}{2 U_N I_N^{1/2}}}, \quad (2)$$

where $L' = (\rho_H/\rho_{H^*})L$, $U_i = 2(\Omega\alpha l)U_N$ [56], and

$$\sigma = \sqrt{\frac{\rho_{H^*} \rho_{H^*}}{\rho_R \rho_H}}. \quad (3)$$

As with classical vortex rings, the formation number depends on the circulation, impulse, and energy, but now also depends on the shock strength and interface density ratio. Equation (2) states that maximal circulation, impulse, and energy generated by the inverting protrusion is contained within the resulting ring. The parameter σ is a formation number multiplier generalizing the classical formation number [compare Eq. (2) in the present Letter to Eqs. (12) and (13) in [56]] to rings generated from shocked interfaces, i.e.,

$$(L/D)_{\text{sat,shock}} = \sigma(L/D)_{\text{sat,classical}}. \quad (4)$$

The formation number multiplier accounts for both the interface density ratio and shock strength. The *multifluid factor* $\sqrt{\rho_{H^*}/\rho_R}$ is the square root of the relative density of the protrusion with respect to the ring after shock interaction. We approximate the ring density as the average of the postshock fluid densities, $\rho_R \approx (\rho_{H^*} + \rho_{L^*})/2$, which is consistent with both simulations and the entrainment properties of saturated Norbury rings [73], though other mixing models could be used. As the density ratio is increased, the relative density of the vortex ring decreases. The resulting decrease in the ring's impulse and energy allows it to support additional impulse and energy from the inverting protrusion, which can be increased by increasing the protrusion depth [Eqs. (1b) and (1c)], thereby producing a larger formation number. The *compression factor* ρ_{H^*}/ρ_H describes shock compression of the protrusion. Because the circulation supplied by the inverting protrusion is proportional to protrusion depth and thus inversely proportional to shock strength [Eq. (1a)], the unshocked depth, and therefore formation number, must increase as the shock strength is increased to supply the same amount of circulation. If a vortex ring is produced with no compression ($\rho_{H^*} \rightarrow \rho_H$) and no density difference ($\rho_H \rightarrow \rho_L$), the classical limit $\sigma = 1$ is recovered. The classical formation number is generally accepted to be $3.0 \leq (L/D)_{\text{sat}} \leq 4.6$ [56], and therefore it is expected that in the present shock-accelerated case the formation number falls within the range

$$3.0\sigma \leq (L/D)_{\text{sat,shock}} \leq 4.6\sigma. \quad (5)$$

To verify these theoretical results, we use an in-house code [74–76] (see Supplemental Material [72]) to solve the three-dimensional Euler equations for the setup in Fig. 2.

TABLE I. Simulation parameters and expected formation numbers and ranges predicted by our theory.

Case	$\rho_{H'}/\rho_H$	M	ρ_H/ρ_L	3.0σ	4.6σ	$(L/D)_{\text{sat,shock}}$
1	1.17	1.1	2	3.94	6.05	5.16
2	1.17	1.1	5	4.27	6.54	5.52
3	1.34	1.2	2	4.42	6.78	5.47
4	1.34	1.2	5	4.65	7.13	6.03
5	1.86	1.5	8	5.54	8.50	6.79
6	2.67	2.0	11	6.33	9.70	7.59

Sets of simulations, detailed in Table I, are performed for different density ratios and shock strengths similar to those in [21,28,29,31,47]. In cases 1–4, L is varied from 1 to 8 while keeping D constant. Cases 5 and 6 explore stronger shocks and larger interface density ratios. The simulations evolve in time long enough for pinch-off to occur, leading to a steady, self-propagating vortex ring identified by the $\lambda_2 = 0$ criterion, a standard method for identifying vortices based on eigenvalues, $\lambda_1 < \lambda_2 < \lambda_3$, related to the velocity gradient tensor [77]. To illustrate the dynamics, Fig. 3 shows out-of-plane vorticity ω and density contours along the centerplane, as well as the three-dimensional surface of $\lambda_2 = 0$, for a shock of strength $\rho_{H'}/\rho_H = 1.34$ with interface density ratio $\rho_H/\rho_L = 2$ and protrusions of aspect ratios $L/D = 1$ and 5. For the $L/D = 1$ case, the protrusion inversion ejects a vortex ring that quickly pinches off and steadily separates from the original interface. Although some vorticity is confined to the interface, most of the vorticity is concentrated within the ring; there are no vortices between the ring and the interface. For the $L/D = 5$ case, which is near the formation number

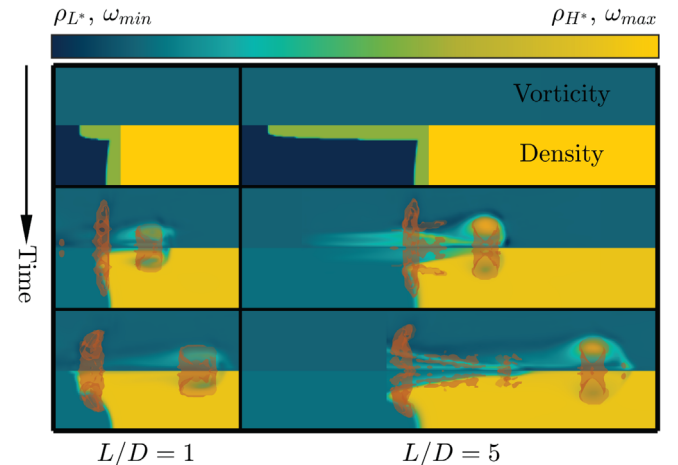


FIG. 3. Evolution of the vorticity (top) and density (bottom) following the interaction of a shock of strength $\rho_{H'}/\rho_H = 1.34$ with heavy-fluid protrusions with aspect ratios $L/D = 1$ (left) and $L/D = 5$ (right) along an interface with $\rho_H/\rho_L = 2$. Orange surfaces enclose vortex rings, as identified by the $\lambda_2 = 0$ criterion [77].

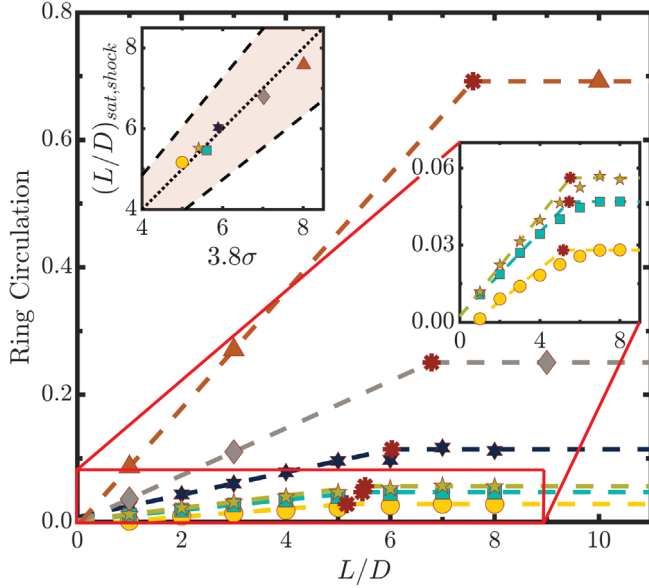


FIG. 4. Ring circulation vs protrusion aspect ratio for cases 1–6 (yellow circles, green pentagons, teal squares, dark blue hexagons, grey diamonds, and orange triangles, respectively) in Table I. The right inset is a close-up of cases 1–3. The horizontal location of each red star identifies the formation number. The upper-left inset shows the formation number from simulations vs our theory. The dotted line indicates where $(L/D)_{\text{sat,shock}} = 3.8\sigma$, and the dashed lines bound the shaded region where $3.0\sigma \leq (L/D)_{\text{sat,shock}} \leq 4.6\sigma$.

predicted by our theory, a pinched-off ring still steadily propagates away from the interface, but a trailing jet with vortices is evident. A known signature of exceeding the formation number, this jet forms because the saturated ring cannot support additional circulation from the vortex ring generator, which accumulates in the trailing jet [55].

These results suggest that the ring circulation saturates at sufficiently large protrusion aspect ratios, consistent with the classical case. To confirm that the formation number has been reached, Fig. 4 shows the circulation of the pinched-off rings for different protrusion aspect ratios L/D . At low aspect ratios, the circulation of the ejected rings increases as the protrusion aspect ratio is increased. As L/D is further increased, however, the circulation saturates; at this point, no further circulation can be imparted to the ring. We take the formation number to be the value of L/D at the intersection of a line fit to points with $L/D \leq 3$, where the ring circulation appears to be growing linearly, and the average circulation of points with $L/D \geq 7$, where the circulation plateaus. The formation number for each set of simulations differs from classical theory but falls near the center of the range predicted by our extended theory, summarized in Table I and the upper-left inset of Fig. 4.

As an example application, we estimate the formation number for the fill-tube jet for experiment N210808 at the National Ignition Facility [4,78] as

$$(L/D)_{\text{sat,shock}} = 3.8\sigma = 3.8\sqrt{\frac{2\rho_{e^*}}{\rho_{a^*} + \rho_{e^*}} \frac{\rho_{e^*}}{\rho_e}} \approx 7, \quad (6)$$

where ρ_{e^*} and ρ_{a^*} are the densities of the epoxy and ablator, respectively, after a strong shock releases off the ablator-ice interface and ρ_e is the unshocked epoxy density. Although a $2\ \mu\text{m}$ fill tube was utilized, the diameter of the epoxy-filled bore hole was $D = 8.4\ \mu\text{m}$ [78], making epoxy the appropriate material to model the protrusion. In this experiment, the ablator thickness was $L = 79\ \mu\text{m}$, and therefore $L/D = 9.4$. Because $L/D > (L/D)_{\text{sat,shock}}$, a saturated vortex ring and a trailing jet are expected, consistent with Fig. 1. The circulation, impulse, and energy injected into the hot spot by the ring can then be calculated from Eq. (1) with $L/D = (L/D)_{\text{sat,shock}}$. Furthermore, by replacing L with an integrated protrusion, or jet, velocity, our theory provides the timescale on which the ring forms. Although the calculated formation number appears to agree with observations of fill-tube jetting, the authors advise caution for direct applications to ICF, which involves fill-tube configurations that differ from our canonical setup, intense radiation, and convergent geometries. These and other effects would likely need to be considered for a precise determination of the fill-tube formation number.

The lack of a trailing jet for rings generated from smaller defects, and in the late nonlinear phase of the RMI, suggests that these rings are unsaturated. Both saturated and unsaturated rings transport kinetic energy, thus giving rise to a reduction in the kinetic energy of the mixing region the ring originates from. Ejected rings could also interact with other flow structures away from the mixing region. While the effects of viscosity and magnetic fields are not presently considered based on the Reynolds number and plasma beta, Re and β , respectively, in ICF, where $\text{Re} \sim 10^4$ [79] and $\beta \sim 10^5$ [80], and for many supernovae, where $\text{Re} \sim 10^{10}$ and $\beta \sim 10^4$ [81], the conservation arguments central to our theory support a path to explore diffusive effects and magnetic fields. Because such mechanisms would tend to inhibit the conversion of circulation, impulse, and energy from the inverting protrusion to the vortex ring, a greater protrusion aspect ratio would likely be required to form a saturated ring. These and other effects, including molecular diffusion and radiation, may be quantifiable with appropriate source terms in Eq. (1).

In this Letter, we demonstrate that vortex rings generated following the interaction of a shock with a cylindrical protrusion originate due to the baroclinic torque deposited along the interface, which ejects the fluid initially in the protrusion as it inverts. The resulting dynamics are consistent with classical piston-cylinder systems, exhibiting ring saturation coincident with the emergence of a trailing jet. However, the formation number is augmented by a factor accounting for the different fluid densities and shock compression. Our analysis thus generalizes the classical theory to compressible multifluid flows and

explains morphological differences observed in practice. While the computations presented support the theory, simulations exploring a wider range of parameters may help elucidate certain details, including late-time shock-induced turbulent mixing, as ejected vortex rings may affect mixing layer development and turbulent transition.

The present findings may have implications in ICF and astrophysics. In ICF, the fill tube may generate a vortex ring that impinges on the hot spot. Our theory also describes a mechanism that may affect mixing following supernovae and explain the presence of stellar core elements in the outer layers of expanding remnants. Furthermore, the results of our analysis could be used to deduce quantities of interest for shock-induced ejecta or jets.

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*mwadas@umich.edu

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