New Diagrammatic Framework for Higher-Spin Gravity

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We consider minimal type-A higher-spin (HS) gravity in four dimensions, at tree level. We propose new diagrammatic rules for this theory, involving both Fronsdal fields and Didenko-Vasiliev particles—linearized versions of HS gravity's "BPS black hole." The vertices include a standard minimal coupling between particle and gauge field, the Sleight-Taronna cubic vertex for HS fields, and a recently introduced vertex coupling two HS fields to a Didenko-Vasiliev particle. We show how these ingredients can be combined to reproduce all n-point functions of the theory's holographic dual—the free $O(N)$ vector model. Our diagrammatic rules interpolate between the usual diagrammatic rules of field theory and those of string theory. Our construction can be viewed as a bulk realization of HS algebra.

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Introduction.—Minimal type-A higher-spin (HS) gravity in $D=4$ spacetime dimensions [1–3] is the interacting theory of an infinite tower of parity-even massless fields, one for each even spin. It is also the conjectured bulk dual [4–7] within AdS/CFT [8–11] of a particularly simple boundary theory: the free $O(N)$ vector model of N real scalar fields φ^I $(I=1,\ldots,N)$. Remarkably, this holographic duality can be extended from AdS to de Sitter space [12], thus offering a window into 4D quantum gravity with positive cosmological constant. In the present Letter, we consider for simplicity the theory in Euclidean AdS, which we represent as a hyperboloid in flat 5D embedding space $\mathbb{R}^{1,4}$:

$$\mathrm{EAdS}_4 = \{x^\mu \in \mathbb{R}^{1,4} | x_\mu x^\mu = -1, x^0 > 0\}, \qquad (1)$$

where the metric of $\mathbb{R}^{1,4}$ is $\eta_{\mu\nu} = \mathrm{diag}(-1,1,1,1,1)$. Since the theory involves infinitely many massless fields interacting at all orders in derivatives, it was always believed to be nonlocal at distances smaller than the AdS radius. Though exotic, this still implies an expectation of locality at *larger* distances. Recent developments, described below, have challenged this expectation. The goal of this Letter is to propose a novel formulation of the theory, in terms of bulk diagrams whose elements' nonlocality is confined, as per the original hopes, by $\sim$1 AdS radius.

A simple formulation of *linearized* HS theory is in terms of Fronsdal fields [13,14]: a totally symmetric rank-s tensor potential for each spin s. We encode these as polynomials in an auxiliary vector u^μ:

$$h^{(s)}(x,u) = \frac{1}{s!} u^{\mu_1}\ldots u^{\mu_s} h^{(s)}_{\mu_1\ldots\mu_s}(x). \qquad (2)$$

In this Letter, the potentials Eq. (2) are always traceless, which can be viewed as either a gauge choice or a self-contained framework [15,16]. An efficient modern approach [17,18] is to treat both x and u as freely varying vectors in $\mathbb{R}^{1,4}$, with a constraint keeping $h^{(s)}_{\mu_1\ldots\mu_s}$ tangential to EAdS_4, and a scaling rule that defines its behavior away from $x\cdot x=-1$. Altogether, the constraints on $h^{(s)}(x,u)$ read:

$$(u\cdot\partial_u)h^{(s)} = s h^{(s)}, \qquad (x\cdot\partial_u)h^{(s)} = 0, \qquad (3)$$

$$(x\cdot\partial_x)h^{(s)} = -(s+1)h^{(s)}, \qquad (\partial_u\cdot\partial_u)h^{(s)} = 0, \qquad (4)$$

where ∂_x^μ and ∂_u^μ denote flat $\mathbb{R}^{1,4}$ derivatives with respect to the specified vectors. An important example of a free HS field is the boundary-bulk propagator [19,20]:

$$\Pi^{(s)}(x,u;\ell,\lambda) = -\frac{2^s s!(2s+\epsilon)}{4\pi^2(2s)!(s+\epsilon)} \times \frac{[(\lambda\cdot x)(\ell\cdot u)-(\ell\cdot x)(\lambda\cdot u)]^s}{(\ell\cdot x)^{2s+1}}. \qquad (5)$$

Here, ℓ^μ is a lightlike vector in $\mathbb{R}^{1,4}$ whose direction represents a point on the AdS boundary, λ^μ is a null polarization vector orthogonal to ℓ^μ, and $\epsilon = D-4$ is a dimensional regulator that serves to unify the $s=0$ and $s>0$ cases. The propagator Eq. (5) obeys the properties Eqs. (3) and (4), while also being transverse $(\partial_u\cdot\partial_x)\Pi^{(s)} = 0$.

In this language, a *cubic vertex* can be written as a differential operator $V^{(s_1,s_2,s_3)}(\partial_{x_1},\partial_{u_1};\partial_{x_2},\partial_{u_2};\partial_{x_3},\partial_{u_3})$ of order s_i in ∂_{u_i} $(i=1,\ 2,\ 3)$ acting on three fields $h_i^{(s_i)}(x_i,u_i)$, where in the end we set the x_i's equal and integrate over EAdS_4:

$$\int_{\mathrm{EAdS}_4} d^4x V^{(s_1,s_2,s_3)} \prod_{i=1}^{3} h^{(s_i)}(x_i,u_i)|_{x_i=x}. \tag{6}$$

When we plug in three propagators Eqs. (5) into (6), we should obtain the boundary three-point correlator $\langle j^{(s_1)} j^{(s_2)} j^{(s_3)} \rangle$ of HS currents. The vertex that accomplishes this is given by a simple formula, found by Sleight and Taronna [18]:

$$\begin{aligned} V^{(s_1,s_2,s_3)} = {} & \frac{8(i\sqrt{2})^{s_1+s_2+s_3}}{\sqrt{N}\Gamma(s_1+s_2+s_3+\epsilon)} \\ & \times (Y_{12}^{s_1} Y_{23}^{s_2} Y_{31}^{s_3} + Y_{13}^{s_1} Y_{21}^{s_2} Y_{32}^{s_3}), \end{aligned} \tag{7}$$

where $Y_{ij} \equiv \partial_{u_i}\cdot\partial_{x_j}$. The symmetrization over the two cyclic structures in Eq. (7) ensures gauge invariance within general traceless gauge [21]. A chiral HS theory based on the self-dual part of the vertex Eq. (7) has been developed in Refs. [22–25].

In contrast to the nice cubic vertex Eq. (7), the derivation of a *quartic* vertex [26] to reproduce the correlator $\langle j^{(0)} j^{(0)} j^{(0)} j^{(0)} \rangle$ yields a result that is nonlocal at all scales [27] (see also Refs. [28,29]). Despite heroic recent efforts in the Vasiliev formalism [30–34], this locality problem at the quartic level still stands. In this Letter, we propose a resolution to the problem, not just for four-point functions, but for all n-point functions. Specifically, we demonstrate that every n-point boundary correlator is equal to a sum of bulk tree diagrams, with *only cubic* vertices that are all local beyond ~1 AdS radius. Beyond cubic order, our rules for constructing these diagrams are different from those of standard bulk field theory, and bear some resemblance to those of string theory. Note that the sufficiency of *tree* diagrams is a general property of a free boundary dual: the correlators in this case lack $1/N$ corrections, which correspond to loop corrections in the bulk. Of course, for different boundary conditions, the bulk loop corrections will not vanish (and even when they do, one would want to compute this explicitly, as was done in Ref. [35] for the simplest diagrams).

Our construction proceeds in two steps. The first is purely within the boundary theory. In the next section, we introduce a simple yet novel diagrammatic description for the n-point correlators, in terms of "single-trace OPE diagrams." These are tree diagrams with cubic vertices, and single-trace operators on all the lines. Since single-trace operators correspond to fundamental bulk fields, such diagrams can be interpreted almost directly in the bulk theory. The complication is that the required operators are not just the local HS currents $j^{(s)}_{\mu_1\ldots\mu_s}(\ell)$ (dual to the bulk HS fields), but rather the *bilocals*:

$$\mathcal{O}(\ell,\ell') = \frac{\varphi^I(\ell)\varphi_I(\ell')}{NG(\ell,\ell')}, \tag{8}$$

whose Taylor expansion around $\ell=\ell'$ yields the $j^{(s)}_{\mu_1\ldots\mu_s}$ and their descendants. Here, the normalization factor $G(\ell,\ell')$ is the propagator of the fundamental boundary fields φ^I:

$$G(\ell,\ell') = \frac{1}{4\pi\sqrt{-2\ell\cdot\ell'}}. \tag{9}$$

Thus, to complete the bulk diagrammatic picture, we need the bulk dual of the bilocals Eq. (8), as well as bulk expressions for their cubic correlators. These were all recently characterized, and satisfy appropriate bulk locality properties [21,36–38]. Farther below, we recall these results, and combine them with the diagrams of the next section to yield the desired bulk rules. A more detailed overview of the recent results [21,36–38] is given in Supplemental Material [39], where we also illustrate the new bulk rules on examples.

Single-trace OPE diagrams.—In the boundary theory, all n-point correlators of local currents $j^{(s)}$ can be obtained by Taylor expanding the correlator of n bilocals $\mathcal{O}(\ell_i,\ell'_i)\equiv\mathcal{O}_i$. The latter is given by a sum of one-loop Feynman diagrams:

$$\langle\mathcal{O}_1\ldots\mathcal{O}_n\rangle = \frac{N^{1-n}}{2n\prod_{p=1}^n G(\ell_p,\ell'_p)} \sum_{\ell_i\leftrightarrow\ell'_i} \sum_{S_n} \prod_{p=1}^{n} G(\ell'_p,\ell_{p+1}), \tag{10}$$

where the last product is cyclic $\ell_{n+1}\equiv\ell_1$, the inner sum is over the $n!$ permutations of $(\mathcal{O}_1,\ldots,\mathcal{O}_n)$, and the outer sum is over the 2^n permutations between the two end points of each $\mathcal{O}_i$.

Now, a defining property of the boundary propagators $G(\ell,\ell')$ is that their conformal Laplacian with respect to each end point ℓ,ℓ' is a boundary δ function:

$$\Box_\ell G(\ell,\ell') = \Box_{\ell'} G(\ell,\ell') = -\delta^3(\ell,\ell'). \tag{11}$$

This allows us to "stitch together" larger one-loop diagrams out of smaller ones. In particular, the general correlator Eq. (10) can be assembled out of cubic correlators, via the following diagrammatic rules. (1) Draw an arbitrary trivalent tree graph $\mathcal{G}$ with n external legs. (2) Assign one of the $\mathcal{O}_i$ to each external leg, and a "dummy" bilocal $\mathcal{O}(\ell,\ell')$ to each internal leg. (3) At each node, compute the cubic correlator of the operators on the three surrounding legs.

(4) Integrate each dummy bilocal's end points over the boundary, as

$$\frac{N}{4}\int d^3\ell d^3\ell'[G(\ell,\ell')(\cdots)]\Box_\ell\Box_{\ell'}[G(\ell,\ell')(\cdots)], \qquad (12)$$

where "$\cdots$" are placeholders for the correlator on each side of the internal leg. (5) Sum over inequivalent permutations of the external legs, multiply by $\mathcal{G}$'s symmetry factor, and divide by $2n$. The freedom to choose the graph $\mathcal{G}$ becomes nontrivial at $n=6$, where two inequivalent graphs first appear. We show the possible graphs for $n\leq 6$ in Fig. 1 in the Supplemental Material [39]. The external bilocals $\mathcal{O}_i$ can be replaced with local currents $j_i^{(s_i)}$, to produce the standard n-point functions. However, the internal-leg integrals Eq. (12) will always feature bilocals. One can think of these rules as constructing operator product expansion (OPE) diagrams, but with the OPE restricted at every step to single-trace operators, where the integral Eq. (12) acts as a projector onto the space of single-trace bilocals Eq. (8). The price for this projection is having to sum over permutations of the external legs, which is not necessary in a standard OPE diagram.

More precisely, the integral Eq. (12) is a projector onto the single-trace sector times a factor of $\frac{1}{2}$. That is, upon inserting quadratic single-trace correlators $\langle\mathcal{O}_1\mathcal{O}(\ell,\ell')\rangle$ and $\langle\mathcal{O}(\ell,\ell')\mathcal{O}_2\rangle$ into the place holders in Eq. (12), the integral evaluates to $\frac{1}{2}\langle\mathcal{O}_1\mathcal{O}_2\rangle$. One way to see that such a factor of $\frac{1}{2}$ is necessary is to compare with Ref. [27], where a straightforward sum of single-trace projections in different channels produces the quartic correlator with a factor of 2.

New bulk rules.—We now translate the above "single-trace OPE" diagrams into the bulk, to produce the bulk diagrams for n-point correlators $\langle j^{(s_1)}\ldots j^{(s_n)}\rangle$. A key step is to identify the bulk dual of the boundary bilocals $\mathcal{O}(\ell,\ell')$ on the diagram's internal legs. As we review in the Supplemental Material [39], this is given by a geodesic particle worldline $\gamma(\ell,\ell')$ stretching between the boundary points ℓ,ℓ' [37,38]. This particle produces HS fields $\phi^{(s)}(x,u;\ell,\ell')$ with all spins s, which solve Fronsdal's linear field equations with sources on $\gamma(\ell,\ell')$, and form the linearized version of the Didenko-Vasiliev (DV) "BPS black hole" [41,42]. In our bulk diagrams (see figures in the Supplemental Material), we depict a DV worldline $\gamma(\ell,\ell')$ as a solid line, and the field $\phi^{(s)}$ as a wavy line emanating from it. The boundary-bulk propagators $\Pi^{(s)}$ corresponding to external currents $j^{(s)}$ are depicted as external wavy lines.

The quadratic correlator of $\mathcal{O}(\ell,\ell')$ with another (local or bilocal) single-trace boundary operator can be computed as a worldline integral, describing a minimal coupling between the DV worldline of $\mathcal{O}(\ell,\ell')$ and the bulk field $h^{(s)}(x,u)$ of the second operator (a boundary-bulk propagator $\Pi^{(s)}$ with fixed spin, or a multiplet $\phi^{(s)}$ with all spins):

$$\frac{4}{\sqrt{N}}\sum_s(i\sqrt{2})^s\int_{\gamma(\ell,\ell')}d\tau[\dot{x}(\tau)\cdot\partial_u]^s h^{(s)}[x(\tau),u]. \qquad (13)$$

Here, τ is the proper time (i.e., length parameter) along $\gamma(\ell,\ell')$, while $x^\mu(\tau),\dot{x}^\mu(\tau)$ are the corresponding position and 4-velocity (i.e., unit tangent). In bulk diagrams, we depict the coupling Eq. (13) as the wavy line depicting $h^{(s)}$ attached to the solid line depicting $\gamma(\ell,\ell')$.

To complete the translation of the previous section diagrams into the bulk, we need bulk expressions for the cubic correlators $\langle jjj\rangle,\langle jj\mathcal{O}\rangle,\langle j\mathcal{O}\mathcal{O}\rangle,\langle\mathcal{O}\mathcal{O}\mathcal{O}\rangle$. For the $\langle jjj\rangle$ correlator, we use the known cubic vertex Eqs. (6) and (9), which we depict as usual as a meeting point of three wavy lines. For the cubic correlators involving bilocals, we need two additional kinds of bulk diagram elements [21]. These couple the DV worldline of a bilocal (summing over the possible choices) to the other two operators' bulk fields $h_1^{(s_1)},h_2^{(s_2)}$. The fields are coupled to the worldline either independently—by multiplying a pair of integrals Eq. (13), each with its worldline-field coupling, or together—via a single worldline integral, and a new worldline-field-field vertex $V_{\text{new}}^{(s_1,s_2)}[\partial_{x_1},\partial_{u_1};\partial_{x_2},\partial_{u_2};\dot{x}(\tau)]$:

$$\sum_{s_1,s_2}\int_{\gamma(\ell,\ell')}d\tau V_{\text{new}}^{(s_1,s_2)}h_1^{(s_1)}(x_1,u_1)h_2^{(s_2)}(x_2,u_2)|_{x_1=x_2=x(\tau)}. \qquad (14)$$

In the bulk diagrams, we depict this new vertex as two wavy lines meeting at a solid line. While the formula for $V_{\text{new}}^{(s_1,s_2)}$ is still unknown, it has been established [21] to be local beyond $\sim$1 AdS radius, as we review in the Supplemental Material [39]. Thus, the boundary bilocals and their cubic correlators can all be described in terms of local bulk objects. Plugging these into the previous section diagrams then yields a bulk-local description to all n-point functions.

As our final step, we notice a simplification when $h^{(s)}$ in the worldline-field coupling Eq. (13) is the field $\phi^{(s)}$ of another bilocal (this will occur in diagrams with $n\geq 5$). Equation (13) then computes the correlator Eq. (10) of two bilocals. When this is acted on in Eq. (12) by boundary Laplacians, Eq. (11) yields δ functions that set the two bilocals (and thus, their bulk worldlines) equal to each other, with a residual factor of $\frac{1}{2}$ (cf. the discussion at the end of the previous section). This trivialization of some of the bilocal integrals Eq. (12) can be incorporated into the diagrammatic rules, whose final form reads as follows. (1) Draw an arbitrary trivalent tree graph $\mathcal{G}$ with n external legs. Write a boundary-bulk propagator Eq. (6) for each external leg. (2) Draw a solid line across each internal leg of $\mathcal{G}$. This visually represents a DV worldline, while also highlighting the fact that it splits $\mathcal{G}$ into two "sides." Integrate over the worldline's end points as in Eq. (12), where the $(\cdots)$ place holders correspond to the diagram elements on either "side" of the worldline. (3) Resolve each cubic vertex of $\mathcal{G}$ into one of the cubic diagrams described above: a usual cubic vertex Eqs. (6) and (7), a pair of worldline-field couplings Eq. (13), or a wordline-field-field coupling Eq. (14). In all of these, a DV worldline is associated with the DV fields $\phi^{(s)}$, with the spin s summed over. (4) Sum over all inequivalent diagrams obtained through steps 2 and 3, sum over inequivalent permutations of the external legs in each diagram, multiply by the symmetry factor of $\mathcal{G}$, and divide by $2n$. (5) *After* evaluating the combinatorics as above, we may identify any two worldlines connected by the minimal coupling Eq. (13), replacing one of the associated integrals Eq. (12) by a factor of $\frac{1}{2}$. In this process, some inequivalent orderings of the external legs may become equivalent; i.e., the diagram's symmetry may increase. Note that the sides of the combined worldline remain unambiguous. In the Supplemental Material, we draw the resulting diagrams for $n \leq 5$, and write out their expressions [39].

Interpreting the integral over worldlines.—Our integral Eq. (12) over DV worldlines is admittedly unusual: instead of integrating over trajectories with fixed end points, we integrate over the end points while keeping the trajectory a geodesic. The following remarks might make this more palatable.

The restriction to geodesics may be considered a consequence of HS symmetry, through the requirement that the worldline's HS currents should all be conserved. Indeed, for spin 2, energy-momentum conservation requires the worldline to be geodesic at leading order in interactions; it is then plausible to expect that the full HS multiplet will enforce a geodesic at all orders.

As for the integration over end points, it is at least consistent with the standard variational principle for bulk gauge fields (HS or not), which is to hold fixed their magnetic boundary data. Our integration over DV worldlines does not violate this aspect of the variational principle, because the asymptotic HS fields induced by such worldlines are purely electric [36].

Relation to string theory.—In several respects, our diagrammatic rules are intermediate between the Feynman rules for fields and strings. First, the fact that any single "seed" graph $\mathcal{G}$ yields the entire correlator is analogous to how tree-level amplitudes in string theory are given by a single string diagram. Second, the factor of $2n$ is the symmetry factor of an (unoriented) open string diagram with disk topology. Thus, the multiplication by $\mathcal{G}$'s symmetry factor and division by $2n$ can be seen as "trading" the combinatorics of field diagrams for those of string diagrams. Third, the fact that each DV worldline in our diagrams has two sides is particularly natural if we imagine it as lying on a string world sheet. Fourth, our use of only cubic vertices can be seen as intermediate between Yang-Mills or general relativity (GR) (which have quartic or higher vertices) and string theory (which has no vertices at all).

Another analogy with string theory is the absence, in a sense, of off-shell fields. In our single-trace OPE diagrams, this manifests as the projection at every step onto the single-trace sector. In our bulk diagrams, the bulk propagators are always attached to a DV worldline, producing the DV solution $\phi^{(s)}$. Thus, instead of arbitrary off-shell fields, we are restricted to the "almost on-shell" space of BPS-like DV fields. In fact, as argued in Ref. [38], the DV particle and its fields can be viewed as a "completion" of the on-shell field space, in the same way that the string is a completion of its associated spectrum of fields. The holographic intuition behind this view is that the DV particle is dual to the boundary bilocal, whose Taylor expansion forms the tower of HS currents, just as the string is dual to a boundary Wilson loop (or line) [43,44], whose Taylor expansion similarly forms the tower of local single-trace operators in super-Yang-Mills theory. Another analogy between the DV particle and the string is that the latter can be discovered as one of the BPS solutions of 10D supergravity [45,46], just like the former is a (linearized) BPS solution of HS gravity [41].

Relation to HS algebra.—One can view our rules as a bulk-local realization of HS algebra [47]—the noncommutative product structure $Y_a \star Y_b = Y_a Y_b + i I_{ab}$ behind the infinite-dimensional symmetry group of HS gravity (here, Y_a is a twistor and I_{ab} is the metric on twistor space). The original cubic vertices found by Fradkin and Vasiliev [48,49] for certain values of spins (s_1, s_2, s_3) were constructed, much like Yang-Mills theory, from the antisymmetric product $\omega_{[\mu} \star \omega_{\nu]}$ acting on the connection master fields $\omega_\mu(x; Y)$. Vasiliev's fully nonlinear equations [1–3] added into the picture a master field $C(x; Y)$ of Weyl curvatures and their derivatives, along with an extra twistor coordinate Z_a. In the original approach to the equations, it was found [50] that the remaining cubic vertices (the ones not covered by Refs. [48,49]) are given by a structure similar to the *symmetric* product $C \star C$, and that this structure is not consistent with bulk locality. The eventual local formula (7) for all on-shell cubic vertices was found in Ref. [18] without any use of HS algebra.

On the other hand, it was noticed in Refs. [51,52] that the boundary n-point functions can all be expressed as traces $\mathrm{tr}_\star(f \star \cdots \star f)$ of symmetrized $\star$ products of a twistor function $f(Y)$, later identified in Ref. [36] as the *Penrose transform* [53,54] of the bulk field $C(x; Y)$. Moreover, the relevant products $f \star \cdots \star f$ are spanned precisely by the Penrose transforms of linearized DV solutions $\phi^{(s)}$. Now, any symmetrized trace $\mathrm{tr}_\star(f \star \cdots \star f)$ can be constructed from two fundamental operations: the symmetrized product $f \star g + g \star f$ and the pairing $\mathrm{tr}_\star(f \star g)$. This precisely corresponds to our construction above, where the n-point correlators are assembled from quadratic and cubic correlators, which in turn are composed of bulk-local elements.

In this view, HS algebra *does* describe local bulk interactions, but one must (a) apply it to the Penrose transform $f(Y)$ and (b) supplement the bulk HS fields with DV particles.

Outlook.—A number of questions are open. First, one should find an explicit expression for the new vertex Eq. (14). Among other benefits, this will allow a direct check of the arguments for its locality made in Ref. [21]. Second, it would be good to find a more natural geometric origin for our diagrammatic rules, such as the world sheet picture of string theory. Finally, our rules should be applied or extended to loop level, which will allow comparison with, e.g., the absence of $1/N$ corrections in the free vector model's correlators.

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