Breaking Down the Magnonic Wiedemann-Franz Law in the Hydrodynamic Regime

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Recent experiments have shown an indication of a hydrodynamic magnon behavior in ultrapure ferromagnetic insulators; however, its direct observation is still lacking. Here, we derive a set of coupled hydrodynamic equations and study the thermal and spin conductivities for such a magnon fluid. We reveal the drastic breakdown of the magnonic Wiedemann-Franz law as a hallmark of the hydrodynamics regime, which will become key evidence for the experimental realization of an emergent hydrodynamic magnon behavior. Therefore, our results pave the way toward the direct observation of magnon fluids.

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Introduction.—Quantum transport has attracted a profound growth of interest owing to its fundamental importance and many applications. Recent significant developments in experimental techniques have further boosted the study of quantum transport. Notably in ultraclean systems, strong interactions between particles drastically affect the transport properties, resulting in an emergent hydrodynamic behavior: examples include electrons [1–3], phonons [4,5], cold atoms [6–8], and quark-gluon plasmas [9,10]. In these systems, the conventional noninteracting description for particles breaks down where momentum-conserving scatterings become dominant, which in turn introduces a novel nonequilibrium state inherent in the so-called hydrodynamic regime. The most-studied example is the hydrodynamic charge transport in metals or semiconductors, which gives rise to an active research field called electron hydrodynamics [11–34]. This concept has revealed various unconventional transport phenomena such as the negative nonlocal resistance [12,13], the Poiseuille flow [19–21], the Hall viscosity [22–24], the geometric control of the flow [25,26], and the violation of the Wiedemann-Franz law [27–34].

Recent experiments on ultrapure ferromagnetic insulators (FMI) have opened up new pathways for magnon hydrodynamics [35–37]. Magnons in FMI attract special attention as a promising candidate for a spin information carrier [38–43] with good coherence and without dissipation of the Joule heating compared to conduction electrons in metals [44–50]. Therefore, hydrodynamic magnon transport implies exhibiting extraordinary features [51–57] as well as electron hydrodynamics and has a potential for innovative functionalities beyond the conventional noninteracting magnon picture. However, the direct observation of magnon fluids remains an open issue due to the lack of probes to access the time and length scales characteristic of this regime.

In this Letter, we derive a set of coupled hydrodynamic equations for a magnon fluid in topologically trivial bulk FMI by focusing on the most dominant timescales. Based on the obtained equations, we investigate the thermal and spin conductivities for magnon systems in the hydrodynamic regime. In the conventional transport regime, the ratio between the two conductivities has a material-independent universal value, which is known as the magnonic Wiedemann-Franz (WF) law [58–62]: a magnon analog of the celebrated WF law [63]. Here, as a hallmark of the hydrodynamic regime, we reveal that the ratio shows a large deviation from the law, implying that magnon-magnon interactions affect the two conductivities in radically different ways. Therefore, our results are expected to become key evidence for the emergence of a hydrodynamic magnon behavior and lead to the direct observation of magnon fluids.

Formulation.—We outline how to derive a set of coupled hydrodynamic equations for a magnon fluid (for details, see the Supplemental Material [64]). We start from the magnon Boltzmann equation [66–76] that governs the evolution of the magnon distribution function $n_{\boldsymbol{k}}(\boldsymbol{r}, t)$ in the phase space:

$$\frac{\partial n_{\boldsymbol{k}}}{\partial t} + \frac{\partial \omega_{\boldsymbol{k}}}{\partial \boldsymbol{k}} \cdot \frac{\partial n_{\boldsymbol{k}}}{\partial \boldsymbol{r}} = \mathcal{C}_N^{\mathrm{mm}}[n_{\boldsymbol{k}}] - \frac{n_{\boldsymbol{k}} - N_{\boldsymbol{k}}}{\tau_U}, \tag{1}$$

where $\omega_{\boldsymbol{k}}$ is the dispersion of magnons with momentum $\boldsymbol{k}$. For ultrapure FMI, we only consider magnon-number-conserving exchange interactions as magnon-magnon scattering processes, which can be divided into the contribution of normal (N) process and Umklapp (U) one in the usual manner, and neglect the dipolar interactions. We have also assumed the absence of external driving forces and relaxation time approximation for U process with τ_U. $\mathcal{C}_N^{\rm mm}$ is the collision integral for N process. Here, N process conserves the momentum, while U process does not. As N scattering rates are very large, an out-of-equilibrium distribution $n_{\boldsymbol{k}}$ will decay first into the drifting distribution $n_{\boldsymbol{k}}^{(0)}$ given by Eq. (3), and from this state it will relax toward static thermal equilibrium. Instead, U process tends to relax the magnon populations toward local thermal equilibrium $N_{\boldsymbol{k}}$, which is given by the Bose-Einstein distribution with a finite chemical potential μ due to the magnon number conservation.

From the collision integral $\mathcal{C}_N^{\rm mm}$ in Eq. (1), we can identify the collision invariants $\mathcal{N}_\lambda$, which are defined as $\int [d\boldsymbol{k}]\mathcal{C}_N^{\rm mm}\mathcal{N}_\lambda = 0$ with $\int [d\boldsymbol{k}] \equiv \int d^3\boldsymbol{k}/(2\pi)^3$. $\mathcal{C}_N^{\rm mm}$ guarantees that the quantities $\mathcal{N}_\lambda = (1, \hbar\boldsymbol{k}, \hbar\omega_{\boldsymbol{k}})$ do not change in the evolution of the distribution function. Following the standard approach [77–83], the conservation laws for number, momentum, and energy densities are obtained as follows:

$$\frac{\partial}{\partial t}\langle\mathcal{N}_0\rangle + \boldsymbol{\nabla}\cdot\boldsymbol{J}^{\mathcal{N}_0} = 0, \tag{2a}$$

$$\frac{\partial}{\partial t}\langle\mathcal{N}_i\rangle + \boldsymbol{\nabla}\cdot\boldsymbol{J}^{\mathcal{N}_i} = -\frac{\langle\mathcal{N}_i\rangle}{\tau_U}, \tag{2b}$$

$$\frac{\partial}{\partial t}\langle\mathcal{N}_4\rangle + \boldsymbol{\nabla}\cdot\boldsymbol{J}^{\mathcal{N}_4} = 0, \tag{2c}$$

where we have defined the fluxes $\boldsymbol{J}^{\mathcal{N}_\lambda}$ corresponding to each invariant density $\langle\mathcal{N}_\lambda\rangle \equiv \int[d\boldsymbol{k}]\mathcal{N}_\lambda n_{\boldsymbol{k}}$. $-\langle\mathcal{N}_i\rangle/\tau_U$ is the momentum relaxation force stems from U process.

In the hydrodynamic regime, the system reaches local equilibrium via N magnon-magnon scatterings, which conserve the total number, momentum, and energy of the magnon system. For this reason, we assume that the distribution functions in the zeroth order are described as

$$n_{\boldsymbol{k}}^{(0)} = \left[\exp\left\{\frac{\hbar\omega_{\boldsymbol{k}} - \hbar\boldsymbol{k}\cdot\boldsymbol{v} - \tilde{\mu}}{k_BT}\right\} - 1\right]^{-1}, \tag{3}$$

which is referred to as the local equilibrium distribution function. Notably, $(-\tilde{\mu}/T, -\boldsymbol{v}/T, 1/T)$ are the position- and time-dependent intensive thermodynamic parameters, which are conjugate to each collision invariant $\mathcal{N}_\lambda = (1, \hbar\boldsymbol{k}, \hbar\omega_{\boldsymbol{k}})$ of the magnon system. Here, the drift velocity $\boldsymbol{v}$ is a Lagrange multiplier enforcing the momentum conservation. $\tilde{\mu}$ is the chemical potential in the frame that moves with the fluid: $\tilde{\mu} = \mu - m^*\boldsymbol{v}^2/2$. We use the quadratic dispersion for magnons $\hbar\omega_{\boldsymbol{k}} = \hbar^2\boldsymbol{k}^2/2m^*$ with the effective mass m^* for simplicity and computability [56]. Under these conditions, $n_{\boldsymbol{k}}^{(0)}$ is transformed into $N_{\boldsymbol{k}}$ under the Galilean transformation from the frame in which the fluid moves with the velocity $\boldsymbol{v}$ to the frame in which it is at rest: $\hbar\boldsymbol{k} \to \hbar\boldsymbol{k} + m^*\boldsymbol{v}$. Note that the nonequilibrium magnon chemical potential μ [36,48–50,73] is present here due to the magnon number conservation under both N and U processes. The magnon chemical potential is an essential ingredient for heat and spin transport.

Deviation from the local equilibrium distribution $n'_{\boldsymbol{k}} = n_{\boldsymbol{k}} - n_{\boldsymbol{k}}^{(0)}$ creates the dissipative number, momentum, and energy fluxes: $\boldsymbol{J} = \boldsymbol{J}^{(0)} + \boldsymbol{J}'$, $\Pi_{ji} = \Pi_{ji}^{(0)} + \Pi'_{ji}$, and $\boldsymbol{Q} = \boldsymbol{Q}^{(0)} + \boldsymbol{Q}'$ with $\boldsymbol{J}^{(0)} = 0$, $\Pi_{ji}^{(0)} = P\delta_{ji}$, and $\boldsymbol{Q}^{(0)} = 0$. By using these quantities, the hydrodynamic equations are obtained as follows:

$$\frac{\partial n}{\partial t} + \boldsymbol{\nabla}\cdot(n\boldsymbol{v} + \boldsymbol{J}') = 0, \tag{4a}$$

$$\rho\left(\frac{\partial}{\partial t} + \boldsymbol{v}\cdot\boldsymbol{\nabla}\right)v_i + \frac{\partial}{\partial x_i}P + \frac{\partial}{\partial x_j}\Pi'_{ji} = -\frac{\rho v_i}{\tau_U}, \tag{4b}$$

$$\frac{\partial u}{\partial t} + \boldsymbol{\nabla}\cdot(u\boldsymbol{v} + \boldsymbol{Q}') + P\boldsymbol{\nabla}\cdot\boldsymbol{v} + \Pi'_{ji}\frac{\partial v_i}{\partial x_j} = \frac{\rho\boldsymbol{v}^2}{\tau_U}, \tag{4c}$$

where n, $\rho\boldsymbol{v} = m^*n\boldsymbol{v}$, and u are the magnon number, momentum, and internal energy densities. The force that drives the magnon flow is the gradient of the magnon pressure P. Heat currents $\boldsymbol{Q} = \boldsymbol{Q}'$ can be obtained only when considering the deviation from the local equilibrium. Here, U process produces not only the momentum relaxation but also the internal energy relaxation to the lattice environment. Although similar hydrodynamic equations are derived in Ref. [56] from the same starting point, Eq. (3), the inequivalence of the momentum density and the number flux stems from fluctuations around the local equilibrium here, which is different from the correction to the particle current operator due to the magnon-magnon interactions [56]. Furthermore, our theory includes the momentum relaxation processes that have a significant importance for the main results.

Entropy production.—The entropy production is one of intriguing aspects of the hydrodynamic approach [84]. After straightforward calculations, the entropy production rate is obtained as the balance equation for the entropy density $s = u + P - \mu n$:

$$\frac{\partial s}{\partial t} + \boldsymbol{\nabla}\cdot\left(s\boldsymbol{v} + \frac{\boldsymbol{Q}' - \mu\boldsymbol{J}'}{T}\right)$$
$$= \boldsymbol{Q}'\cdot\boldsymbol{\nabla}\left(\frac{1}{T}\right) + \frac{\Pi'_{ji}}{T}\partial_j(-v_i) + \boldsymbol{J}'\cdot\boldsymbol{\nabla}\left(-\frac{\mu}{T}\right) + \frac{1}{T}\frac{\rho\boldsymbol{v}^2}{\tau_U}. \tag{5}$$

The entropy production rate is given by the product of thermodynamic driving forces and their conjugate dissipative currents. We should note that the adiabatic evolution condition, $\boldsymbol{Q}' = \boldsymbol{J}' = \Pi'_{ji} = 0$, reproduces the dissipationless equations in Eq. (4). Furthermore, these pairs obey the Onsager reciprocity relations [85,86]. These relations describe the effects on an irreversible flux of an extensive conserved quantity by intensive thermodynamic variables. For example, gradients in the magnon chemical potential can be treated as conjugate forces for spin transport in the Onsager relations. By analogy with the thermoelectric Onsager relations, thermo-spin Onsager relations connect magnonic spin and heat currents.

Thermal and spin conductivities.—Based on the obtained hydrodynamic equations, Eq. (4), and the entropy production rate, Eq. (5), we investigate the spin and heat currents in the hydrodynamic regime. In the following analysis, we consider the linear response under constant gradients of the temperature and chemical potential. In order to calculate the dissipative heat and magnon currents, we also perform the relaxation time approximation for N process,

$$\mathcal{C}_N^{\mathrm{mm}}[n_{\boldsymbol{k}}] = -\frac{n_{\boldsymbol{k}} - n_{\boldsymbol{k}}^{(0)}}{\tau_N}. \tag{6}$$

For the purpose of obtaining the linear thermal conductivity, we assume the deviation from the local equilibrium distribution $\delta n_{\boldsymbol{k}} = n_{\boldsymbol{k}} - n_{\boldsymbol{k}}^{(0)}$ is small to justify the use of the linearized Boltzmann equation. Then, $\delta n_{\boldsymbol{k}}$ is obtained as

$$\delta n_{\boldsymbol{k}} \simeq -\left(\frac{1}{\tau_N} + \frac{1}{\tau_U}\right)^{-1} \times \left[\left(\frac{\partial}{\partial t} + \frac{\partial \omega_{\boldsymbol{k}}}{\partial \boldsymbol{k}} \cdot \boldsymbol{\nabla}\right) n_{\boldsymbol{k}}^{(0)} + \frac{n_{\boldsymbol{k}}^{(0)} - N_{\boldsymbol{k}}}{\tau_U}\right]. \tag{7}$$

We further assume that $\delta n_{\boldsymbol{k}}$ satisfies the conditions: $\int [d\boldsymbol{k}] \mathcal{N}_\lambda \delta n_{\boldsymbol{k}} = 0$. Under these conditions, the densities of conserved quantities $\langle \mathcal{N}_\lambda(\boldsymbol{r}, t)\rangle = \langle \mathcal{N}_\lambda(\boldsymbol{r}, t)\rangle^{(0)}$ are determined by the local equilibrium distribution $n_{\boldsymbol{k}}^{(0)}$. This assumption is necessary for not violating the conservation laws under the relaxation time approximation [87].

In the following analysis, we ignore $\partial/\partial t$ because we are interested in steady state transport properties. Substituting $\delta n_{\boldsymbol{k}}$ into hydrodynamic variables, we obtain the magnon and heat currents in the linear response regime, respectively:

$$\boldsymbol{J} = -\frac{\tau_N}{\tau_N + \tau_U} n_m^{(0)} \boldsymbol{v} + \left(\frac{1}{\tau_N} + \frac{1}{\tau_U}\right)^{-1} \frac{T}{m^*} \times \left[n_m^{(0)} \boldsymbol{\nabla}\left(-\frac{\mu}{T}\right) + (u^{(0)} + P)\boldsymbol{\nabla}\left(\frac{1}{T}\right)\right], \tag{8a}$$

$$\boldsymbol{Q} = -\frac{\tau_N}{\tau_N + \tau_U} (u^{(0)} + P)\boldsymbol{v} + \left(\frac{1}{\tau_N} + \frac{1}{\tau_U}\right)^{-1} \frac{T}{m^*} \times \left[(u^{(0)} + P)\boldsymbol{\nabla}\left(-\frac{\mu}{T}\right) + \frac{7}{3}\langle \mathcal{N}_4^2\rangle_{\mathrm{eq}} \boldsymbol{\nabla}\left(\frac{1}{T}\right)\right], \tag{8b}$$

where $\langle \mathcal{N}_4^2\rangle_{\mathrm{eq}} \equiv \int [d\boldsymbol{k}] (\hbar\omega_{\boldsymbol{k}})^2 N_{\boldsymbol{k}}$. The transport coefficients $\mathcal{L}_{ij}$ in the linear response regime are defined as

$$\begin{pmatrix} \boldsymbol{J} \\ \boldsymbol{Q} \end{pmatrix} = \begin{pmatrix} \mathcal{L}_{11} & \mathcal{L}_{12} \\ \mathcal{L}_{21} & \mathcal{L}_{22} \end{pmatrix} \begin{pmatrix} \boldsymbol{\nabla}(-\mu/T) \\ \boldsymbol{\nabla}(1/T) \end{pmatrix}.$$

In the hydrodynamic regime, the transport coefficients are described by the drift velocity $\boldsymbol{v}$. In our hydrodynamic equations, $\boldsymbol{\nabla} P = -n_m^{(0)} T[\boldsymbol{\nabla}(-\mu/T) + \alpha \boldsymbol{\nabla}(1/T)]$ drives magnon flows with $\alpha = (u^{(0)} + P)/n_m^{(0)}$. By solving Eq. (4b) as $\rho_m^{(0)} \boldsymbol{v} = \tau_U \mathcal{V} n_m^{(0)} T[\boldsymbol{\nabla}(-\mu/T) + \alpha \boldsymbol{\nabla}(1/T)]$, the components of $\mathcal{L}_{ij}$ are obtained as,

$$\mathcal{L}_{11} = \left(\frac{1}{\tau_N} + \frac{1}{\tau_U}\right)^{-1} n_m^{(0)} \frac{T}{m^*}(1 - \mathcal{V}), \tag{9a}$$

$$\mathcal{L}_{12} = \left(\frac{1}{\tau_N} + \frac{1}{\tau_U}\right)^{-1} (u^{(0)} + P)\frac{T}{m^*}(1 - \mathcal{V}) \tag{9b}$$

$$= \mathcal{L}_{21} = \alpha \mathcal{L}_{11}, \tag{9c}$$

$$\mathcal{L}_{22} = \left(\frac{1}{\tau_N} + \frac{1}{\tau_U}\right)^{-1} (u^{(0)} + P)\frac{T}{m^*} \times \left(\frac{7}{2} k_B T \frac{\mathrm{Li}_{7/2}(z)}{\mathrm{Li}_{5/2}(z)} - \mathcal{V}\alpha\right), \tag{9d}$$

where we have introduced the polylogarithm function $\mathrm{Li}_s(z) \equiv \sum_{n=1}^{\infty} z^n/n^s$, the fugacity $z \equiv e^{\mu/k_B T}$, and the renormalized velocity $\mathcal{V}$. $\mathcal{V}$ deviates from unity when viscous effects are switched on. As can be seen from Eq. (9), the transport coefficients $\mathcal{L}_{ij}$ obey the Onsager reciprocity relations.

In the following, we consider magnon fluids in bulk FMI in order to justify neglecting nonlocal effects. Substituting $\mathcal{V} \approx 1$ in Eq. (9), we obtain the spin and thermal conductivities as

$$\sigma_m = \tau_U \frac{1}{m^*} \frac{\hbar}{\Lambda_T^3} \mathrm{Li}_{3/2}(z), \tag{10a}$$

$$\kappa_m = \left(\frac{1}{\tau_N} + \frac{1}{\tau_U}\right)^{-1} \frac{1}{m^*} \frac{k_B^2 T}{\Lambda_T^3} \left[\frac{35}{4}\mathrm{Li}_{7/2}(z) - \frac{25}{4}\frac{\mathrm{Li}_{5/2}^2(z)}{\mathrm{Li}_{3/2}(z)}\right], \tag{10b}$$

where $\Lambda_T \equiv \sqrt{2\pi\hbar/m^* k_B T}$ is the thermal de Broglie wavelength. Here, we have defined the spin and thermal conductivities as $\hbar(n_m^{(0)} \boldsymbol{v} + \boldsymbol{J}) = \sigma_m \boldsymbol{\nabla}(-\mu)$ and $\boldsymbol{Q} = \kappa_m \boldsymbol{\nabla}(-T)$. While the spin conductivity is not affected by momentum-conserving N process because the spin current here is the momentum flow of magnons, the thermal conductivity is drastically changed by both processes as a hallmark of the hydrodynamic regime.

Breakdown of the magnonic WF law.—We are now ready to discuss the breakdown of the magnonic WF law in the hydrodynamic regime. The magnonic WF law claims that the thermal and spin conductivities satisfy the relation [58–62]

$$\left.\frac{\kappa_m}{\sigma_m T}\right|_{\text{non-int}} = \frac{k_B^2}{\hbar}\left[\frac{35}{4}\frac{\zeta(7/2)}{\zeta(3/2)} - \frac{25}{4}\frac{\zeta^2(5/2)}{\zeta^2(3/2)}\right], \tag{11}$$

where this ratio shows the material-independent universal value so-called the magnonic Lorenz number: a magnon analog of the well-known WF law [63]. Here, we have used the fact that the polylogarithm function $\mathrm{Li}_s(z)$ can be approximated by the Riemann zeta function $\zeta(s) = \sum_{n=1}^{\infty} 1/n^s$ for $z \approx 1$. In the hydrodynamic regime, the ratio becomes

$$\left.\frac{\kappa_m}{\sigma_m T}\right|_{\text{hydro}} = \left(\frac{\tau_N}{\tau_N + \tau_U}\right)\left.\frac{\kappa_m}{\sigma_m T}\right|_{\text{non-int}}, \tag{12}$$

which indicates that the standard form of the magnonic WF law is reduced by a factor $\tau_N/(\tau_N + \tau_U)$ owing to the difference in the relaxation processes between spin and heat currents in Eq. (10). Equation (12) is the main result of this Letter. Especially in the hydrodynamic regime, $\tau_N \ll \tau_U$, large deviations from the law are implied by this factor.

Discussion.—Finally, we will discuss the experimental feasibility of the breakdown of the magnonic WF law. According to Refs. [88–91], relaxation times are estimated as $\tau_N^{-1} \sim (T/T_e)^3 k_B T/\hbar$ and $\tau_U^{-1} \sim \sqrt{T/T_e}\exp(-12T_e/T)k_B T/\hbar$ with $T_e = 2SJ/k_B \sim 37$ K for yttrium iron garnet. The difference in temperature dependences of these two relaxation times originates from possible regions in reciprocal space under the scattering processes limited by overall conservation of energy and momentum modulo reciprocal lattice vectors [53,54,92,93]. Figure 1 illustrates the temperature dependences of scattering rates $1/\tau$. The red box dictates the area satisfying $\tau_N \ll \tau_U$, namely, the hydrodynamic regime. The ratio $\kappa_m/\sigma_m T|_{\text{hydro}}$ in units of the magnonic Lorenz number $\kappa_m/\sigma_m T|_{\text{non-int}}$ is depicted in the inset of Fig. 1. When the magnon system enters the hydrodynamic regime, the ratio shows a large deviation from unity, which in turn results in the drastic breakdown of the magnonic WF law. We should discuss the effect of the dipolar interactions in realistic magnets that violates the magnon number conservation. The magnon dumping rate due to the dipolar interactions is in the order of 10^6 s^{-1} [56] and is much slower than τ_N^{-1} and τ_U^{-1} (see Fig. 1). Therefore, we neglect the dipolar interactions here and the hydrodynamic description including the number conservation law is valid under the dynamics we are interested in.

As the system enters the collisionless regime where the hydrodynamic description is invalid, an emergent sound mode referred to as the zero-magnon mode appears in low dimensional Heisenberg ferromagnets [94]. Although it needs microscopic calculations and is beyond the scope of our study, the impact of the zero-magnon mode on the magnonic Lorenz number may be an interesting future work.

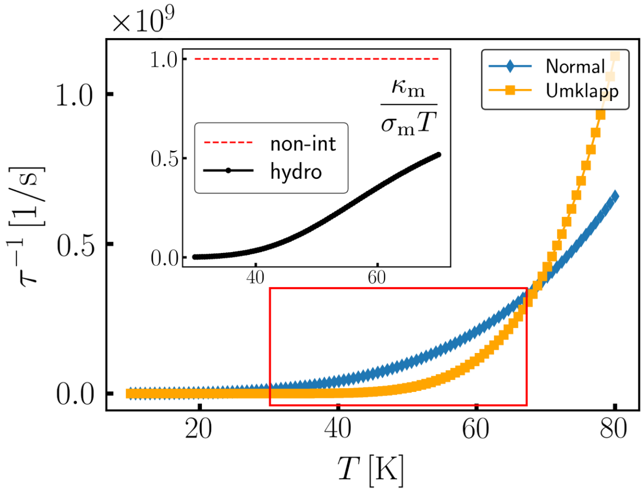

FIG. 1. Breaking down the magnonic Wiedemann-Franz law. Temperature dependence of relaxation rates τ^{-1} for both Normal and Umklapp magnon-magnon scatterings are depicted. The red box shows the area satisfying $\tau_N \ll \tau_U$, namely, the hydrodynamic regime. The inset shows the ratio $\kappa_m/\sigma_m T|_{\text{hydro}}$, calculated from Eq. (12) in units of the magnonic Lorenz number $\kappa_m/\sigma_m T|_{\text{non-int}}$. When the system enters the hydrodynamic regime, the ratio shows a large deviation from unity.

Conclusion.—In summary, we have developed a basic framework of magnon hydrodynamics for topologically trivial bulk FMI, which is composed of the hydrodynamic equations, Eq. (4), and the entropy production rate, Eq. (5). Based on these equations, we have first investigated the dissipative magnon and heat currents, which obey the Onsager reciprocity relations, and then the thermal and spin conductivities. As a hallmark of the hydrodynamic regime, we have revealed that the ratio between the two conductivities shows a large deviation from the standard form, the so-called magnonic WF law. Here, we have identified an origin of the drastic breakdown as the difference in relaxation processes between spin and heat currents, which is unique to the hydrodynamic regime. The violation of the magnonic WF law does not depend on the details of the magnon dispersion: a universal feature of the hydrodynamic regime. Therefore, our results may yield key evidence for the experimental realization and detection of magnon fluids in a wide range of materials.

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