

Higher-Dimensional Origin of Extended Black Hole Thermodynamics

Antonia M. Frassino¹, Juan F. Pedraza², Andrew Svesko³, and Manus R. Visser⁴

¹*Departament de Física Quàntica i Astrofísica and Institut de Ciències del Cosmos, Universitat de Barcelona, 08028 Barcelona, Spain*

²*Instituto de Física Teórica UAM/CSIC Calle Nicolás Cabrera 13-15, Cantoblanco, 28049 Madrid, Spain*

³*Department of Physics and Astronomy, University College London, London WC1E 6BT, United Kingdom*

⁴*Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Cambridge CB3 0WA, United Kingdom*

 (Received 16 January 2023; accepted 27 March 2023; published 19 April 2023)

Holographic braneworlds are used to present a higher-dimensional origin of extended black hole thermodynamics. In this framework, classical, asymptotically anti-de Sitter black holes map to quantum black holes in one dimension less, with a conformal matter sector that backreacts on the brane geometry. Varying the brane tension alone leads to a dynamical cosmological constant on the brane, and, correspondingly, a variable pressure attributed to the brane black hole. Thus, standard thermodynamics in the bulk, including a work term coming from the brane, induces extended thermodynamics on the brane, exactly, to all orders in the backreaction. A microscopic interpretation of the extended thermodynamics of specific quantum black holes is given via double holography.

DOI: [10.1103/PhysRevLett.130.161501](https://doi.org/10.1103/PhysRevLett.130.161501)

Introduction.—Black holes remain an important laboratory for testing ideas in quantum gravity. This is owed to the discovery they behave as thermal systems with an entropy proportional to their horizon area [1,2]. Though, black holes are peculiar as their thermodynamic first law lacks a pressure-volume work term. This is because there is no clear notion of pressure or volume for a general black hole. For anti-de Sitter (AdS) black holes the viewpoint dramatically changes. There, the cosmological constant Λ can be identified as pressure, and its variation appears in a generalized first law with a conjugate quantity dubbed the thermodynamic volume [3–5]. Thus, AdS black holes give rise to *extended* black hole thermodynamics, offering a rich gravitational perspective on everyday phenomena such as Van der Waals fluids [6,7], polymers [8], and heat engines [9].

Despite numerous explorations into extended black hole thermodynamics, there is criticism for identifying the cosmological constant as a varying thermodynamic pressure. A historical inspiration follows from treating Λ as a dynamical parameter, leading to additional black hole “hair” [10]. Further, for consistency, the Smarr relation for AdS black holes requires a pressure-volume term, with pressure proportional to Λ [3,11], however, this does not imply that the pressure should be varied. Outside of these considerations, introducing variations of Λ is not *a priori* clear from a purely gravitational viewpoint.

In this Letter we provide a new motivation for a varying cosmological constant using holographic braneworlds, where a dynamical Λ on a brane naturally arises from tuning the brane tension. In braneworld holography [12] one couples a d -dimensional brane to a gravitational theory in an asymptotically $(d + 1)$ -dimensional AdS background, which has a dual interpretation as a conformal field theory (CFT) living on the asymptotic AdS boundary. Upon integrating out the ultraviolet (UV) degrees of freedom of the CFT, a specific theory of gravity is induced on the brane with an effective cosmological constant controlled by the brane tension. From the higher-dimensional perspective, this tension is a physical tunable parameter, controlling both the coupling to the bulk gravity theory and the position of the brane, leading to variations of the cosmological constant on the brane.

Specific braneworld models are also useful to investigate the microscopic interpretation of extended thermodynamics. To this end, one may consider AdS braneworlds with a second holographic description in terms of a $(d - 1)$ -dimensional CFT. Via AdS/CFT, variations in Λ correspond to variations in the number of degrees of freedom of the dual theory, or the CFT central charge c [3,9,13–16]. While AdS/CFT duality is often discussed in the classical $c \rightarrow \infty$ limit, it is expected to hold even when all $1/c$ quantum corrections are included. In fact, going beyond infinite c could be worthwhile since in the classical limit all extensive quantities scale with the number of degrees of freedom [16,17] and, when properly organized, extended black hole thermodynamics appears to be a “trivial” modification of standard thermodynamics. It is only when $1/c$ corrections are included that true new physics could arise, and we can access them via braneworld holography.

Published by the American Physical Society under the terms of the Creative Commons Attribution 4.0 International license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP³.

In this setting, solving classical equations of a braneworld in $(d+1)$ dimensions amounts to solving semiclassical gravitational equations in d dimensions, giving rise to so-called “quantum” black holes on the brane [18]. Therefore, braneworld holography provides a natural framework to generalize extended thermodynamics of holographic CFTs to all orders in $1/c$.

Inducing a variation of the cosmological constant.— Consider an asymptotically AdS_{d+1} spacetime \mathcal{M} of curvature scale L_{d+1} , with a dual description in terms of a CFT_d on the asymptotic boundary $\partial\mathcal{M}$. The quantum fluctuations of the CFT lead to UV divergences which may be removed by adding appropriate counterterms [19,20], leading to a holographically renormalized theory. In braneworld holography [12] (see also [21,22]), one replaces the regulator surface inside the bulk with a d -dimensional brane \mathcal{B} , as in the Randall-Sundrum braneworld construction [23], which acts as a physical cutoff and renders the UV divergences finite. Further, the metric on the brane is dynamical, and is governed by a holographically induced higher curvature theory of gravity coupled to a CFT with a UV cutoff.

Precisely, let the bulk be characterized by the action

$$I_{\text{bulk}}[\mathcal{M}] + I_{\text{GHY}}[\partial\mathcal{M}] + I_{\text{brane}}[\mathcal{B}], \quad (1)$$

where

$$I_{\text{bulk}} = \frac{1}{16\pi G_{d+1}} \int_{\mathcal{M}} d^{d+1}x \sqrt{-g} (\hat{R} - 2\Lambda_{d+1}), \quad (2)$$

$$I_{\text{GHY}} = \frac{1}{8\pi G_{d+1}} \int_{\partial\mathcal{M}} d^d x \sqrt{-h} K, \quad (3)$$

$$I_{\text{brane}} = -\tau \int_{\mathcal{B}} d^d x \sqrt{-h}. \quad (4)$$

Here \hat{R} is the bulk Ricci scalar, $\Lambda_{d+1} = -d(d-1)/2L_{d+1}^2$ is the cosmological constant, in the Gibbons-Hawking-York (GHY) term K is the trace of the extrinsic curvature of $\partial\mathcal{M}$, and τ is the tension of the brane.

Integrating out the bulk between $\partial\mathcal{M}$ up to \mathcal{B} (typically taken to be near the boundary) amounts to removing CFT degrees of freedom above the UV cutoff, leading to the brane effective action I (see Fig. 1 for a depiction)

$$I = I_{\text{Bgrav}}[\mathcal{B}] + I_{\text{CFT}}[\mathcal{B}], \quad (5)$$

with the induced gravity theory on the brane

$$I_{\text{Bgrav}} = \frac{1}{16\pi G_d} \int_{\mathcal{B}} d^d x \sqrt{-h} \left[R - 2\Lambda_d + \frac{L_{d+1}^2}{(d-4)(d-2)} (R^2 \text{ terms}) + \dots \right], \quad (6)$$

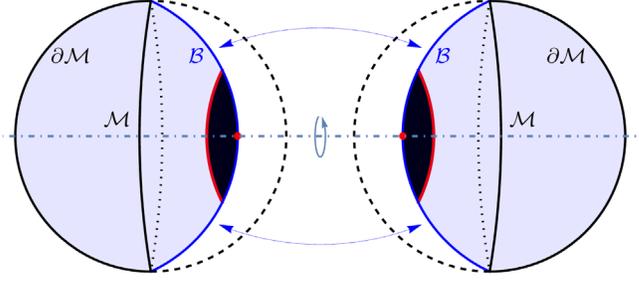


FIG. 1. A \mathbb{Z}_2 -symmetric, double-sided braneworld construction. The bulk region in white is excised down to the brane \mathcal{B} (blue line), and glued to another copy. A bulk black hole with event horizon (red line) is attached to the brane, and induces a horizon on the brane.

and two effective parameters endowed from the higher-dimensional parent theory

$$G_d = \frac{d-2}{2L_{d+1}} G_{d+1}, \quad (7)$$

$$\frac{1}{L_d^2} = \frac{2}{L_{d+1}^2} \left(1 - \frac{4\pi G_{d+1} L_{d+1}}{d-1} \tau \right). \quad (8)$$

Here L_d is the AdS radius on the brane and the $R^2 + \dots$ terms refer to higher curvature contributions that may be solved for perturbatively, in principle to any order. The action I_{CFT} arises from integrating out normalizable modes, which accounts for the dual CFT state.

We can understand Eq. (1) as a theory of a finite $(d+1)$ -dimensional system with dynamics ruled by general relativity and a brane. Alternatively, from the brane perspective, one has a specific higher-curvature theory in d spacetime dimensions coupled to a cutoff CFT which backreacts on the brane metric. This means classical solutions to the bulk Einstein equations correspond to solutions to the semiclassical equations of motion on the brane. Specifically, classical black holes map to quantum black holes, accounting for all orders of backreaction [18].

From the bulk perspective, it is very natural to tune the tension of the brane τ since it is a physical parameter of the system. The tension determines the position of the brane via Israel junction conditions, thence varying the tension changes the position. Accordingly, by virtue of Eq. (8), varying the tension alone (keeping the other bulk parameters L_{d+1} and G_{d+1} fixed) corresponds to varying the cosmological constant on the brane Λ_d :

$$\delta\tau = \frac{\delta\Lambda_d}{8\pi G_d}. \quad (9)$$

Notably, this argument holds for general geometric setups—we have not selected particular solutions in the bulk or brane. However, in anticipation of what is to come, it is this observation which leads to our central claim: classical black

hole thermodynamics in the bulk, including work done by the brane, induces extended thermodynamics of quantum black holes on the brane.

Quantum black holes on the brane.—For an explicit realization of the map between classical and quantum black holes, consider the setup of [24–26], with an AdS₃ Karch-Randall brane [27] in a bulk AdS₄ C -metric

$$ds^2 = \frac{\ell^2}{(\ell + xr)^2} \left[-H(r)dt^2 + \frac{dr^2}{H(r)} + r^2 \left(\frac{dx^2}{G(x)} + G(x)d\phi^2 \right) \right], \quad (10)$$

with metric functions

$$H(r) = \kappa + \frac{r^2}{\ell_3^2} - \frac{\mu\ell}{r}, \quad G(x) = 1 - \kappa x^2 - \mu x^3. \quad (11)$$

Here $\kappa = \pm 1, 0$ corresponds to the types of slicings on the brane, where $\kappa = -1$ recovers a classical Banados-Teitelboim-Zanelli (BTZ) black hole [28,29] on the brane, but we will leave κ arbitrary corresponding to a family of brane black holes. The positive parameter ℓ is the inverse acceleration associated with the accelerating black hole interpretation of the C -metric and is related to the AdS₄ length scale via $L_4^{-1} = [(1/\ell^2) + (1/\ell_3^2)]^{1/2}$, and τ by

$$\tau = \frac{1}{2\pi G_4 \ell}. \quad (12)$$

Further, ℓ_3 is a length scale related to the curvature scale L_3 of the AdS₃ brane via

$$\frac{1}{L_3^2} = \frac{2}{L_4^2} \left(1 - \frac{L_4}{\ell} \right) = \frac{1}{\ell_3^2} \left[1 + \frac{\ell^2}{4\ell_3^2} + \mathcal{O}\left(\frac{\ell^4}{\ell_3^4}\right) \right]. \quad (13)$$

Note it is L_3 which appears in the brane action but the solutions are characterized by the AdS radius ℓ_3 , as standard in higher curvature gravities. Lastly, μ is a positive parameter related to the mass of the black hole.

Holographically, excising the bulk region between the AdS₄ boundary and the brane (Fig. 1) is dual to integrating out UV degrees of freedom of the CFT to a cutoff energy $1/\ell$. This procedure induces a quantum BTZ (qBTZ) black hole on the brane at $x = 0$ [26],

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\phi^2, \quad f(r) = \frac{r^2}{\ell_3^2} - 8\mathcal{G}_3 M - \frac{\ell F(M)}{r}, \quad (14)$$

where the largest root of $f(r)$ yields the qBTZ horizon $r = r_+$. Here M is the black hole mass, $\mathcal{G}_3 = G_3/\sqrt{1 + (\ell/\ell_3)^2}$ is a “renormalized” Newton’s constant, and $F(M)$ a form function which can be explicitly determined by solving the semiclassical gravitational equations on the brane. In fact, the renormalized stress tensor of the coupled holographic CFT has the same radial

dependence and tensorial structure as the stress tensor of a free massless scalar field conformally coupled to three-dimensional Einstein gravity in the classical BTZ or conical AdS₃ backgrounds [30–32], differing only in $F(M)$. Notably, the qBTZ black hole is guaranteed to be a solution to the semiclassical theory to all orders in backreaction.

Thus, ℓ controls the strength of backreaction from the brane perspective. As such, one typically treats $\ell \ll \ell_3$, such that induced quantum corrections are perturbatively small compared to the leading order Einstein-Hilbert contribution in the brane effective action. In the bulk this is equivalent to taking the limit where the brane is close to the AdS₄ boundary. We emphasize, however, our construction works equally well when ℓ is nonzero and finite, where, however, the gravity theory on the brane has a massive graviton [27].

The parameter ℓ also features into the coupling of the gravity theory to the cutoff CFT on the brane. To see this, note the dictionary for the central charge c_3 of the holographic CFT₃ is [26]

$$c_3 = \frac{L_4^2}{G_4} = \frac{\ell}{2G_3\sqrt{1 + \nu^2}}, \quad (15)$$

where we introduced the positive dimensionless parameter $\nu \equiv \ell/\ell_3$. Even without knowing the explicit form of I_{CFT} , its typical value is c_3 , while the typical value of the gravity action is L_3/G_3 , leading to the dimensionless effective coupling $g_{\text{eff}} = G_3 c_3/L_3$ [33]. The central charge is related to the tension via $\tau = 1/(8\pi\ell G_3^2 c_3)$. Generically, variations in τ induce changes in G_3 , c_3 , and ℓ , though below we restrict ourselves to a fixed c_3 and G_3 ensemble, since that corresponds to keeping L_4 and G_4 fixed.

Inducing extended thermodynamics.—Despite the fact that the C -metric may be interpreted as an accelerating black hole, it nonetheless has a sensible thermodynamic description. Indeed, the C -metric has a time-translation Killing symmetry, which can be used to derive a thermodynamic first law for the black hole horizon. There is no acceleration horizon in our setup, since we are working in the “small acceleration” regime $\ell > L_4$ [34].

With the brane, the bulk black hole thermodynamics is identified with the thermodynamics of the quantum black hole. To see this, first consider the case when the tension is fixed. It is convenient to introduce the dimensionless parameter $z = \ell_3/(r_+ x_1) \in [0, \infty)$, where x_1 is the only positive root of $G(x)$ remaining after excising the bulk region at $x = 0$. Then, the mass M , temperature T , and entropy S of the classical bulk black hole are [25,26]

$$M = \frac{\sqrt{1 + \nu^2} z^2 (1 - \nu z^3) (1 + \nu z)}{2G_3 (1 + 3z^2 + 2\nu z^3)^2}, \quad (16)$$

$$T = \frac{1}{2\pi\ell_3} \frac{z(2 + 3\nu z + \nu z^3)}{1 + 3z^2 + 2\nu z^3}, \quad (17)$$

$$S = \frac{\pi \ell_3 \sqrt{1 + \nu^2}}{G_3} \frac{z}{1 + 3z^2 + 2\nu z^3}. \quad (18)$$

Each of these expressions follows from the identification of the free energy with the Euclidean action of the bulk solution [35], and, for fixed ν , they obey the first law, $dM = TdS$. From the brane perspective, the classical entropy is identified with the generalized entropy $S \equiv S_{\text{gen}}$ [26,36], and has the same temperature T . Thus, the qBTZ first law, valid to all orders in backreaction, is

$$dM = TdS_{\text{gen}}. \quad (19)$$

A similar relation holds for black holes in semiclassical two-dimensional dilaton gravity [37,38].

Let us now consider the case when the brane tension is treated as a thermodynamic variable, similar to the surface tension of liquids. The effect is that the brane will do work on the black hole system, such that the bulk first law becomes

$$dM = TdS + A_\tau d\tau, \quad (20)$$

where $A_\tau \equiv (\partial M / \partial \tau)_S$ is the variable conjugate to τ with scaling dimension of bulk area. We note that the black hole mass plays the role of enthalpy $H = E + \tau A_\tau$, since enthalpy in standard thermodynamics satisfies a first law of the form Eq. (20), whereas the internal energy E satisfies the usual first law $dE = TdS - \tau dA_\tau$. Further, the thermodynamic quantities obey a Smarr relation, which we derive in the Supplemental Material [39] (generalizing [40])

$$M = 2TS - 2P_4 V_4 - \tau A_\tau. \quad (21)$$

This includes a τA_τ term coming from the brane and the standard $P_4 V_4$ due to the bulk cosmological constant, with “pressure” $P_4 = -\Lambda_4 / (8\pi G_4)$, and its conjugate “thermodynamic volume” V_4 . The first law (20) corresponds to working in a thermal ensemble of fixed pressure P_4 , such that we fix the bulk scales L_4 and G_4 . Our approach contrasts previous work on (extended) thermodynamics of accelerating black holes [41–45] as we include a brane and work in an ensemble where P_4 is fixed. These authors also find a mechanical work term similar to ours, though the tension corresponds to that of a cosmic string generating the acceleration of the black hole.

As anticipated, $d\tau$ induces extended thermodynamics on the brane. Particularly, the bulk first law (20) maps to the brane first law

$$dM = TdS_{\text{gen}} + V_3 dP_3, \quad (22)$$

generalizing the quantum first law (19), where $P_3 = -\Lambda_3 / (8\pi G_3)$ is the pressure of the qBTZ black hole and V_3 its thermodynamic volume. Moreover, the

corresponding Smarr relation of the qBTZ solution also includes a contribution from the central charge c_3

$$0 = TS_{\text{gen}} - 2P_3 V_3 + \mu_3 c_3, \quad (23)$$

where μ_3 is the chemical potential conjugate variable to the central charge with scaling dimension $[L]^{-1}$. Notably, the mass term is absent from the three-dimensional semiclassical Smarr relation (23), since $G_3 M$ has vanishing scaling dimension, consistent with the extended thermodynamics of the classical BTZ black hole ($\nu \rightarrow 0$) [46,47]. Further, note that fixing L_4 (and G_4) in the bulk has restricted us to the ensemble where c_3 is kept fixed. The crucial point here is that the variation of the pressure on the brane follows directly from variation of the tension.

We derive exact expressions for the conjugate variables V_3 and μ_3 in the Supplemental Material, however, for small backreaction $\nu \ll 1$, we find

$$V_3 = V_{\text{BTZ}} + 8\pi \ell_3^2 \frac{z^3(1+z^2)}{(1+3z^2)^3} \nu + \dots, \quad (24)$$

$$\mu_3 = \frac{z^3(1-z^2)}{\ell_3(1+3z^2)^2} - \frac{z^2(3+9z^2+8z^4-8z^6)}{2\ell_3(1+3z^2)^3} \nu + \dots, \quad (25)$$

where V_{BTZ} is the thermodynamic volume of the classical BTZ black hole, when $\nu = 0$. We see that backreaction of the holographic CFT modifies the classical relation between the volume and entropy, $V_{\text{BTZ}} \sim S_{\text{BTZ}}^2$, where S_{BTZ} is the classical BTZ entropy. The effect of this is that the reverse isoperimetric inequality is obeyed [5],

$$\mathcal{R} \equiv \left(\frac{V_3}{\pi}\right)^{1/2} \left(\frac{\pi}{2S_{\text{gen}}}\right) = 1 + z\nu + \mathcal{O}(\nu^2) \geq 1. \quad (26)$$

Hence, in the limit of small backreaction, the qBTZ is said to be “subentropic,” i.e., a black hole with volume V_3 whose entropy is less than a classical BTZ black hole (where $\mathcal{R} = 1$) of the same volume. This inequality is violated, however, for general z and ν . In fact, the volume is bounded above and below, with the lower bound taking a negative value due to a large Casimir effect. A similar feature appears in the mass of the qBTZ solution, which has a finite range and eventually becomes negative as Casimir effects dominate [25,26]. These new features are most easily observed in a pressure-volume plot at constant temperature [48].

Doubly holographic interpretation.—Since the brane geometry is asymptotically AdS₃, a third description of the bulk-brane system emerges. Namely, we may replace the induced gravity theory on the brane with a defect CFT₂ coupled to the CFT₃, hence, our construction exhibits “double holography” [49]. Per the AdS₃/CFT₂ dictionary [50], the defect CFT has central charge

$$c_2 = \frac{3L_3}{2G_3}. \quad (27)$$

For fixed L_4 and G_4 , we see variations in the brane tension cascade down to variations in c_2 due to the proportionality with L_3 . This is consistent with the standard microscopic interpretation of a dynamical cosmological constant [3,9,13–16].

We can now ascribe a dual field theory interpretation to qBTZ thermodynamics. Following [17,51], we introduce a conjugate variable to the central charge c_2 , namely, a chemical potential μ_2 . We may then recast the brane extended first law (20) in terms of CFT₂ quantities

$$dE = TdS_2 - P_2dV_2 + \mu_2dc_2. \quad (28)$$

Here we identified the energy E and entropy S_2 of the two-dimensional field theory with M and S_{gen} , respectively. Further, $V_2 = 2\pi L_3$ and $P_2 = M/V_2$ are the volume and pressure in the CFT₂. Unlike the brane first law, notice we have a PdV and not VdP term. This suggests the mass M should be understood in the dual CFT as internal energy [17], not enthalpy, as is common in extended black hole thermodynamics [3]. By comparing (22) and (28) and using the Smarr relation (23) one finds that the chemical potential is proportional to the grand canonical free energy W

$$\mu_2c_2 = M - TS_{\text{gen}} - \mu_3c_3 \equiv W. \quad (29)$$

This can be interpreted as the Euler relation in the CFT₂,

$$E = TS_2 + \mu_2c_2 + \mu_n n, \quad (30)$$

where the coupling g_3 is dual to a chemical potential μ_n associated to $c_3 = n$, the number of the light degrees of freedom that are excited in the CFT₂ (corresponding to the cutoff CFT on the brane). Thus, we have found a consistent thermodynamic field theory description of quantum black holes, and generalized the Euler equation in holographic CFTs to higher order in $1/c$.

Discussion.—We have shown braneworld holography offers a natural framework to explore extended black hole thermodynamics: varying brane tension induces a variable pressure of a quantum black hole on the brane to all orders in $1/c$. As proof of concept, we focused on a specific type of black hole, the qBTZ, exhibiting markedly different behavior from the classical BTZ. In particular, the qBTZ system is subentropic, and the thermodynamic volume attains a maximum value.

The investigation here opens up a plethora of future directions. First, our analysis suggests the mass of the bulk is interpreted as an enthalpy, consistent with the interpretation of black hole mass on the brane. It would be interesting to verify this using the on-shell Euclidean action of the bulk black hole, along the lines of [35,45,52].

Further, our case study can be extended to other types of quantum black holes, e.g., rotating and charged qBTZ [26,53], and quantum de Sitter black holes [54,55]. Extra work terms in these models are expected to enrich the thermodynamic phase structure, for which one may build doubly holographic and quantum heat engines, semiclassical versions of [9,56], where universal features of heat engine efficiency [57,58] will be modified due to backreaction.

Another interesting extension would be to consider more general braneworld models. In our analysis, we assumed a simple brane action with only a tension term. Upon integrating out UV degrees of freedom we thus arrive at a theory where the gravitational strength on the brane and the graviton mass are controlled by the same parameter τ . This in turn imposes an interdependence on various scales in the worldvolume theory, which control the strength of the different (higher curvature) gravity couplings. More generally one may consider situations with induced scale separation. One could modify the brane action, for instance, by additional matter fields or via a Dvali-Gabadadze-Porrati (DGP) term [59], e.g.,

$$I_{\text{brane}} = \int_B d^d x \sqrt{-h} \left(-\tau + \frac{1}{16\pi G_B} R \right). \quad (31)$$

The DGP term separates the graviton mass from the gravitational strength on the brane, and has proven insightful for calculations of semiclassical entropy [21,60]. Generally, the separation of scales will be induced because the Einstein-Hilbert term will change the position of the brane by altering the brane stress tensor and also modify the effective brane cosmological constant. Alternately, by tuning the tension such that the brane position is unchanged, the Newton's constant on the brane G_B directly influences the effective Newton's constant via [21]

$$G_d^{-1} = 2L_{d+1}/[(d-2)G_{d+1}] + G_B^{-1}, \quad (32)$$

such that $\delta G_B/G_B^2 = \delta G_d/G_d^2$. Thus, even if the gravitational parameters are held fixed in the bulk, the DGP term could allow for richer induced brane thermodynamics. The DGP term induces a variation of Newton's constant, which in turn yields a variation of the central charge c_3 , kept fixed in this Letter [16,17,61]. One may also consider including topological gravitational terms, such as Gauss-Bonnet or Lovelock densities, or theories coupled with matter, all of which introduce additional tunable parameters. Adding such DGP-like terms allows one to study extended first laws of entanglement in the presence of additional central charges, as in [14,62–64], but beyond their classical limits.

Lastly, while we have provided a higher-dimensional origin of extended thermodynamics using a bottom-up construction, it is desired to have a more fundamental string theoretic underpinning. Doing so would grant us access to

stringy corrections beyond the semiclassical regime considered here. Such a program has been initiated in [65]. It is also worth constructing braneworld black holes in string or M theory and studying their doubly holographic description, in the spirit of [66–70].

We are grateful to Roberto Emparan, Robie Hennigar, and Clifford Johnson for useful discussions and correspondence. A.M.F. is supported by ERC Advanced Grant No. GravBHs-692951, MICINN Grant No. PID2019–105614 GB-C22, AGAUR Grant No. 2017-SGR 754, and State Research Agency of MICINN through the “Unit of Excellence María de Maeztu 2020-2023” award to the Institute of Cosmos Sciences (CEX2019-000918-M). J. F. P. is supported by the “Atracción de Talento” program Grant No. 2020-T1/TIC-20495 and by the Spanish Research Agency through the Grants No. CEX2020-001007-S and No. PID2021-123017NB-I00, funded by MCIN/AEI/10.13039/501100011033 and by ERDF A way of making Europe. A. S. is supported by the Simons Foundation via *It from Qubit Collaboration* and by EPSRC. M. R. V. is supported by SNF Postdoc Mobility Grant No. P500PT-206877 “Semi-classical thermodynamics of black holes and the information paradox.”

-
- [1] J. D. Bekenstein, *Phys. Rev. D* **7**, 2333 (1973).
 [2] S. W. Hawking, *Commun. Math. Phys.* **43**, 199 (1975); **46**, 206(E) (1976).
 [3] D. Kastor, S. Ray, and J. Traschen, *Classical Quantum Gravity* **26**, 195011 (2009).
 [4] B. P. Dolan, *Classical Quantum Gravity* **28**, 125020 (2011).
 [5] M. Cvetič, G. W. Gibbons, D. Kubiznak, and C. N. Pope, *Phys. Rev. D* **84**, 024037 (2011).
 [6] A. Chamblin, R. Emparan, C. V. Johnson, and R. C. Myers, *Phys. Rev. D* **60**, 064018 (1999).
 [7] D. Kubiznak and R. B. Mann, *J. High Energy Phys.* **07** (2012) 033.
 [8] D. Kubiznak and R. B. Mann, *Can. J. Phys.* **93**, 999 (2015).
 [9] C. V. Johnson, *Classical Quantum Gravity* **31**, 205002 (2014).
 [10] C. Teitelboim, *Phys. Lett.* **158B**, 293 (1985).
 [11] M. M. Caldarelli, G. Cognola, and D. Klemm, *Classical Quantum Gravity* **17**, 399 (2000).
 [12] S. de Haro, K. Skenderis, and S. N. Solodukhin, *Classical Quantum Gravity* **18**, 3171 (2001).
 [13] B. P. Dolan, *J. High Energy Phys.* **10** (2014) 179.
 [14] D. Kastor, S. Ray, and J. Traschen, *J. High Energy Phys.* **11** (2014) 120.
 [15] E. Caceres, P. H. Nguyen, and J. F. Pedraza, *J. High Energy Phys.* **09** (2015) 184.
 [16] A. Karch and B. Robinson, *J. High Energy Phys.* **12** (2015) 073.
 [17] M. R. Visser, *Phys. Rev. D* **105**, 106014 (2022).
 [18] R. Emparan, A. Fabbri, and N. Kaloper, *J. High Energy Phys.* **08** (2002) 043.
 [19] P. Kraus, F. Larsen, and R. Siebelink, *Nucl. Phys.* **B563**, 259 (1999).
 [20] I. Papadimitriou and K. Skenderis, *IRMA Lect. Math. Theor. Phys.* **8**, 73 (2005).
 [21] H. Z. Chen, R. C. Myers, D. Neuenfeld, I. A. Reyes, and J. Sandor, *J. High Energy Phys.* **10** (2020) 166.
 [22] P. Bueno, R. Emparan, and Q. Llorens, *Phys. Rev. D* **106**, 044012 (2022).
 [23] L. Randall and R. Sundrum, *Phys. Rev. Lett.* **83**, 4690 (1999).
 [24] R. Emparan, G. T. Horowitz, and R. C. Myers, *J. High Energy Phys.* **01** (2000) 007.
 [25] R. Emparan, G. T. Horowitz, and R. C. Myers, *J. High Energy Phys.* **01** (2000) 021.
 [26] R. Emparan, A. M. Frassino, and B. Way, *J. High Energy Phys.* **11** (2020) 137.
 [27] A. Karch and L. Randall, *J. High Energy Phys.* **05** (2001) 008.
 [28] M. Banados, C. Teitelboim, and J. Zanelli, *Phys. Rev. Lett.* **69**, 1849 (1992).
 [29] M. Banados, M. Henneaux, C. Teitelboim, and J. Zanelli, *Phys. Rev. D* **48**, 1506 (1993); **88**, 069902(E) (2013).
 [30] T. Souradeep and V. Sahni, *Phys. Rev. D* **46**, 1616 (1992).
 [31] A. R. Steif, *Phys. Rev. D* **49**, 585 (1994).
 [32] M. Casals, A. Fabbri, C. Martínez, and J. Zanelli, *Phys. Lett. B* **760**, 244 (2016).
 [33] R. Emparan, A. M. Frassino, M. Sasieta, and M. Tomašević, *J. High Energy Phys.* **02** (2022) 204.
 [34] J. Podolsky, *Czech. J. Phys.* **52**, 1 (2002).
 [35] H. Kudoh and Y. Kurita, *Phys. Rev. D* **70**, 084029 (2004).
 [36] R. Emparan, *J. High Energy Phys.* **06** (2006) 012.
 [37] J. F. Pedraza, A. Svesko, W. Sybesma, and M. R. Visser, *J. High Energy Phys.* **12** (2021) 134.
 [38] A. Svesko, E. Verheijden, E. P. Verlinde, and M. R. Visser, *J. High Energy Phys.* **08** (2022) 075.
 [39] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/PhysRevLett.130.161501> for a derivation of the bulk and brane Smarr relations.
 [40] J. Armas, N. A. Obers, and M. Sanchioni, *J. High Energy Phys.* **09** (2016) 124.
 [41] M. Appels, R. Gregory, and D. Kubiznak, *Phys. Rev. Lett.* **117**, 131303 (2016).
 [42] A. Anabalón, M. Appels, R. Gregory, D. Kubizňák, R. B. Mann, and A. Ovgün, *Phys. Rev. D* **98**, 104038 (2018).
 [43] A. Anabalón, F. Gray, R. Gregory, D. Kubizňák, and R. B. Mann, *J. High Energy Phys.* **04** (2019) 096.
 [44] R. Gregory and A. Scoins, *Phys. Lett. B* **796**, 191 (2019).
 [45] A. Ball and N. Miller, *Classical Quantum Gravity* **38**, 145031 (2021).
 [46] A. M. Frassino, R. B. Mann, and J. R. Mureika, *Phys. Rev. D* **92**, 124069 (2015).
 [47] A. M. Frassino, R. B. Mann, and J. R. Mureika, *J. High Energy Phys.* **11** (2019) 112.
 [48] A. Frassino, J. F. Pedraza, A. Svesko, and M. R. Visser (to be published).
 [49] A. Almheiri, R. Mahajan, J. Maldacena, and Y. Zhao, *J. High Energy Phys.* **03** (2020) 149.
 [50] J. D. Brown and M. Henneaux, *Commun. Math. Phys.* **104**, 207 (1986).
 [51] W. Cong, D. Kubiznak, R. B. Mann, and M. R. Visser, *J. High Energy Phys.* **08** (2022) 174.

- [52] S. W. Hawking and S. F. Ross, *Phys. Rev. Lett.* **75**, 3382 (1995).
- [53] A. Climent, R. Emparan, and R. Hennigar (to be published).
- [54] R. Emparan, J. F. Pedraza, A. Svesko, M. Tomašević, and M. R. Visser, *J. High Energy Phys.* 11 (2022) 073.
- [55] E. Panella and A. Svesko, [arXiv:2303.08845](https://arxiv.org/abs/2303.08845).
- [56] C. V. Johnson, *Classical Quantum Gravity* **37**, 034001 (2020).
- [57] C. V. Johnson, *Entropy* **18**, 120 (2016).
- [58] M. C. DiMarco, S. L. Jess, R. A. Hennigar, and R. B. Mann, *Phys. Rev. D* **107**, 044001 (2023).
- [59] G. R. Dvali, G. Gabadadze, and M. Porrati, *Phys. Lett. B* **485**, 208 (2000).
- [60] H. Z. Chen, R. C. Myers, D. Neuenfeld, I. A. Reyes, and J. Sandor, *J. High Energy Phys.* 12 (2020) 025.
- [61] W. Cong, D. Kubiznak, and R. B. Mann, *Phys. Rev. Lett.* **127**, 091301 (2021).
- [62] E. Caceres, P. H. Nguyen, and J. F. Pedraza, *Phys. Rev. D* **95**, 106015 (2017).
- [63] C. V. Johnson and F. Rosso, *Classical Quantum Gravity* **36**, 015019 (2019).
- [64] F. Rosso and A. Svesko, *J. High Energy Phys.* 08 (2020) 008.
- [65] P. Meessen, D. Mitsios, and T. Ortín, *J. High Energy Phys.* 12 (2022) 155.
- [66] A. Karch and L. Randall, *J. High Energy Phys.* 06 (2001) 063.
- [67] A. Karch and L. Randall, *Phys. Rev. Lett.* **87**, 061601 (2001).
- [68] E. D'Hoker, J. Estes, and M. Gutperle, *J. High Energy Phys.* 06 (2007) 021.
- [69] O. Aharony, L. Berdichevsky, M. Berkooz, and I. Shamir, *Phys. Rev. D* **84**, 126003 (2011).
- [70] A. Karch, H. Sun, and C. F. Uhlemann, *J. High Energy Phys.* 10 (2022) 012.