What It Takes to Solve the Hubble Tension through Modifications of Cosmological Recombination

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We construct data-driven solutions to the Hubble tension which are perturbative modifications to the fiducial ΛCDM cosmology, using the Fisher bias formalism. Taking as proof of principle the case of a time-varying electron mass and fine structure constant, and focusing first on Planck CMB data, we demonstrate that a modified recombination *can* solve the Hubble tension and lower S_8 to match weak lensing measurements. Once baryonic acoustic oscillation and uncalibrated supernovae data are included, however, it is not possible to fully solve the tension with perturbative modifications to recombination.

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Introduction.—The standard Λ cold dark matter (ΛCDM) model has been providing an astonishing fit to a wide variety of cosmological data. Yet, the precise value of a very basic parameter of the model, the present-time expansion rate of the Universe (or Hubble constant) H_0, remains the subject of intense debate. On the one hand, this parameter can be inferred indirectly from early-Universe probes, under the assumption of standard physics and ΛCDM cosmology. The most precise early-Universe measurement is that inferred from the Planck satellite's Cosmic Microwave Background (CMB) anisotropy data, $H_0 = 67.36 \pm 0.54$ km/s/Mpc [1]. On the other hand, H_0 can be obtained from local (or late-Universe) measurements. The most precise local measurement is that provided by the SH0ES Collaboration, which directly measures the current expansion rate from supernovae, using Cepheid variables as calibrators, arriving at $H_0 = 73.04 \pm 1.04$ km/s/Mpc [2,3]. Depending on the specific datasets considered, the discrepancy between early-Universe and local measurements has reached 4–6σ [4,5], enough to make this "Hubble tension" one of the most pressing issues in recent cosmology.

Although one cannot exclude multiple unknown systematic errors as a reason for the discrepancy [6–16], it may also hint at new physics, or extensions of the ΛCDM model. To resolve this Hubble tension, an enormous number of models have been proposed. Late-time solutions, which include late dark energy, emergent dark energy, interacting dark energy, and decaying CDM, have been shown to be less effective [17–26]. This is because postrecombination solutions do not change the sound horizon at baryon decoupling, r_d, and can therefore not fit baryonic acoustic oscillation (BAO) data and uncalibrated type Ia supernovae (SNIa) data while increasing the Hubble constant (so-called "sound horizon problem"). This implies that a modification in early-time cosmology is needed to solve the Hubble tension [22,27–30] (see also Ref. [31] for a newly proposed quantity in the context of H_0 tension: the age of the Universe, and see Ref. [32] for a study of the degeneracy of H_0 with the CMB monopole temperature T_0). Early-time solutions focus on the reduction of the sound horizon at recombination, through either an increase in energy density, e.g., via early dark energy (EDE) [33–38] or additional dark radiation [39–41], or a modification of the recombination history itself by, for example, introducing primordial magnetic fields (PMFs) [42–44] (see Refs. [45,46] for whether small-scale baryon clumping due to PMF can resolve the Hubble tension together with Ref. [47] for a general formalism to estimate the effect of small-scale baryon perturbations on CMB anisotropies) or varying fundamental constants [48–51] (see also Refs. [52,53] for nonstandard recombination). Reducing the size of the sound horizon at recombination naturally leads to larger H_0 value, in order to keep the angular size of the sound horizon measured at $\mathcal{O}(0.1\%)$ precision with Planck CMB data unaffected [21]. However, none of the proposed solutions have been robustly detected in the variety of cosmological data, and, further, the reduction of the Hubble tension is partly due to an increased uncertainty. See Refs. [30,54] for a recent summary and comparison of proposed models.

In this Letter, we move beyond the model-by-model approach as an effort to resolve the Hubble tension, and for the first time, make use of the Fisher-bias formalism to find minimal *data-driven* extensions to the ΛCDM model producing desired shifts in cosmological parameters (in this case, an increase in H_0), while not worsening the fit to a given dataset. We cast this question as a well-defined simple mathematical problem. With this formalism, as examples, we extract the shape of a time-varying electron mass $m_e(z)$ or fine structure constant $\alpha(z)$ modifications that would result in a better agreement of a given early-Universe dataset with SH0ES. Let us stress that our primary goal is not to find a compelling physical solution to the tension, i.e., that can easily be realized via a simple theoretical model [for modeling of $\alpha(z)$ see, e.g., Refs. [55,56]]. Instead, we focus on establishing whether such solutions *exist*, which is already a nontrivial question. Indeed, it could very well be that the relevant observables are only sensitive to a few integrals of the ionization history, and that arbitrary modifications of the latter could only project on a limited subspace of observable variations.

In short, we show that while one can find *small* time-varying perturbations to m_e or α that would entirely solve the Hubble tension between Planck and SH0ES, once BAO and uncalibrated SNIa data are included, one can only lower the Hubble tension down to $\sim 2.4\sigma$ with perturbative modifications to recombination, not being able to entirely resolve the tension.

Setting up the problem.—We denote general observables by $\boldsymbol{X}$, a vector which may contain multiple observables, such as CMB angular power spectra, BAO measurements, or any other. Specifically, we denote the observed data by $\boldsymbol{X}^{\rm obs}$ and the corresponding theoretical prediction by $\boldsymbol{X}(\vec{\Omega})$, where $\vec{\Omega} \equiv \{\omega_c, \omega_b, H_0, \tau, \ln(10^{10}A_s), n_s\}$ is a set of cosmological parameters. One can obtain the best-fit parameters by maximizing the likelihood of the data $\mathcal{L}[\boldsymbol{X}(\vec{\Omega}); \boldsymbol{X}^{\rm obs}]$, or equivalently minimizing $\chi^2 \equiv -2\ln\mathcal{L}$. Importantly, the best-fit parameters $\vec{\Omega}_{\rm BF}$ and best-fit chi-squared $\chi^2_{\rm BF} \equiv \chi^2[\boldsymbol{X}(\vec{\Omega}_{\rm BF}); \boldsymbol{X}^{\rm obs}]$ both depend on the underlying theoretical model $\boldsymbol{X}(\vec{\Omega})$. In particular, if we consider a model $\boldsymbol{X}'(\vec{\Omega}) = \boldsymbol{X}(\vec{\Omega}) + \Delta\boldsymbol{X}(\vec{\Omega})$ that differs from the standard model $\boldsymbol{X}(\vec{\Omega})$ by a small amount $\Delta\boldsymbol{X}(\vec{\Omega})$, the resulting best-fit parameters and chi-squared are shifted.

More specifically, we will consider changes in the theoretical model resulting from perturbations to a smooth function $f(z)$ on which it depends. The resulting changes to the best-fit parameters $\Delta\vec{\Omega}_{\rm BF}[\Delta f(z)]$ and chi-squared $\Delta\chi^2_{\rm BF}[\Delta f(z)]$ are both *functionals* of $\Delta f(z)$. Our general goal, then, is to find the smallest possible perturbations $\Delta f(z)$ allowing to shift the best-fit parameters to a target value $\vec{\Omega}_{\rm target}$, while not worsening the quality of fit [57]. In other words, we want to solve the following constrained optimization problem, for different datasets and different functions $f(z)$:

$$\text{minimize}(\|\Delta f\|^2)\text{with}\begin{cases}\vec{\Omega}_{\rm BF}[\Delta f(z)] = \vec{\Omega}_{\rm target},\\ \Delta\chi^2_{\rm BF}[\Delta f(z)] \le 0,\end{cases} \quad (1)$$

where $\|\cdots\|$ is the L^2 norm, $\|\Delta f\|^2 \equiv \int dz[\Delta f(z)]^2$. In principle, this optimization problem can be solved exactly if combined with Markov Chain Monte Carlo (MCMC) analysis (or a minimization process) to estimate $\vec{\Omega}_{\rm BF}[\Delta f(z)]$ and $\Delta\chi^2_{\rm BF}[\Delta f(z)]$ for each given $\Delta f(z)$. However, this exact method would be heavily computationally expensive. Hence, to keep the optimization problem tractable, we will first derive simple approximations for $\vec{\Omega}_{\rm BF}$ and $\chi^2_{\rm BF}$, relying on the Fisher approximation (e.g., Ref. [58]). We approximate the data as Gaussian distributed, with inverse-covariance matrix $\boldsymbol{M} = \boldsymbol{\Sigma}^{-1}$. In general, this matrix depends on $\boldsymbol{X}(\vec{\Omega})$ itself; we will denote $\boldsymbol{M}(\vec{\Omega}) \equiv \boldsymbol{M}[\boldsymbol{X}(\vec{\Omega})]$ for short. We define the chi-squared of a given cosmology $\vec{\Omega}$ as

$$\chi^2(\vec{\Omega}) \equiv [\boldsymbol{X}(\vec{\Omega}) - \boldsymbol{X}^{\rm obs}]\cdot\boldsymbol{M}(\vec{\Omega})\cdot[\boldsymbol{X}(\vec{\Omega}) - \boldsymbol{X}^{\rm obs}]. \quad (2)$$

By Taylor expanding the chi-squared to second order around a fiducial cosmology $\vec{\Omega}_{\rm fid}$, which we assume to be reasonably close to the best-fit, and minimizing it, we find the (approximate) best-fit cosmology $\vec{\Omega}_{\rm BF}$,

$$\Omega^i_{\rm BF} = \Omega^i_{\rm fid} - \frac{1}{2}(F^{-1})_{ij}\frac{\partial\chi^2}{\partial\Omega^j}\bigg|_{\rm fid}, \quad (3)$$

where F is the Fisher matrix defined and approximated as

$$F_{ij} \equiv \frac{1}{2}\frac{\partial^2\chi^2}{\partial\Omega^i\partial\Omega^j}\bigg|_{\rm fid} \approx \left(\frac{\partial\boldsymbol{X}}{\partial\Omega^i}\cdot\boldsymbol{M}\cdot\frac{\partial\boldsymbol{X}}{\partial\Omega^j}\right)\bigg|_{\rm fid}. \quad (4)$$

Provided that the fiducial model is sufficiently close to the observations, we can approximate $\partial\chi^2/\partial\Omega^i$ to include only the leading contribution, which then implies

$$\Omega^i_{\rm BF} \approx \Omega^i_{\rm fid} - (F^{-1})_{ij}\frac{\partial\boldsymbol{X}}{\partial\Omega^j}\bigg|_{\rm fid}\cdot\boldsymbol{M}(\vec{\Omega}_{\rm fid})\cdot[\boldsymbol{X}(\vec{\Omega}_{\rm fid}) - \boldsymbol{X}^{\rm obs}]. \quad (5)$$

Inserting this solution into the Taylor-expanded chi-squared we find the approximate best-fit chi-squared

$$\chi^2(\vec{\Omega}_{\rm BF}) \approx [\boldsymbol{X}(\vec{\Omega}_{\rm fid}) - \boldsymbol{X}^{\rm obs}]\cdot\tilde{\boldsymbol{M}}\cdot[\boldsymbol{X}(\vec{\Omega}_{\rm fid}) - \boldsymbol{X}^{\rm obs}], \quad (6)$$

where $\tilde{\boldsymbol{M}}$ is defined as

$$\tilde{M}_{\alpha\beta} \equiv M_{\alpha\beta} - M_{\alpha\gamma}\frac{\partial X^\gamma}{\partial\Omega^i}(F^{-1})_{ij}\frac{\partial X^\sigma}{\partial\Omega^j}M_{\sigma\beta}, \quad (7)$$

where $\boldsymbol{M}$ and $\partial\boldsymbol{X}/\partial\Omega^i$ are evaluated at the fiducial cosmology, and repeated indices are to be summed over. A simple property of the matrix $\tilde{\boldsymbol{M}}$ is that it admits $\partial\boldsymbol{X}/\partial\Omega^i$ as null eigenvectors. We can thus think of $\tilde{\boldsymbol{M}}$ as the inverse-covariance matrix of the data after marginalization over shifts in standard cosmological parameters.

Introducing new physics.—Our main results so far, Eqs. (5) and (6), apply to an arbitrary theoretical model, provided that it gives a reasonable fit to the data for the chosen fiducial cosmological parameters $\vec{\Omega}_{\rm fid}$. The best-fit parameters and chi-squared of a new theoretical model $\boldsymbol{X}'(\vec{\Omega}) = \boldsymbol{X}(\vec{\Omega}) + \Delta\boldsymbol{X}(\vec{\Omega})$ differ from those of the standard model $\boldsymbol{X}(\vec{\Omega})$ by small amounts $\Delta\Omega^i_{\rm BF}$ and $\Delta\chi^2_{\rm BF}$, respectively. Assuming $\Delta\boldsymbol{X}(\vec{\Omega}_{\rm fid})$ and $\boldsymbol{X}(\vec{\Omega}_{\rm fid}) - \boldsymbol{X}^{\rm obs}$ are approximately of the same order of magnitude, and by writing a change in the theoretical model due to changes in a smooth function $f(z)$ as

$$\Delta\boldsymbol{X} = \int dz \frac{\delta\boldsymbol{X}}{\delta f(z)} \Delta f(z), \tag{8}$$

we obtain the resulting changes in the best-fit parameters and chi-squared

$$\Delta\Omega^i_{\rm BF} = \int dz \frac{\delta\Omega^i_{\rm BF}}{\delta f(z)} \Delta f(z), \tag{9}$$

$$\Delta\chi^2_{\rm BF} = \int dz \frac{\delta\chi^2_{\rm BF}}{\delta f(z)} \Delta f(z) + \frac{1}{2} \iint dz\, dz' \frac{\delta^2\chi^2_{\rm BF}}{\delta f(z)\delta f(z')} \Delta f(z)\Delta f(z'), \tag{10}$$

where

$$\frac{\delta\Omega^i_{\rm BF}}{\delta f(z)} = -(F^{-1})_{ij} \frac{\partial\boldsymbol{X}}{\partial\Omega^j} \cdot \boldsymbol{M} \cdot \frac{\delta\boldsymbol{X}}{\delta f(z)}, \tag{11}$$

$$\frac{\delta\chi^2_{\rm BF}}{\partial f(z)} = 2[\boldsymbol{X}(\vec{\Omega}_{\rm fid}) - \boldsymbol{X}^{\rm obs}] \cdot \tilde{\boldsymbol{M}} \cdot \frac{\delta\boldsymbol{X}}{\delta f(z)}, \tag{12}$$

$$\frac{\delta^2\chi^2_{\rm BF}}{\delta f(z)\delta f(z')} = 2\frac{\delta\boldsymbol{X}}{\delta f(z)} \cdot \tilde{\boldsymbol{M}} \cdot \frac{\delta\boldsymbol{X}}{\delta f(z')}, \tag{13}$$

where F_{ij}, $\boldsymbol{M}$, $\tilde{\boldsymbol{M}}$, $\partial\boldsymbol{X}/\partial\Omega_i$, and $\delta\boldsymbol{X}/\delta f(z)$ are all to be evaluated at the fiducial cosmology and in the standard model. With the simplified expressions of Eqs. (9)–(13), the optimization problem of Eq. (1) becomes tractable. The equations above are known as the Fisher-bias formalism [59–63], used in Refs. [64,65] to constrain arbitrary functions. While the formalism is well known, the *application* we make of it is completely novel.

While our formalism is general and could be applied to any function $f(z)$ on which observables depend, in this Letter, we will consider modifications to the cosmological ionization history. Specifically, we will consider time-dependent relative variations of the electron mass $[f(z) = \ln m_e(z)]$ in the main text, generalizing the constant change to the electron mass which has been shown to be a promising solution [49,50,54]. We also consider time-dependent variations of the fine structure constant $[f(z) = \ln\alpha(z)]$, in Appendix H of the Supplemental Material [66] [90].

The functional derivatives $\delta\boldsymbol{X}/\delta f(z)$ are obtained numerically by adding narrow (Dirac-delta-like) changes to the smooth function $f(z)$, at different redshifts. This is done by modifying the recombination code HYREC-2 [91–93] implemented in CLASS [94] (see Appendix B of the Supplemental Material [66] for details). This part of the calculation is similar to what has been done in principal component analyses (PCAs) of recombination perturbations [95,96]. Despite this technical similarity, the mathematical problem we solve is very different from the one considered in PCAs, which search the eigenmodes of the (discretized) matrix $\delta^2\chi^2/\delta f(z)\delta f(z')$ with the largest eigenvalues. In words, PCAs look for perturbations to recombination to which the data is *most* sensitive, while in contrast, our goal is to find the *smallest* perturbations producing a desired shift in best-fit cosmological parameters while *not* increasing the best-fit χ^2. See Appendix D of the Supplemental Material [66] for the differences in two analyses.

We will now apply this general formalism to Planck CMB anisotropy data and then to the combined Planck + BAO, Planck + BAO + PantheonPlus [97] dataset, with the goal of finding data-driven solutions to the Hubble tension. Note that by BAO we denote BOSS DR12 anisotropic BAO measurements [98].

Result I: Application to Planck CMB data.—Here, the vector $\boldsymbol{X}$ consists of the binned lensed temperature and polarization power spectra, $\boldsymbol{X} \equiv \{D^{\rm TT}_\ell, D^{\rm TE}_\ell, D^{\rm EE}_\ell\}$. For $\ell \geq 30$, we use the Planck-lite foreground-marginalized binned spectra and covariance matrix. For $\ell < 30$, we adopt the compressed log-normal likelihood of Prince and Dunkley [99], which has been shown to give virtually the same constraints as the exact low-ℓ Planck likelihood (and therefore we use $\boldsymbol{X} \equiv \{\ln D^{\rm TT}_\ell, \ln D^{\rm TE}_\ell, \ln D^{\rm EE}_\ell\}$ for $\ell < 30$). We set our fiducial cosmology to the Planck best-fit ΛCDM parameters [1].

Using our formalism, we find variations of the time-varying electron mass $m_e(z)$ that cause the value inferred from Planck CMB anisotropies to be equal to a given target Hubble constant $H_0^{\rm BF}$ while not deteriorating the best fit chi-squared $\Delta\chi^2_{\rm BF} \leq 0$. These deviations of $m_e(z)$ from its standard value are shown in Fig. 1 with a range of $H_0^{\rm BF}$ values whose upper bound is the recent SH0ES best-fit [2]. It is striking that our solution exhibits three large oscillations offset from zero between $z \simeq 700$–1500. This behavior is significantly different than what has been modeled in past literature, namely, either a constant shift in m_e or a power-law dependence on redshift [48,49,54], explaining why these studies did not find as good a solution as we do, and illustrating the power of our formalism which can further be used to guide model-building (e.g., [56]). In particular, such a solution would also avoid big bang nucleosynthesis (BBN) constraints [100]. We keep it for future work to investigate possible physical mechanisms that may generate the required oscillations. We confirm that

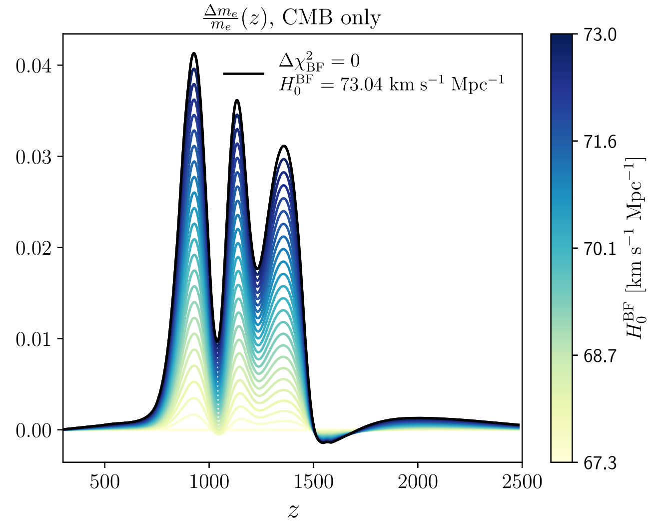

FIG. 1. Solutions for $(\Delta m_e/m_e)(z)$ given target values of the CMB-only best-fit Hubble constant H_0, using Planck data [1] alone. All solutions are constructed to keep the Planck best-fit chi-squared unaffected. The solution with a best-fit consistent with SH0ES [2] $H_0^{\rm BF}=H_0^{\rm SH0ES}\equiv 73.04\ {\rm km\,s^{-1}\,Mpc^{-1}}$ (black curve), is denoted as ΛCDM + $m_e(z)$ model in the text.

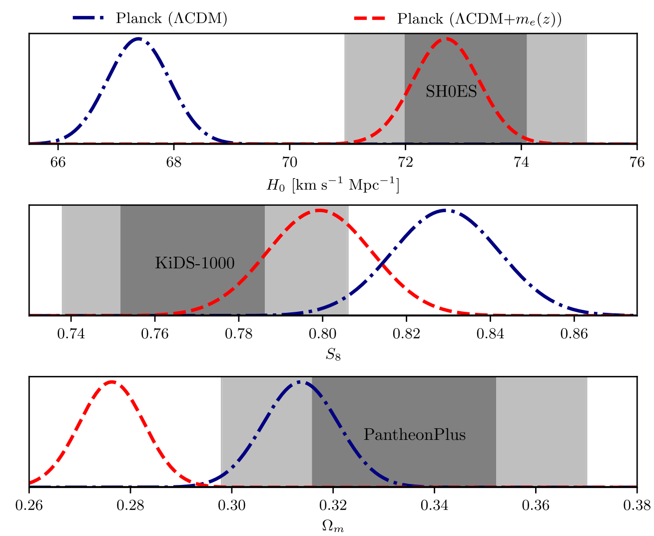

FIG. 2. Posteriors of H_0 (top), S_8 (middle), and Ω_m (bottom) inferred from Planck full likelihood, together with SH0ES, KiDS-1000, and PantheonPlus results, shown as gray bands.

the obtained $m_e(z)$ does indeed result in the expected parameter shifts by performing a MCMC analysis using MontePython v3.0 [101,102] with the full Planck TT, TE, EE + low E + lensing likelihood (see Appendix E of the Supplemental Material [66] for this validation test). This also attests that our use of the high-ℓ Planck-lite Gaussian likelihood combined with the low-ℓ compressed likelihood of Ref. [99], as well as our noninclusion of the lensing potential likelihood, is accurate enough to derive a solution $m_e(z)$. In Appendix G of the Supplemental Material [66], we further quantify the accuracy of the other two approximations we make to derive the $m_e(z)$ solution, namely, the Taylor expansion of χ^2 around a fiducial cosmology, and the linearity of observables in $\Delta f(z)$.

Our main result is shown in the top panel of Fig. 2, which displays the posterior distributions for H_0. In short, the ΛCDM model with a time-varying $m_e(z)$ given by the black curve in Fig. 1 is a solution to the Hubble tension between Planck CMB and SH0ES data. It lowers the discrepancy to 0.29σ resulting in $H_0=72.70\pm 0.57\ {\rm km\,s^{-1}\,Mpc^{-1}}$, and $\Delta\chi^2=-1.15$ compared to the ΛCDM model [103]. Interestingly, this solution tailored to solve the H_0 tension also happens to mostly solve another infamous tension in cosmology, the so-called S_8 tension ($S_8=\sigma_8\sqrt{\Omega_m/0.3}$), which is a discrepancy in measurements of the amplitude of matter clustering at the scale of 8 Mpc/h between weak lensing probes and the value inferred from CMB anisotropies [104–106]. Indeed, the ΛCDM + $m_e(z)$ model brings the Planck best-fit S_8 value to 1.1σ from the recent DES-Y3 constraint $S_8=0.776\pm0.017$ [107] and within $\sim1.4\sigma$ from KiDS-450 ($S_8=0.745\pm0.039$) [108] and KiDS-1000 ($S_8=0.766^{+0.020}_{-0.014}$) [109], down from the $2-2.6\sigma$ tension in ΛCDM, as shown in the middle panel of Fig. 2.

However, this extension to the ΛCDM model is less consistent with two other crucial cosmological data, BAO [98] and PantheonPlus [97]. The bottom panel of Fig. 2 shows that this model is inconsistent with the PantheonPlus result for Ω_m with $\sim3\sigma$ tension. This is fundamentally due to the well-known dependence of θ_s on $\Omega_m h^3$, which requires the best-fit h and Ω_m to change in opposite ways, as we describe in further detail in Appendix C of the Supplemental Material [66]. In addition, the agreement with BOSS DR12 BAO data is worsened resulting in an increase of the chi-squared of BOSS DR12 anisotropic measurements, $\Delta\chi^2_{\rm BAO}=+5.37$ (see also Ref. [110] for a similar result [111]).

Result II: Application to Planck CMB + BAO *or Planck* CMB + BAO + PatheonPlus.—We include either BAO or BAO + PantheonPlus data in our data vector $\boldsymbol{X}$, in order to see if we can obtain solutions to the Hubble tension that do not violate the agreement with these data that is present in the ΛCDM model. For BAO, we include BOSS DR12 anisotropic measurements at three effective redshifts $z_{\rm eff}=0.38,\ 0.51,\ 0.61$ [98], $\{[D_M(z_{\rm eff})r_d^{\rm fid}/r_d],[H(z_{\rm eff})r_d/r_d^{\rm fid}]\}\subset \boldsymbol{X}$, and for PantheonPlus [97] we include its constraint on the fractional energy density of the total matter $\{\Omega_m\}\subset\boldsymbol{X}$. We find that, in order to solve the Hubble tension either together with BAO or BAO + PantheonPlus, variations of $m_e(z)$ with larger amplitude are required together with larger shifts in best-fit cosmology. We note that these required more radical changes in recombination history and best-fit cosmology induce larger errors from the approximations taken in our formalism, preventing us from finding a self-consistent solution with a target $H_0^{\rm BF} = 73.04\ {\rm km\,s^{-1}\,Mpc^{-1}}$. Yet, we find that one can still *partially* ease the Hubble tension with our current method while remaining within the regime of validity of our perturbative treatment. Explicitly, we find that one could lower the Hubble tension down to $\sim 1.3\sigma$ with BAO data included, while satisfying $\Delta\chi^2 < 1$ and $|\Omega_{\rm BF}^{\rm formalism} - \Omega_{\rm BF}^{\rm MCMC}|/\sigma < 1$ for all six cosmological parameter Ω's. However, the reconstructed $\Omega_m \simeq 0.287$ is again less consistent with PantheonPlus compared to the standard ΛCDM model. Further, when PantheonPlus data are added along with CMB and BAO, we find that one could lower the tension down to at most $\sim 2.4\sigma$ while remaining within the region of validity of the formalism. As such, even with arbitrary perturbative modifications to $\alpha(z)$ or $m_e(z)$ around the time of recombination, the Hubble tension can only be partially eased once BAO and supernovae data are accounted for.

Conclusions.—We have built on the Fisher bias formalism to systematically search for data-driven solutions to any tension between any given datasets, by looking for the smallest possible change to an arbitrary function leading to a desired small shift in cosmological parameters [112]. We applied our formalism to find a time-dependent function for the electron mass (and for the fine structure constant in Appendix F of the Supplemental Material [66]) leading to a Hubble constant consistent with SH0ES while providing an equally good fit to Planck CMB data. We show that as a remarkable byproduct it happens to also solve the S_8 tension. However, this extended model is less consistent with BAO [98] and PantheonPlus [97].

Once BAO and PantheonPlus data are included in the formalism, we find that larger changes in recombination history are required to achieve the same target value of $H_0^{\rm BF}$, making the assumed linearity of observables, and the validity of Taylor expansion of χ^2, break down. We note that these limitations can in principle be removed if one approaches the optimization problem with an exact method, which we defer to future work. In practice, we find that small perturbations to recombination through a time-varying electron mass can only reduce the tension down to 2.4σ, and decreasing it further would likely require nonperturbative changes to recombination.

While we focus on perturbations to recombination in this Letter, our formalism can be applied more generally to any quantity impacting the prediction of a cosmological observable, e.g., the Hubble rate $H(z)$. We trust that the phenomenological framework we laid out, and the specific examples we provide here in terms of a modified recombination, will inspire a model-building effort from the cosmology and particle physics community with potential implications well beyond the mere study of cosmological tensions.

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[111] Note that, however, our results cannot be directly compared with those of Ref. [110] due to the fixed relation between two sound horizon scales at baryon decoupling and recombination in Ref. [110], which is not satisfied in our case.

[112] While we find minimal extensions not worsening the fit to a given dataset, one could also seek solutions to the tension with other strategies. For example, rather than aiming for a specific value of H_0, one can look for extensions minimizing the fit to all datasets including SH0ES, Pantheon, and BAO (putting an additional constraint on the norm of the solution). We defer exploring these different strategies to future work.