Universal Framework for Record Ages under Restart

Aanjaneya Kumar[1] and Arnab Pal[2,3,*]

[1]*Department of Physics, Indian Institute of Science Education and Research, Dr. Homi Bhabha Road, Pune 411008, India*
[2]*The Institute of Mathematical Sciences, CIT Campus, Taramani, Chennai 600113, India*
[3]*Homi Bhabha National Institute, Training School Complex, Anushakti Nagar, Mumbai 400094, India*

(Received 7 September 2022; accepted 16 March 2023; published 11 April 2023)

We propose a universal framework to compute record age statistics of a stochastic time series that undergoes random restarts. The proposed framework makes minimal assumptions on the underlying process and is furthermore suited to treat generic restart protocols going beyond the Markovian setting. After benchmarking the framework for classical random walks on the 1D lattice, we derive a universal criterion underpinning the impact of restart on the age of the nth record for generic time series with nearest-neighbor transitions. Crucially, the criterion contains a penalty of order n that puts strong constraints on restart expediting the creation of records, as compared to the simple first-passage completion. The applicability of our approach is further demonstrated on an aggregation-shattering process where we compute the typical growth rates of aggregate sizes. This unified framework paves the way to explore record statistics of time series under restart in a wide range of complex systems.

DOI: 10.1103/PhysRevLett.130.157101

Introduction.—How long will it take for the price of a stock to cross its current all-time-high value? When will another human being cover 100 meters faster than Usain Bolt? These questions pertain to computing record ages, a quantity that lies at the heart of the subject of record statistics [1–6]. The study of record-breaking events has generated immense research interest since the pioneering work of Chandler in 1952 [7], owing to its applications in fields including finance [8–10], climate studies [11–14], hydrology [15], sports [16,17], and also physics [18–23].

The prototypical setting in the study of records consists of a discrete time series $\bar{x} = \{x_0, x_1, x_2, \ldots\}$, where the entries could represent the daily temperatures of a city, the number of people infected in a day during a pandemic, or any other observable of interest that is being measured at discrete time points. The jth entry, x_j, of the time series $\bar{x}$ is called a record if its numerical value exceeds the values of all preceding entries, x_i, i.e., $x_j > x_i$ for all $i < j$. Important insight into the persistence of a record x_i is obtained through the record age $L(x_i)$, which denotes the number of time steps needed for a new record to be created, after x_i. Concretely, for two consecutive records x_i and x_j, the record age $L(x_i)$ is defined to be $j - i$, as depicted in Fig. 1(a), where the two yellow symbols denote record events.

While most of the efforts have been focused toward studying the case where the entries of the time series $\bar{x}$ are independent and identically distributed random variables, oftentimes the entries obtained from real-world scenarios are in fact correlated. Moreover, as we live in a world where global catastrophes seem to be inevitable, consequently their signatures appear in most datasets of practical relevance, with examples including a sudden fall in the price of a stock [24], a sharp layoff of individual jobs due to postpandemic recession, or a massive extinction of population due to catastrophe [25]. This ubiquitous feature of resetting events is not limited to economics [24,26], operations research [27], or ecology [28]. It can also be observed in microscopic out-of-equilibrium physical [29–33], chemical [34,35], or biological [36,37] systems. More recently, restart has also emerged as an efficient strategy

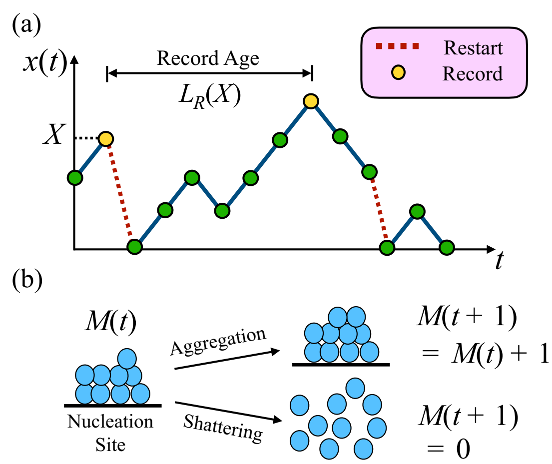

FIG. 1. (a) Schematic for an observable $x(t)$ that evolves and undergoes stochastic resetting. Yellow points denote record events (X). Dashed red lines denote restart events, and blue lines denote nonrestart transitions in the evolution of the process. (b) A mass aggregation-shattering process. Masses aggregate on a nucleation site until a shattering event occurs, resetting the mass to zero. Aggregation resumes and continues to grow until the next shattering event.

to speed up complex search processes with potential applications in optimization problems [38–41] and search theory [28,29,31,42–55]. A natural question then arises: How do such restart events ramify the record statistics—in particular, the record ages? Quite remarkably, our answer to this question also sheds light on a seemingly unrelated problem, namely the lifetime statistics in the mass aggregation-shattering models (see Fig. 1).

The central theme of this Letter is to build a unified formalism that allows us to obtain record ages for time series generated by arbitrary stochastic processes that are subjected to intermittent collapse-restart events. Employing ideas and techniques from the first passage under restart description [44,56,57], we distill the core principles that underpin the universal behavior of record ages under arbitrary restart. This allows us to probe record ages in a very generic setting covering both Markov and non-Markov processes, with minimal assumptions. In particular, we derive a universal criterion that dictates the effect of restart on the record ages. Notably, the statistics of the number of records (average properties) have been studied recently for random walk (RW) models under the assumption of geometric restart steps [58,59]. However, the observable of interest herein is the record ages that have not been studied hitherto. After demonstrating the formalism for a biased RW, we apply it to the widely applicable aggregation-fragmentation models [Fig. 1(b)]. To be specific, we compute the growth rate for mass aggregates that requires us to generalize the formalism to arbitrary shattering and restart events that are not necessarily rate limiting process but can also have intrinsic temporal heterogeneity.

General formalism.—We start by considering an extremely general case, where we have an arbitrary discrete time series $\bar{x} = \{x_1, x_2, x_3 \ldots\}$ generated by a stochastic process. Corresponding to this time series, we have the set of records $\bar{X} = \{X_1, X_2, X_3, \ldots\}$, where X_i denotes the numerical value of the ith record-breaking event in the time series $\bar{x}$. For each X_i, we define the record age $L(X_i)$ to be the time taken for the next record-breaking event X_{i+1} to occur following X_i [60].

Now, suppose the stochastic process generating the time series $\bar{x}$ is subjected to random restart events, whose occurrence bring the numerical value of the subsequent entry in the time series to a predetermined value that is assumed to be 0. Note, however, a generalization to this assumption (i.e., restart from another arbitrary value or from an ensemble) is feasible within our framework. Let us denote by X an entry that is a record-breaking event in the time series generated by the stochastic process under restart events. For simplicity, let us assume that these restart events take place after some geometrically distributed random time step R (generalization to arbitrary distributions is considered later). The age of the record X (under restart) is denoted by $L_R(X)$. If the record-breaking event subsequent to the formation of record X occurs prior to any restart, we have $L_R(X) = L(X)$. Otherwise, the process resets to 0 after time R, and from there the resultant process has to be observed until it crosses the record X. Combining these two possibilities, one has

$$L_R(X) = \begin{cases} L(X) & \text{if } L(X) < R \\ R + T^R_{X,0} & \text{otherwise,} \end{cases} \tag{1}$$

where $T^R_{X,0}$ is the time taken for the time series to cross the threshold X for the first time, given that it starts from 0, in the presence of restart events. Equation (1) is central to our analysis. Indeed, noting that $L_R(X) = \min\{L(X), R\} + \mathbb{1}[R \le L(X)]T^R_{X,0}$, where $\min\{z_1, z_2\}$ is the minimum of z_1 and z_2 and $\mathbb{1}(z_1 \le z_2)$ is an indicator random variable that is unity if $z_1 \le z_2$ and zero otherwise, we find the mean record age as follows:

$$\langle L_R(X)\rangle = \langle \min\{L(X), R\}\rangle + \Pr[R \le L(X)]\langle T^R_{X,0}\rangle, \tag{2}$$

where $\langle T^R_{X,0}\rangle$ is the mean first-passage time under restart [56,57,62]. Given the statistics of individual terms, one can then compute the mean record age using Eq. (2). Notably, Eq. (1) serves as a backbone to provide the full statistics of the record age, which is also a perceived challenge. To gain further insights, we first illustrate our formalism on the 1D lattice RW and then show how the generalized theory applies to more complex scenarios.

Random walks on 1D lattice.—A major advancement in our understanding of record statistics beyond independent and identically distributed random variables has come through the examples of RW that were popularized following Pólya's seminal work [63]. A major advantage of using random walk models is that we can gain much insight by solving them analytically [64–67]. To proceed further, we assume that the 1D RW evolves with the dynamics $x_i = x_{i-1} + \eta_{i-1}$, where x_i denotes the position of the RW at ith step and η_i is the increment. The walker is biased so that $\eta_i = +1$ with probability p, and $\eta_i = -1$ with probability $1-p$, for all i. Positions of the RW (x_i) represent a *strongly correlated* time series. Furthermore, the walker experiences sharp transitions with probability r to the origin after which it restarts its dynamics [42,56].

For the case of random walk on a 1D lattice with nearest-neighbor jumps, the time series of the position of the walker $\bar{x}$ is a sequence of integers, and consequentially the same holds for the sequence of records $\bar{X}$. We denote by $L_R(n)$ the time taken for a record-breaking event to occur, after the last record was created at position n. In the absence of resetting, the record age $L(n)$ is simply $T_{n+1,n}$—the first-passage time to go from n to $n+1$, and it is independent of n. However, restart introduces an inherent heterogeneity in the problem so that the record ages depend on the record number or the magnitude of the last record. To see this, we first obtain the mean from Eq. (2):

$$\langle L_R(n)\rangle = \langle \min\{T_{n+1,n}, R\}\rangle + \Pr[R \le T_{n+1,n}]\langle T^R_{n+1,0}\rangle, \tag{3}$$

where each component of the rhs can be computed given the distribution of T and R. In particular, for geometric distribution of restart steps, we have [61]

$$\langle L_R(n)\rangle = \frac{\tilde{Q}_{1-r}(n+1|n)}{1 - r\tilde{Q}_{1-r}(n+1|0)}, \tag{4}$$

where $\tilde{Q}_z(i|j) = \sum_{k=0}^{\infty} z^k Q_k(i|j)$ is the generating function of the survival probability $Q_k(i|j)$ that denotes the probability for a RW starting from a site $j < i$ and *not* reaching i until the kth time step. It is important to note that $\langle L_R(n)\rangle$ is expressed solely in terms of the survival properties for the bare process. Furthermore, the survival probability equals $\tilde{Q}_z(i|j) = \{[1 - \tilde{F}_z(i|j)]/(1-z)\}$, where $\tilde{F}_z(i|j)$ denotes the generating function of the first-passage time distribution $F_k(i|j) = \mathrm{Prob}[T_{i,j} = k]$ that the walker starts from state j and reaches i for the first time exactly in k steps. For the biased RW, this can be expressed as [65]

$$\tilde{F}_z(i|j) = \left(\frac{1 - \sqrt{1 - 4p(1-p)z^2}}{2(1-p)z}\right)^{i-j} \tag{5}$$

for $i - j > 0$. Replacing the expressions in Eq. (4), one finds the mean record age for the RW.

Several comments are in order now. In the case of symmetric walkers for the bare process ($p = 1/2$, $r = 0$), the mean record age is infinite, and any restart probability $r < 1$ would render the mean finite. In other words, collapse-restart events will always expedite record-breaking events for symmetric RW. However, this need not be the case in general as restart could also result in longer record ages. Consider, for example, the biased RW, where the mean record ages are finite for the bare process. Thus, it is essential to pinpoint the transition point that can be understood by the introduction of an infinitesimal resetting probability. Indeed, expanding mean record age Eq. (4) with respect to $\delta r \to 0$, one finds $\langle L_{\delta r}(n)\rangle = \langle L(n)\rangle - (\delta r/2)\langle T_{n+1,n}\rangle^2\{CV^2_{n+1,n} + [(\langle T_{n+1,n}\rangle - 1 - 2\langle T_{n+1,0}\rangle)/\langle T_{n+1,n}\rangle]\}$, where $CV^2_{n+1,n} = \mathrm{Var}(T_{n+1,n})/\langle T_{n+1,n}\rangle^2$ is the squared coefficient of variation of $T_{n+1,n}$. For restart to reduce the record age, one should have $\langle L_{\delta r}(n)\rangle < \langle L(n)\rangle$, resulting in [61]

$$CV^2_{n+1,n} > \frac{2\langle T_{n+1,0}\rangle + 1}{\langle T_{n+1,n}\rangle} - 1. \tag{6}$$

Equation (6) remarkably holds for any underlying stochastic process, and sets up a universal criterion for the effect of restart on record ages. In the paradigmatic case of biased RW, the criterion in Eq. (6) reduces to

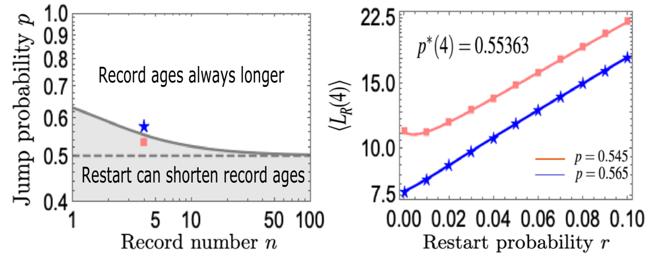

FIG. 2. Left: Phase diagram for record ages in a biased RW on a 1D lattice. Right: Mean age of the 4th record $\langle L_R(4)\rangle$ as a function of restart probability r for $p = 0.565[> p^*(4)]$ (blue) and $p = 0.545[< p^*(4)]$ (orange), where $p^*(n)$ is the value of the bias p (taken from the phase separatrix on the left panel for a given n [see Sec. S2 in [61]]), beyond which restart cannot shorten record ages. Here, $p^*(4) = 0.55363$. The solid lines are obtained from our *analytical* formula, while symbols represent values obtained from *numerical simulations*.

$$CV^2_{n+1,n} > 2n + \left\{1 + \frac{1}{\langle T_{n+1,n}\rangle}\right\}, \tag{7}$$

where the second term on the rhs is the criterion for the mean first passage solely [57]. Thus, the additional term of $2n$ corresponds to a "penalty" for resetting to a point further away from the target, compared to the initial condition, setting up a very strict criterion on the relative fluctuations of the underlying first-passage process in order for restart to expedite record-breaking events. For large n, the criterion is dominated by the penalty term $2n$, as both $\langle T_{n+1,n}\rangle$ and $CV^2_{n+1,n}$ are independent of n, resulting in an invalid inequality. Thus, restart never shortens the record ages for large n. Based on Eq. (7), in Fig. 2(a) we illustrate the particular phase space region spanned by p and n where restart can expedite the creation of records (gray shaded). Note that for values of p below the dashed line ($p = 0.5$), restart renders $\langle L_R(n)\rangle$ finite for all n, and thus *always* leads to shorter record ages. In the right panel, we further plot $\langle L_R(n)\rangle$ for $n = 4$, as a function of restart probability r for two different values of p: (i) $p = 0.565$, which lies above the critical value $p^*(4) \approx 0.55363$ beyond which Eq. (7) is not satisfied for $n = 4$ [61] and (ii) $p = 0.545$, which lies below the critical value $p^*(4)$, demonstrating the validity of the criterion.

Record ages under arbitrary restart step.—So far, we had restricted our discussion to geometric restarts. However, while going beyond this Markovian case is an important step, as evident through the first-passage literature [44,68–70], it is also quite challenging. The key issue here is to know the statistics of the time required for a restart event to occur right after a record. While for Markovian setup, this time coincides with the restart time (R) itself (due to the memoryless property of geometric restart events), it is generically different for arbitrary restart steps (see Fig. 3 for a timeline illustration).

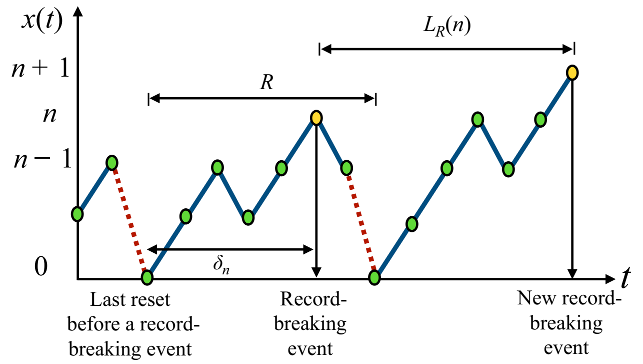

FIG. 3. Schematic for record age statistics under nongeometric restarts. Unlike the Markovian case, the time taken for a restart event to occur after the creation of a record (in this figure, a record is created when the time series reaches n for the first time) is not simply given by the distribution of R, and instead is given by $R_{\delta_n} = R - \delta_n$.

For such a stochastic process, we can identify a renewal structure for the record ages as the following:

$$L_R(n) = \begin{cases} L(n), & \text{if } L(n) < R_{\delta_n} \\ R_{\delta_n} + T^R_{n+1,0} & \text{otherwise}, \end{cases} \tag{8}$$

where $R_{\delta_n} = \{R - \delta_n | R > \delta_n\}$ is the forward recurrence time, and δ_n is the backward recurrence time so that $\delta_n = \{T_{n,0} | T_{n,0} < R\}$ and $R_{\delta_n} + \delta_n = R$ (see Fig. 3). The latter is distributed according to

$$P_{\delta_n}(k) = F_k(n|0) \frac{\sum_{m=k+1}^{\infty} P_R(m)}{\Pr(T_{n,0} < R)}, \tag{9}$$

where $P_R(m)$ is the restart time density (not necessarily geometric). We stress that while Eq. (8) is written in terms of the variable n, keeping in mind discrete-state stochastic processes (e.g., RW), generalization to continuous state processes is straightforward.

For geometrically distributed restarts, R_{δ_n} and R have statistically identical distribution and hence one recovers Eq. (1). However, generically, R_{δ_n} pertains to a different distribution [61],

$$P_{R_{\delta_n}}(k) = \frac{\Pr(R - T_{n,0} = k)}{\Pr(R > T_{n,0}) \cdot \Pr(R > \delta_n)}, \tag{10}$$

where k takes strictly positive values. Together, Eqs. (8), (9), and (10) allow us to write a closed set of equations to obtain the record age statistics of a time series generated by an arbitrary stochastic process that undergoes possibly nongeometric restarts. For instance, the mean record age reads

$$\langle L_R(n) \rangle = \langle \min\{L(n), R_{\delta_n}\} \rangle + \Pr[R_{\delta_n} \le L(n)] \langle T^R_{n+1,0} \rangle, \tag{11}$$

where we show that the mean record age under a nongeometric restart protocol can be expressed completely in terms of quantities related to the underlying process. This property holds also for all the subsequent moments of $L_R(n)$.

Aggregation-shattering processes.—An important application of studying nongeometric (non-Markovian) restarts arises in the study of aggregation-fragmentation-shattering processes. Apart from being a paradigmatic model to probe nonequilibrium behavior [71–75], models of aggregation and fragmentation have found diverse applications, ranging from modeling socioeconomic phenomena [76,77] and neurodegenerative diseases [78–80] to explaining the particle size distribution in Saturn's rings [81], the distribution of sizes of animal groups in nature [82–84], and raindrops [85]. In particular, in the case of neurodegenerative diseases, where it is argued that diseases like Alzheimer's or Parkinson's disease are caused by the pathological aggregation of certain proteins, it is suggested that some clearance mechanisms must also be at play, which keep these proteins from forming large aggregates in healthy individuals. These clearance mechanisms play the role of "shattering," which bring down the size of aggregates [78]. Figure 1(b) is a schematic of such an aggregation-shattering process, where masses arrive, possibly nongeometrically, on a nucleation site, and form a larger aggregate. However, shattering of the aggregate can reset the mass at the nucleation site to zero.

Let us consider a time series $M(t)$, which tracks the size of the aggregate at the nucleation site. Clearly, $M(t)$ is a stochastic process that undergoes restarts at random times. To delve deeper, let us assume that the interarrival times between two masses or monomers follow a geometric distribution. Upon the arrival of a monomer, it sticks to the cluster of masses at the nucleation site (an aggregation event), leading to an increase in the mass at the site by unit one. However, "clearance" occurs at random times following possibly nongeometric distributions indicating a restart protocol with temporal memory. Clearance leads to the shattering of the cluster at the nucleation site, rendering $M(t) = 0$ in that time step. In this context, record age

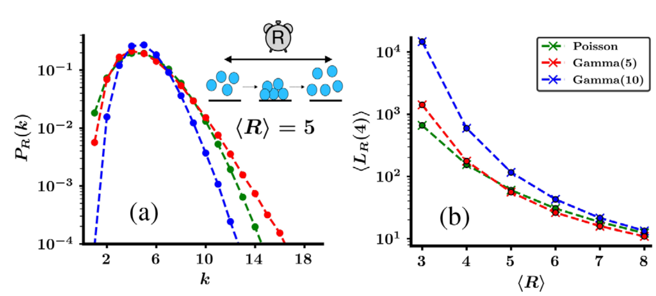

FIG. 4. Mean record age in aggregation-shattering process. (a) Different non-Markovian restart distributions characterizing the time between shattering events. (b) Variation of $\langle L_R(4) \rangle$, under the restart distributions plotted in (a), as a function of mean restart time. In panel (b), the cross symbols denote our numerical results and circles denote the theoretical estimates from Eq. (11).

statistics of the aggregate size carries valuable insight into the lifetime of these masses, and the rate at which they grow.

In Fig. 4 we demonstrate the dependence of $\langle L_R(4)\rangle$, i.e., the time until the formation of a mass aggregate with $n = 5$ units after an aggregate of size 4 has been created for the first time, as a function of the mean shattering time $\langle R\rangle$, for nongeometric shattering times (drawn from Poisson distribution and discretized Gamma distribution [61]). The plot shows an excellent agreement between our theoretical prediction from Eq. (11) and the simulations. It is evident that shattering (restart) events slow down the process of creation of records, which concurs with our physical intuition. In [61], we show how this is consistent with the criterion derived in Eq. (6) and discuss other contrasting scenarios where such shattering like mechanisms can expedite the creation of records despite the underlying time series taking only non-negative values. Furthermore, note that for higher mass index (i.e., increasing n), the record age $\langle L_R(n)\rangle$ also keeps increasing, which is a highlighted feature as well.

Conclusions.—Record statistics has been a longstanding focal point of research due to its numerous interdisciplinary applications that go beyond physics. In this Letter, we focus on understanding record statistics of a time series that may contain signatures of catastrophes or sharp intermittent changes in the observed values. Modeling these events as restart, we build a unified framework to estimate record age statistics in a generic scenario. As such, our framework advances in encompassing arbitrary stochastic processes that undergo general non-Markovian restart events. Quite importantly, our framework reveals a universal criterion, Eq. (6), that can predict the conditions under which restart could shorten the mean record ages of stochastic time series with nearest-neighbor transitions. The application of this criterion is demonstrated not only for RW models where the underlying variable can be both positive and negative, but also for the mass aggregation model where the random variable remains strictly non-negative (see Ref. [61] for additional discussion).

Our work brings forward new insights on the intricate interplay between the inherent stochasticity pertaining to the system and the restart events. Although the focus has been on the average quantities, such ideas can also be extended to study fluctuations and higher moments. Finally, we highlight that sharp catastrophe in time series [24] is a key signature of extreme events [86–88] across complex systems. Thus, our formalism paves the way for building an improved understanding of rare events in natural systems and their consequences.

The authors gratefully acknowledge M. S. Santhanam for fruitful discussions. A. K. acknowledges the Prime Minister's Research Fellowship of the Government of India for financial support. A. P. gratefully acknowledges research support from the Department of Science and Technology, India, SERB Start-up Research Grant No. SRG/2022/000080 and Department of Atomic Energy, India.

*Corresponding author.
arnabpal@imsc.res.in

[1] V. B. Nevzorov, Theory Probab. Appl. **32**, 201 (1988).
[2] S. Gulati and W. J. Padgett, *Parametric and Nonparametric Inference from Record-Breaking Data* (Springer Science & Business Media, Berlin, 2003), Vol. 172.
[3] G. Wergen, J. Phys. A **46**, 223001 (2013).
[4] G. Schehr and S. N. Majumdar, in *First-Passage Phenomena and Their Applications* (World Scientific, Singapore, 2013), pp. 226–251.
[5] C. Godrèche, S. N. Majumdar, and G. Schehr, J. Phys. A **50**, 333001 (2017).
[6] S. Sabhapandit, arXiv:1907.00944.
[7] K. N. Chandler, J. R. Stat. Soc. Ser. B **14**, 220 (1952).
[8] G. Wergen, M. Bogner, and J. Krug, Phys. Rev. E **83**, 051109 (2011).
[9] B. Sabir and M. S. Santhanam, Phys. Rev. E **90**, 032126 (2014).
[10] M. Santhanam and A. Kumar, in *Econophysics and Sociophysics: Recent Progress and Future Directions* (Springer, New York, 2017), pp. 103–112.
[11] D. V. Hoyt, Clim. Change **3**, 243 (1981).
[12] B. Schmittmann and R. K. P. Zia, Am. J. Phys. **67**, 1269 (1999).
[13] R. E. Benestad, Clim. Res. **25**, 3 (2003).
[14] S. Redner and M. R. Petersen, Phys. Rev. E **74**, 061114 (2006).
[15] R. M. Vogel, A. Zafirakou-Koulouris, and N. C. Matalas, Water Resour. Res. **37**, 1723 (2001).
[16] D. Gembris, J. G. Taylor, and D. Suter, Nature (London) **417**, 506 (2002).
[17] D. Gembris, J. G. Taylor, and D. Suter, J. Appl. Stat. **34**, 529 (2007).
[18] B. Alessandro, C. Beatrice, G. Bertotti, and A. Montorsi, J. Appl. Phys. **68**, 2901 (1990).
[19] S. Sabhapandit, Europhys. Lett. **94**, 20003 (2011).
[20] S. N. Majumdar, G. Schehr, and G. Wergen, J. Phys. A **45**, 355002 (2012).
[21] C. Godrèche, S. N. Majumdar, and G. Schehr, J. Phys. A **47**, 255001 (2014).
[22] C. Godrèche, S. N. Majumdar, and G. Schehr, J. Stat. Mech. (2015) P07026.
[23] S. N. Majumdar, P. von Bomhard, and J. Krug, Phys. Rev. Lett. **122**, 158702 (2019).
[24] V. Stojkoski, P. Jolakoski, A. Pal, T. Sandev, L. Kocarev, and R. Metzler, Phil. Trans. R. Soc. A **380**, 20210157 (2022).
[25] A. Di Crescenzo, V. Giorno, A. G. Nobile, and L. M. Ricciardi, Stat. Probab. Lett. **78**, 2248 (2008).
[26] X. Gabaix, J.-M. Lasry, P.-L. Lions, and B. Moll, Econometrica **84**, 2071 (2016).
[27] O. L. Bonomo, A. Pal, and S. Reuveni, PNAS Nexus **1**, pgac070 (2022).
[28] A. Pal, L. Kusmierz, and S. Reuveni, Phys. Rev. Res. **2**, 043174 (2020).

[29] M. R. Evans and S. N. Majumdar, Phys. Rev. Lett. **106**, 160601 (2011).
[30] A. Pal, Phys. Rev. E **91**, 012113 (2015).
[31] M. R. Evans, S. N. Majumdar, and G. Schehr, J. Phys. A **53**, 193001 (2020).
[32] D. Gupta, C. A. Plata, and A. Pal, Phys. Rev. Lett. **124**, 110608 (2020).
[33] S. Gupta, S. N. Majumdar, and G. Schehr, Phys. Rev. Lett. **112**, 220601 (2014).
[34] S. Reuveni, M. Urbakh, and J. Klafter, Proc. Natl. Acad. Sci. U.S.A. **111**, 4391 (2014).
[35] T. Robin, S. Reuveni, and M. Urbakh, Nat. Commun. **9**, 1 (2018).
[36] É. Roldán, A. Lisica, D. Sánchez-Taltavull, and S. W. Grill, Phys. Rev. E **93**, 062411 (2016).
[37] S. Budnar, K. B. Husain, G. A. Gomez, M. Naghibosadat, A. Varma, S. Verma, N. A. Hamilton, R. G. Morris, and A. S. Yap, Dev. Cell **49**, 894 (2019).
[38] M. Luby, A. Sinclair, and D. Zuckerman, Inf. Proc. Lett. **47**, 173 (1993).
[39] A. Montanari and R. Zecchina, Phys. Rev. Lett. **88**, 178701 (2002).
[40] C. P. Gomes, B. Selman, H. Kautz *et al.*, AAAI/IAAI **98**, 431 (1998).
[41] J. Huang *et al.*, in *The International Joint Conferences on Artificial Intelligence.* (Morgan Kaufmann Publishers, San Francisco, 2007), Vol. 7, pp. 2318–2323.
[42] L. Kusmierz, S. N. Majumdar, S. Sabhapandit, and G. Schehr, Phys. Rev. Lett. **113**, 220602 (2014).
[43] S. Reuveni, Phys. Rev. Lett. **116**, 170601 (2016).
[44] A. Pal and S. Reuveni, Phys. Rev. Lett. **118**, 030603 (2017).
[45] S. Ray, D. Mondal, and S. Reuveni, J. Phys. A **52**, 255002 (2019).
[46] A. Pal and V. V. Prasad, Phys. Rev. E **99**, 032123 (2019).
[47] A. Chechkin and I. Sokolov, Phys. Rev. Lett. **121**, 050601 (2018).
[48] S. Gupta and A. M. Jayannavar, Front. Phys. **10**, 130 (2022).
[49] P. Singh and A. Pal, Phys. Rev. E **103**, 052119 (2021).
[50] B. De Bruyne, J. Randon-Furling, and S. Redner, Phys. Rev. Lett. **125**, 050602 (2020).
[51] F. Huang and H. Chen, Phys. Rev. E **103**, 062132 (2021).
[52] A. Ray, A. Pal, D. Ghosh, S. K. Dana, and C. Hens, Chaos **31**, 011103 (2021).
[53] A. P. Riascos, D. Boyer, P. Herringer, and J. L. Mateos, Phys. Rev. E **101**, 062147 (2020).
[54] Y. Ye and H. Chen, J. Stat. Mech. (2022) 053201.
[55] D. Campos and V. Méndez, Phys. Rev. E **92**, 062115 (2015).
[56] O. L. Bonomo and A. Pal, Phys. Rev. E **103**, 052129 (2021).
[57] O. L. Bonomo and A. Pal, arXiv:2106.14036.
[58] S. N. Majumdar, P. Mounaix, S. Sabhapandit, and G. Schehr, J. Phys. A **55**, 034002 (2021).
[59] C. Godrèche and J.-M. Luck, J. Stat. Mech. (2022) 063202.
[60] We note that $L(X_i)$ differs from another working definition of record age ℓ_i, which is defined as the time taken for the $(i+1)$th record to be created, after the creation of the ith record. Notably, the latter definition does not depend on the record magnitude *per se*, while in our case it does. See Supplemental Material [61] for further discussions.
[61] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.130.157101 for detailed additional derivations and numerical results along with other related discussions.
[62] J. M. Flynn and S. S. Pilyugin, arXiv:2204.07422.
[63] G. Pólya, Math. Ann. **84**, 149 (1921).
[64] E. W. Montroll and G. H. Weiss, J. Math. Phys. (N.Y.) **6**, 167 (1965).
[65] J. Klafter and I. M. Sokolov, *First Steps in Random Walks: From Tools to Applications* (OUP, Oxford, 2011).
[66] B. D. Hughes, *Random Walks and Random Environments: Random Walks* (Oxford University Press, New York, 1995), Vol. 1.
[67] L. Giuggioli, Phys. Rev. X **10**, 021045 (2020).
[68] A. Pal, A. Kundu, and M. R. Evans, J. Phys. A **49**, 225001 (2016).
[69] V. P. Shkilev, Phys. Rev. E **96**, 012126 (2017).
[70] A. S. Bodrova and I. M. Sokolov, Phys. Rev. E **101**, 062117 (2020).
[71] P. L. Krapivsky, S. Redner, and E. Ben-Naim, *A Kinetic View of Statistical Physics* (Cambridge University Press, Cambridge, England, 2010).
[72] P. L. Krapivsky, W. Otieno, and N. V. Brilliantov, Phys. Rev. E **96**, 042138 (2017).
[73] S. A. Matveev, P. L. Krapivsky, A. P. Smirnov, E. E. Tyrtyshnikov, and N. Brilliantov, Phys. Rev. Lett. **119**, 260601 (2017).
[74] A. S. Bodrova, V. Stadnichuk, P. L. Krapivsky, J. Schmidt, and N. V. Brilliantov, J. Phys. A **52**, 205001 (2019).
[75] N. V. Brilliantov, W. Otieno, and P. L. Krapivsky, Phys. Rev. Lett. **127**, 250602 (2021).
[76] S. Ispolatov, P. L. Krapivsky, and S. Redner, Eur. Phys. J. B **2**, 267 (1998).
[77] D. T. Robson, A. C. Baas, and A. Annibale, J. Stat. Mech. (2021) 053203.
[78] T. B. Thompson, G. Meisl, T. Knowles, and A. Goriely, J. Chem. Phys. **154**, 125101 (2021).
[79] Y. Wang, M. Martinez-Vicente, U. Krüger, S. Kaushik, E. Wong, E.-M. Mandelkow, A. M. Cuervo, and E. Mandelkow, Hum. Mol. Genet. **18**, 4153 (2009).
[80] S. Fornari, A. Schäfer, E. Kuhl, and A. Goriely, J. Theor. Biol. **486**, 110102 (2020).
[81] N. Brilliantov, P. L. Krapivsky, A. Bodrova, F. Spahn, H. Hayakawa, V. Stadnichuk, and J. Schmidt, Proc. Natl. Acad. Sci. U.S.A. **112**, 9536 (2015).
[82] H.-S. Niwa, J. Theor. Biol. **224**, 451 (2003).
[83] Q. Ma, A. Johansson, and D. J. T. Sumpter, J. Theor. Biol. **283**, 35 (2011).
[84] G. G. Nair, A. Senthilnathan, S. K. Iyer, and V. Guttal, Phys. Rev. E **99**, 032412 (2019).
[85] R. Srivastava, J. Atmos. Sci. **39**, 1317 (1982).
[86] S. N. Majumdar, A. Pal, and G. Schehr, Phys. Rep. **840**, 1 (2020).
[87] V. Kishore, M. S. Santhanam, and R. E. Amritkar, Phys. Rev. Lett. **106**, 188701 (2011).
[88] N. Malik and U. Ozturk, Chaos **30**, 090401 (2020).