Spin-Liquid Insulators Can Be Landau's Fermi Liquids

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The long search for insulating materials that possess low-energy quasiparticles carrying electron's quantum numbers except charge—inspired by the neutral spin-$1/2$ excitations, the so-called spinons, exhibited by Anderson's resonating-valence-bond state—seems to have reached a turning point after the discovery of several Mott insulators displaying the same thermal and magnetic properties as metals, including quantum oscillations in a magnetic field. Here, we show that such anomalous behavior is not inconsistent with Landau's Fermi liquid theory of quasiparticles at a Luttinger surface. That is the manifold of zeros within the Brillouin zone of the single-particle Green's function at zero frequency, and which thus defines the spinon Fermi surface conjectured by Anderson.

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Common sense would suggest that Mott insulators and Landau's Fermi liquids are antinomic phases of matter that can turn one into the other only through a Mott transition. However, there is growing, intriguing evidence of *quasiparticlelike* excitations in some Mott insulating materials. For instance, the Kondo insulators SmB_6 and YbB_{12} show quantum oscillations in a magnetic field, finite specific heat, C_v/T, and thermal conductivity, κ/T, coefficients for $T \to 0$ [1–6], though $\kappa \sim T$ is still debated in SmB_6 [7,8]. Evidence of finite C_v/T and κ/T for $T \to 0$ is also found in candidate spin-liquid insulators: $1T$-TaS_2 [9–11], and, with some caveats, in the organic salts $EtMe_3Sb[Pd(dmit)_2]_2$ [12–17] and κ-$(BEDT\text{-}TTF)_2Cu_2(CN)_3$ [18,19]. Quantum oscillations in the magnetothermal conductivity of the field induced spin-liquid state of α-$RuCl_3$ have also been reported [20], even though their origin is controversial [21].

All the above properties, at odds with the conventional view of insulators, are commonly interpreted by the existence of *neutral quasiparticles* [22–28], not necessarily gapless [26], although alternative explanations have been proposed [29–31]. Those quasiparticles are dubbed *spinons* [32,33] when they only carry the spin quantum number, which is the case of systems whose low energy behavior is determined by just a single band, as we shall assume hereafter.

Despite the observed Fermi-liquid-like thermal and magnetic properties of spinons, their emergence from spin-charge deconfinement [34] is at first sight incompatible with Landau's Fermi liquid theory [35–37]. This is obviously the case of conventional Landau's quasiparticles at a Fermi surface, the location of poles of the single-particle Green's function at zero frequency and temperature, since these poles entail metallicity. However, it has been recently shown [38] that Landau's quasiparticles also exist at a Luttinger surface, the manifold of zeros of the single-particle Green's function at zero frequency and temperature. These quasiparticles are invisible in the single-particle spectrum, and are also incompressible [39], thus perfectly allowed in insulators. Nonetheless, the insulating character poses constraints to Landau's Fermi liquid theory, most notably the vanishing of Drude weight and of charge compressibility. Here, we show that these constraints can be fulfilled. We conclude that a Landau Fermi liquid can well be insulating, and analyze its physical properties with special emphasis on the quantum oscillations in a magnetic field.

Uncovering Landau quasiparticles.—In what follows, we consider a periodic model with a single band of interacting electrons, and assume that neither translational symmetry nor a spin rotational one are broken. The single-particle Green's function is therefore diagonal in momentum $\mathbf{k}$ and spin $\sigma = \uparrow, \downarrow$, and independent of the latter. In Matsubara frequencies, $\epsilon = (2n+1)\pi T$, the Green's function satisfies Dyson's equation

$$G(i\epsilon, \mathbf{k}) = \frac{1}{i\epsilon - \epsilon(\mathbf{k}) - \Sigma(i\epsilon, \mathbf{k})}, \tag{1}$$

where $\epsilon(\mathbf{k})$ is the noninteracting energy dispersion in momentum space measured with respect to the chemical potential, and $\Sigma(i\epsilon, \mathbf{k})$ the self-energy that, like $G(i\epsilon, \mathbf{k})$, has a real part even in ϵ, while

$$\mathrm{Im}\Sigma(i\epsilon, \mathbf{k}) = -\mathrm{Im}\Sigma(-i\epsilon, \mathbf{k}) \begin{cases} < 0 & \epsilon > 0, \\ > 0 & \epsilon < 0. \end{cases} \tag{2}$$

We define the real function

$$Z(\epsilon, \mathbf{k}) = Z(-\epsilon, \mathbf{k}) = \left(1 - \frac{\mathrm{Im}\Sigma(i\epsilon, \mathbf{k})}{\epsilon}\right)^{-1}, \tag{3}$$

which, because of (2), varies in the interval [0, 1]. Through $Z(\epsilon, \mathbf{k})$ we can rewrite Eq. (1) as

$$G(i\epsilon, \mathbf{k}) = \frac{Z(\epsilon, \mathbf{k})}{i\epsilon - \epsilon_*(\epsilon, \mathbf{k})}, \tag{4}$$

with real

$$\epsilon_*(\epsilon, \mathbf{k}) = \epsilon_*(-\epsilon, \mathbf{k}) = Z(\epsilon, \mathbf{k})(\epsilon(\mathbf{k}) + \mathrm{Re}\Sigma(i\epsilon, \mathbf{k})). \tag{5}$$

Landau's Fermi liquid theory can be formally derived under the assumption that $\epsilon_*(\epsilon, \mathbf{k})$ and $Z(\epsilon, \mathbf{k})$ are analytic, at least to leading order, in ϵ around $\epsilon = 0$, as well as in $\mathbf{k}$ close to the surface defined by $\epsilon_*(0, \mathbf{k}) = 0$ [38]. This assumption is equivalent to assuming that $\Sigma(i\epsilon, \mathbf{k})$ is analytic at any nonzero ϵ, which includes conventional Fermi liquids as the special case of $\Sigma(i\epsilon, \mathbf{k})$ analytic also at $\epsilon = 0$, but also allows for poles of $\Sigma(i\epsilon, \mathbf{k})$ for $\epsilon \to 0$.

The actual *quasiparticles* have energy dispersion $\epsilon_*(\mathbf{k}) \equiv \epsilon_*(0, \mathbf{k})$ and residue $Z(\mathbf{k}) \equiv Z(0, \mathbf{k})$. The roots of $\epsilon_*(\mathbf{k})$ in momentum space define the *quasiparticle Fermi surface* that, because of the definition (5), correspond (i) either to the roots of $\epsilon(\mathbf{k}) + \mathrm{Re}\Sigma(0, \mathbf{k})$, the conventional Fermi surface, (ii) or those of $Z(0, \mathbf{k})$, the so-called Luttinger surface [40]. Therefore, well-defined quasiparticles exist at Fermi as well at Luttinger surfaces, and that despite the vanishing quasiparticle residue $Z(\mathbf{k})$ at the Luttinger surface implies the absence of quasiparticle peaks in the physical electron density of states.

Fermi liquid properties.—We recall that Landau's Fermi liquid theory allows calculating linear response functions at low temperature, low frequency, and long wavelength in terms of two unknown functions: the quasiparticle dispersion $\epsilon_*(\mathbf{k})$ and the Landau parameters $f_{\mathbf{k}\sigma,\mathbf{k}'\sigma'}$, where σ and σ' are the spins of the quasiparticles with momentum $\mathbf{k}$ and $\mathbf{k}'$, respectively. In reality, this huge simplification just applies to densities of conserved quantities and their currents defined through the continuity equation. Indeed, only in those cases one can exploit the Ward-Takahashi identities and relate vertex to self-energy corrections [36].

In a single-band periodic model, the conserved quantities are the electron number $N = N_\uparrow + N_\downarrow$, the energy E, and the magnetization along a given axis, e.g., $M = N_\uparrow - N_\downarrow$. We denote by $\chi_{\rho_Q}(\omega, \mathbf{q})$ and $\chi_{J_Q}(\omega, \mathbf{q})$, the proper response functions, respectively, of the density, ρ_Q, and current, J_Q, operators associated to the conserved quantity $Q = N$, E, M, i.e., the response functions irreducible with respect to cutting a Coulomb interaction line. The thermodynamic susceptibilities are simply obtainable through $\chi_Q = -\chi^q_{\rho_Q}$, where $\chi^q_{\rho_Q} \equiv \chi_{\rho_Q}(\omega = 0, \mathbf{q} \to \mathbf{0})$ is the so-called q limit of the density response function. We recall that the specific heat is actually defined through $C_v = \chi_E/T$.

In the absence of impurities, the low-temperature conductivities have the standard Drude-like expression $\sigma_Q(\omega) = iD_Q/(\omega + i0^+)$, where the Drude weights D_Q coincide with the so-called ω limit of the corresponding current response functions: $D_Q = \chi^\omega_{J_Q} \equiv \chi_{J_Q}(\omega \to 0, \mathbf{q} = \mathbf{0})$. Similarly to the specific heat, the thermal conductivity is defined by $\sigma_E(\omega)/T$.

According to Landau's Fermi-liquid theory [36,37]

$$\chi_{N/M} = -2 \int \frac{d\mathbf{k}}{(2\pi)^d} \frac{\partial f(\epsilon_*(\mathbf{k}))}{\partial \epsilon_*(\mathbf{k})} (1 - \mathrm{A}_{S/A}(\mathbf{k})),$$
$$D_{N/M} = -\frac{2}{d} \int \frac{d\mathbf{k}}{(2\pi)^d} \frac{\partial f(\epsilon_*(\mathbf{k}))}{\partial \epsilon_*(\mathbf{k})} \boldsymbol{v}_*(\mathbf{k}) \cdot \boldsymbol{v}_{S/A}(\mathbf{k}), \tag{6}$$

where $d > 1$ is the dimension (in $d = 1$ Landau's Fermi liquid theory is not applicable [41]), $f(x)$ the Fermi distribution function, $\boldsymbol{v}_*(\mathbf{k}) = \partial \epsilon_*(\mathbf{k})/\partial \mathbf{k}$ the quasiparticle group velocity, and

$$\mathrm{A}_{S/A}(\mathbf{k}) = -\int \frac{d\mathbf{k}'}{(2\pi)^d} \frac{\partial f(\epsilon_*(\mathbf{k}'))}{\partial \epsilon_*(\mathbf{k}')} \mathrm{A}^{S/A}_{\mathbf{k},\mathbf{k}'},$$
$$\bar{\boldsymbol{v}}_{S/A}(\mathbf{k}) = \boldsymbol{v}_*(\mathbf{k}) + \int \frac{d\mathbf{k}'}{(2\pi)^d} \frac{\partial f(\epsilon_*(\mathbf{k}'))}{\partial \epsilon_*(\mathbf{k}')} \boldsymbol{v}_*(\mathbf{k}') f^{S/A}_{\mathbf{k},\mathbf{k}'}. \tag{7}$$

The parameters $\mathrm{A}^{S/A}_{\mathbf{k},\mathbf{k}'}$ correspond to the q limit of the quasiparticle scattering amplitudes in the spin-singlet (S) and spin-triplet (A) particle-hole channels, and are related to the f parameters, the ω-limit counterparts,

$$f_{S\mathbf{k},\mathbf{k}'} = f_{\mathbf{k}\uparrow,\mathbf{k}'\uparrow} + f_{\mathbf{k}\uparrow,\mathbf{k}'\downarrow},$$
$$f_{A\mathbf{k},\mathbf{k}'} = f_{\mathbf{k}\uparrow,\mathbf{k}'\uparrow} - f_{\mathbf{k}\uparrow,\mathbf{k}'\downarrow}, \tag{8}$$

through the Bethe-Salpeter equation

$$\mathrm{A}^{S/A}_{\mathbf{k},\mathbf{k}'} = f^{S/A}_{\mathbf{k},\mathbf{k}'} + \int \frac{d\mathbf{p}}{(2\pi)^d} \frac{\partial f(\epsilon_*(\mathbf{p}))}{\partial \epsilon_*(\mathbf{p})} f^{S/A}_{\mathbf{k},\mathbf{p}} \mathrm{A}^{S/A}_{\mathbf{p},\mathbf{k}'}.$$

Similarly, the specific heat C_v and the Drude weight K of the thermal conductivity read

$$C_v = -\frac{2}{T} \int \frac{d\mathbf{k}}{(2\pi)^d} \frac{\partial f(\epsilon_*(\mathbf{k}))}{\partial \epsilon_*(\mathbf{k})} \epsilon_*(\mathbf{k})^2 - \frac{2}{T} \int \frac{d\mathbf{k} d\mathbf{k}'}{(2\pi)^{2d}} \frac{\partial f(\epsilon_*(\mathbf{k}))}{\partial \epsilon_*(\mathbf{k})} \frac{\partial f(\epsilon_*(\mathbf{k}'))}{\partial \epsilon_*(\mathbf{k}')} \epsilon_*(\mathbf{k}) \epsilon_*(\mathbf{k}') \mathrm{A}^S_{\mathbf{k},\mathbf{k}'},$$
$$K = -\frac{2}{dT} \int \frac{d\mathbf{k}}{(2\pi)^d} \frac{\partial f(\epsilon_*(\mathbf{k}))}{\partial \epsilon_*(\mathbf{k})} \epsilon_*(\mathbf{k})^2 |\boldsymbol{v}_*(\mathbf{k})|^2 + \frac{2}{dT} \int \frac{d\mathbf{k} d\mathbf{k}'}{(2\pi)^{2d}} \frac{\partial f(\epsilon_*(\mathbf{k}))}{\partial \epsilon_*(\mathbf{k})} \frac{\partial f(\epsilon_*(\mathbf{k}'))}{\partial \epsilon_*(\mathbf{k}')} \epsilon_*(\mathbf{k}) \epsilon_*(\mathbf{k}') \boldsymbol{v}_*(\mathbf{k}) \cdot \boldsymbol{v}_*(\mathbf{k}') f^{\mathrm{S}}_{\mathbf{k},\mathbf{k}'}.$$

The first term on the right-hand side of both equations is linear in temperature T. Conversely, the second terms give a finite contribution at low T only upon expanding $\mathrm{A}^S_{\mathbf{k},\mathbf{k}'}$ and $f^{\mathrm{S}}_{\mathbf{k},\mathbf{k}'}$ in $\epsilon_*(\mathbf{k})$ and $\epsilon_*(\mathbf{k}')$, as well as including higher order corrections in the heat vertex as obtained through the Ward-Takahashi identity. All those corrections yield at first sight terms of order T^3. In reality, the expansion is not regular. For instance, the corrections to the linear term of the specific heat are actually of order T^d [42,43], with logarithmic corrections in $d = 3$, $T^3 \ln 1/T$. Nonetheless, at leading order in T only the first terms contribute, and thus

$$C_v \simeq \frac{2\pi^2}{3} T \rho_*, \quad K \simeq C_v \frac{v_*^2}{d}, \tag{9}$$

where

$$\rho_* \equiv \int \frac{d\mathbf{k}}{(2\pi)^d} \delta(\epsilon_*(\mathbf{k})), \tag{10}$$

is the quasiparticle density of states at the chemical potential, and

$$v_*^2 \equiv \frac{1}{\rho_*} \int \frac{d\mathbf{k}}{(2\pi)^d} \delta(\epsilon_*(\mathbf{k})) |\boldsymbol{v}_*(\mathbf{k})|^2. \tag{11}$$

Mott insulators with a Luttinger surface.—Let us now consider a hypothetical model that has only a Luttinger surface in the Brillouin zone, with finite quasiparticle density of states at the chemical potential, $\rho_* \neq 0$ in Eq. (10). Since quasiparticles at the Luttinger surface are invisible in the single-particle density of states and incompressible [39], the system describes a nonsymmetry breaking Mott insulator that may only occur at half-filling in a single-band model. In a Mott insulator with localized electrons, we expect that $f_{\mathbf{k}\uparrow,\mathbf{k}'\uparrow} \simeq 0$, which implies $f^S_{\mathbf{k},\mathbf{k}'} \simeq -f^A_{\mathbf{k},\mathbf{k}'}$ and $\mathrm{A}^S_{\mathbf{k},\mathbf{k}'} \simeq -\mathrm{A}^A_{\mathbf{k},\mathbf{k}'}$. However, for the system to be a charge insulator, we need to impose that the compressibility χ_N and charge Drude weight D_N in Eq. (6) vanish, which implies, through Eq. (7), that $\mathrm{A}_S(\mathbf{k}) = 1$ plus a correction that averages to zero on the Luttinger surface, as well as that the flux of $\boldsymbol{v}_S(\mathbf{k})$ out of the Luttinger surface is zero. In turn, since $\mathrm{A}_A(\mathbf{k}) \simeq -\mathrm{A}_S(\mathbf{k}) = -1$ and $\boldsymbol{v}_A(\mathbf{k}) \simeq 2\boldsymbol{v}_*(\mathbf{k}) - \boldsymbol{v}_S(\mathbf{k})$, then, through Eqs. (6) and (11), the spin susceptibility χ_M and Drude weight D_M become simply

$$\chi_M \simeq 4\rho_*, \quad D_M \simeq \frac{4}{d} \rho_* v_*^2. \tag{12}$$

Comparing (12) with (9), we find that the Wilson ratio, which measures the effective correlation strength, is

$$R_\mathrm{W} = \frac{\pi^2 T}{3 C_v} \chi_M \simeq 2. \tag{13}$$

Therefore, a Landau Fermi liquid characterized by a Luttinger surface without Fermi pockets may indeed have charge properties of an insulator, while spin and thermal ones of a metal, very much alike a spin-liquid insulator with gapless spinons.

We mention that conventional Fermi liquids often do not survive down to $T = 0$, since they may encounter an instability at $T_c > 0$ towards a different phase that, most of the time, breaks symmetries and opens gaps in the quasiparticle spectrum. Well-known examples are the superconducting and superfluidity instabilities in normal metals and ^{3}He, respectively. A Fermi liquid description of such an instability is justified when quasiparticles have already reached quantum degeneracy at T_c, which implies that T_c must be much smaller than the quasiparticle Fermi energy ϵ_F.

Similarly, we cannot exclude that also quasiparticles at a Luttinger surface, the gapless spinons, may become unstable at $T_c \ll \epsilon_F$ towards, e.g., a magnetically ordered phase, and eventually acquire a gap. In this case, which presumably corresponds to highly frustrated magnets, the above Fermi liquid properties would still be observable for $T_c \ll T \ll \epsilon_F$. On the contrary, if $T_c \sim \epsilon_F$, likely the case of unfrustrated magnets, the quantum degenerate behavior of quasiparticles at the Luttinger surface cannot set in before the instability.

Quantum oscillations.—The next relevant question to be addressed is whether quasiparticles at a Luttinger surface contribute to quantum oscillations in a magnetic field B. On one hand, the semiclassical approach to the de Haas–van Alphen (dHvA) effect by Lifshitz and Kosevich [44], which just relies on the existence of quasiparticles, would suggest a positive answer. However, the vanishing Drude weight implies, through (6) and (7), that

$$\begin{aligned} 0 &= -\int d\mathbf{k} \frac{\partial f(\epsilon_*(\mathbf{k}))}{\partial \mathbf{k}} \cdot \boldsymbol{v}_S(\mathbf{k}) \\ &= \int d\mathbf{k} f(\epsilon_*(\mathbf{k})) \boldsymbol{\nabla}_\mathbf{k} \cdot \boldsymbol{v}_S(\mathbf{k}) \\ &= \int d\mathbf{k} f(\epsilon_*(\mathbf{k})) \mathrm{Tr}(\hat{m}_c(\mathbf{k})^{-1}), \end{aligned}$$

where $\hat{m}_c(\mathbf{k})$ is the cyclotron mass tensor as it emerges from the Landau-Boltzmann transport equation. Considering, for simplicity, an isotropic $\hat{m}_c(\mathbf{k}) = m_c(\mathbf{k})\hat{I}$, it follows that vanishing Drude weight is equivalent to vanishing $1/m_c(\mathbf{k})$, or, equivalently, vanishing cyclotron frequency, once integrated over the volume enclosed by the Luttinger surface. That hints at the absence of quantum oscillations, in contrast to the previous observation.

To resolve this issue, we resort to Luttinger's theory of the de Haas–van Alphen effect in interacting electron systems [45]. Luttinger showed that the leading oscillatory part of the free energy derives from

$$\Delta F_\mathrm{osc} = -T \sum_\epsilon e^{i\epsilon 0^+} \mathrm{Tr} \ln(i\epsilon - \hat{H}_0 - \hat{\Sigma}(i\epsilon)), \tag{14}$$

where $\hat{H}_0$ is the noninteracting Hamiltonian, which includes the static and uniform magnetic field B, represented in a generic basis of single particle wave functions. The self-energy matrix $\hat{\Sigma}(i\epsilon)$ in (14) must include any polynomial in B but not oscillatory terms in $1/B$ [45]. In matrix notations, we now define

$$\hat{Z}(\epsilon)^{-1} \equiv 1 - \frac{\mathrm{Im}\hat{\Sigma}(\epsilon)}{\epsilon},$$

which is a positive-definite matrix with eigenvalues ≥ 1, and the Hermitian matrix

$$\hat{H}_*(\epsilon) = \sqrt{\hat{Z}(\epsilon)}(\hat{H}_0 + \mathrm{Re}\hat{\Sigma}(i\epsilon))\sqrt{\hat{Z}(\epsilon)}.$$

With these definitions that generalize (3) and (5), the free energy component (14) becomes

$$\begin{aligned}\Delta F_{\mathrm{osc}} &= -T\sum_{\epsilon} e^{i\epsilon 0^+}\mathrm{Tr}\ln(i\epsilon - \hat{H}_*(\epsilon)) \\ &\quad + T\sum_{\epsilon} e^{i\epsilon 0^+}\mathrm{Tr}\ln\hat{Z}(\epsilon) \\ &\equiv \Delta F^{(1)}_{\mathrm{osc}} + \Delta F^{(2)}_{\mathrm{osc}}.\end{aligned} \tag{15}$$

In conventional Fermi liquids, where $\hat{Z}(0)$ has no null eigenvalue, the first term, $\Delta F^{(1)}_{\mathrm{osc}}$, is the only that contributes and yields the Lifshitz and Kosevich theory of the dHvA effect, as shown by Luttinger [45]. Indeed, in the semiclassical limit, $\hat{H}_*(\epsilon)$ becomes the representation in the chosen basis of the operator $\epsilon_*(\epsilon, \mathbf{K}(\mathbf{r}))$, Eq. (5) with $\mathbf{k}$ replaced by

$$\mathbf{K}(\mathbf{r}) = -i\hbar\frac{\partial}{\partial\mathbf{r}} + \frac{e}{2c}\mathbf{B}\wedge\mathbf{r}, \tag{16}$$

and thus

$$\Delta F^{(1)}_{\mathrm{osc}} \simeq -T\sum_{\epsilon} e^{i\epsilon 0^+}\mathrm{Tr}\ln(i\epsilon - \epsilon_*(\mathbf{K}(\mathbf{r}))). \tag{17}$$

After that, one can simply follow Lifshitz and Kosevich [44] and derive the expression of the dHvA oscillations. However, in the present case of a Luttinger surface, also $\Delta F^{(2)}_{\mathrm{osc}}$ in (15) may contribute since $\hat{Z}(\epsilon)$ has zero eigenvalues at $\epsilon = 0$. To assess their role, we note that $\hat{Z}(\epsilon)$ in the semiclassical limit is the representation of the operator $Z(\epsilon, \mathbf{K}(\mathbf{r}))$, i.e., of $Z(\epsilon, \mathbf{k})$ in Eq. (3) with $\mathbf{k} \to \mathbf{K}(\mathbf{r})$. Moreover, the contribution of $\Delta F^{(2)}_{\mathrm{osc}}$ to quantum oscillations only derives from the region around the zeros of $Z(\epsilon, \mathbf{k})$ [46], i.e., small ϵ and $\mathbf{k}$ close to the Luttinger surface. In that region, we can write, without loss of generality and consistently with the analytic assumption, that [38,47]

$$\Sigma(i\epsilon, \mathbf{k}) \underset{\epsilon\to 0}{\simeq} \frac{\Delta(\mathbf{k})^2}{i\epsilon - E(\mathbf{k})}, \tag{18}$$

where $\mathbf{k}_L$: $E(\mathbf{k}_L) = 0$ defines the Luttinger surface provided $\Delta(\mathbf{k}_L) \neq 0$, so that, for $\epsilon \simeq 0$ and $\mathbf{k} \simeq \mathbf{k}_L$,

$$\begin{aligned} Z(\epsilon, \mathbf{k}) &= \frac{\epsilon^2 + E(\mathbf{k})^2}{\epsilon^2 + E(\mathbf{k})^2 + \Delta(\mathbf{k})^2} \\ &\simeq \frac{\epsilon^2 + E(\mathbf{k})^2}{\Delta(\mathbf{k})^2}, \\ \epsilon_*(\epsilon, \mathbf{k}) &= \frac{\epsilon(\mathbf{k})(\epsilon^2 + E(\mathbf{k})^2) - E(\mathbf{k})\Delta(\mathbf{k})^2}{\epsilon^2 + E(\mathbf{k})^2 + \Delta(\mathbf{k})^2} \\ &\simeq -E(\mathbf{k}),\end{aligned} \tag{19}$$

which, as anticipated, are analytic. Therefore,

$$\begin{aligned} Z(\epsilon, \mathbf{K}(\mathbf{r})) &\simeq \epsilon^2 + \epsilon_*(\mathbf{K}(\mathbf{r}))^2 \\ &= (i\epsilon - \epsilon_*(\mathbf{K}(\mathbf{r})))(-i\epsilon - \epsilon_*(\mathbf{K}(\mathbf{r}))),\end{aligned}$$

and, correspondingly,

$$\begin{aligned}\Delta F^{(2)}_{\mathrm{osc}} &\simeq T\sum_{\epsilon} e^{i\epsilon 0^+}[\ln(i\epsilon - \epsilon_*(\mathbf{K}(\mathbf{r}))) \\ &\quad + \ln(-i\epsilon - \epsilon_*(\mathbf{K}(\mathbf{r})))],\end{aligned} \tag{20}$$

so that, through (17) and (20), Eq. (15) becomes

$$\begin{aligned}\Delta F_{\mathrm{osc}} &\simeq T\sum_{\epsilon} e^{i\epsilon 0^+}\ln(-i\epsilon - \epsilon_*(\mathbf{K}(\mathbf{r}))) \\ &\simeq -\Delta F^{(1)}_{\mathrm{osc}},\end{aligned} \tag{21}$$

as can be readily verified following Lifshitz and Kosevich [44]. As a result, quasiparticles at the Luttinger surface of a Mott insulator do yield dHvA oscillations in the magnetization $-\partial\Delta F_{\mathrm{osc}}/\partial B$ alike conventional quasiparticles with dispersion $\epsilon_*(\mathbf{k})$, apart from a π shift.

Concluding remarks.—A few remarks are now in order. Conventional theories of spin liquids [48–54] predict that a spinon Fermi surface is most likely associated with so-called $U(1)$ spin liquids, apart from a few known exceptions [55–58]. In that $U(1)$ case, the specific heat behaves at low temperature as $T^{2/3}$ and $T\ln 1/T$ in $d = 2$ and $d = 3$, respectively [53,59,60], and, correspondingly, κ/T diverges for $T \to 0$ [22]. These thermal properties, different from the observed ones, challenge the spin-liquid interpretation. Finite C_v/T and κ/T for $T \to 0$ may be, for instance, attributed to magnetic impurities, assuming a gapped spin liquid phase lacking a spinon Fermi surface [26]. However, this explanation implies that also quantum oscillations are not due to spinons, and thus that all intriguing thermal and magnetic properties observed in experiments are unrelated to the purported spin liquid nature of the material, which is a bit disappointing.

On the contrary, the Fermi liquid properties of a Mott insulator with a Luttinger surface seem to account for all experimental evidences. Nonetheless, the analyticity assumption on the self-energy underlying Landau's Fermi liquid theory is evidently incompatible with the above-mentioned nonanalytic behavior of $U(1)$ spin liquids with a spinon Fermi surface. Therefore, either that analytic behavior never occurs in physical models, or Mott insulators with a Luttinger surface realize one of the above-mentioned exceptions [55–58] of spin liquids with a spinon Fermi surface. Indeed, an example of a spin liquid with $C_v \sim T$ is very well known: the half-filled Hubbard model in one dimension. Even though interacting electrons in $d = 1$ behave as Luttinger liquids [61], their low-frequency, low-temperature, and long-wavelength properties are just like conventional Fermi liquids [41,61,62], including the specific heat that, as we mentioned, is obtainable by the q limit of the heat-heat response function. In particular, the half-filled Hubbard model in $d = 1$ is an insulator that has a Luttinger surface at $k = \pm\pi/2$ as well as gapless spinons that yield a finite spin susceptibility, a finite C_v/T, apart from corrections vanishing as powers of $1/\ln T$, and a Wilson ratio $R_W = 2$ for $T \to 0$ [63]. That is precisely what our Fermi-liquid analysis predicts.

In conclusion, we have shown that non symmetry breaking Mott insulators with a Luttinger surface realize gapless spin liquids, where the spinons are actually Landau's quasiparticles at the Luttinger surface, which thus provides the rigorous definition of Anderson's spinon Fermi surface [32,33]. These quasiparticles contribute to thermal and magnetic properties, including quantum oscillations, just like conventional quasiparticles do, despite that the system is a charge insulator.

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