Mean-Field Phase Transitions in Tensorial Group Field Theory Quantum Gravity

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Controlling the continuum limit and extracting effective gravitational physics are shared challenges for quantum gravity approaches based on quantum discrete structures. The description of quantum gravity in terms of tensorial group field theory (TGFT) has recently led to much progress in its application to phenomenology, in particular, cosmology. This application relies on the assumption of a phase transition to a nontrivial vacuum (condensate) state describable by mean-field theory, an assumption that is difficult to corroborate by a full RG flow analysis due to the complexity of the relevant TGFT models. Here, we demonstrate that this assumption is justified due to the specific ingredients of realistic quantum geometric TGFT models: combinatorially nonlocal interactions, matter degrees of freedom, and Lorentz group data, together with the encoding of microcausality. This greatly strengthens the evidence for the existence of a meaningful continuum gravitational regime in group-field and spin-foam quantum gravity, the phenomenology of which is amenable to explicit computations in a mean-field approximation.

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Introduction.—The main challenge for quantum gravity, in all its candidate formulations [1–18], is to show the existence of a regime of the fundamental theory that reproduces gravitational physics together with new observable consequences. This is highly nontrivial because we expect the new quantum gravity effects to become dominant at rather extreme energy and length scales. Still, early-universe cosmology, black holes, and several astrophysical phenomena might provide many testing grounds for quantum gravity, also thanks to the recent improvement of observational techniques [19]. In quantum gravity formulations based on fundamentally discrete quantum structures, the challenge of extracting an effective continuum gravitational physics is intertwined with that of controlling the continuum limit of the fundamental quantum dynamics, i.e., its renormalization group flow and its continuum phase structure.

Black hole [20,21] and cosmological [22–25] physics have been studied in the framework of tensorial group field theory (TGFT) [14–16]. This is a quantum field theory (QFT) generating spacetime geometries from discrete geometric building blocks given by combinatorially nonlocal interactions. This framework also provides a completion of the quantum dynamics encoded in spin foam models [9,10], which is a covariant counterpart of canonical loop quantum gravity [12,13] and a reformulation of specific simplicial lattice gravity path integrals [26–30].

The underlying assumption in all these works is that there is a condensate phase, the dynamics of which can be captured by a mean-field approximation. However, this assumption is difficult to corroborate by a full RG flow analysis due to the complexity of the relevant TGFT models. On the other hand, this formal complexity of the fundamental quantum dynamics is the result of trying to incorporate the conditions required for a geometric interpretation at the discrete level, some seed of the causal structure we expect to emerge in the continuum approximation, and appropriate matter degrees of freedom.

In this Letter, we show that an appropriate mean-field description of a condensate phase exists in TGFT and that the quantum geometric and physical ingredients even improve the mean-field theory behavior. To this aim, we apply a Landau-Ginzburg analysis, in particular, the Ginzburg criterion, to models with (1) nonlocal interactions [31,32], (2) additional matter degrees of freedom [33], and (3) the Lorentz group $\mathrm{SL}(2,\mathbb{C})$, together with the implementation of geometricity constraints [34]. For the first time, we show how an effective dimension can be deduced from the Landau-Ginzburg setting; as such a notion relates to renormalization group results [35,36], it allows us to argue for the validity of the result of two phases, even beyond mean-field theory.

Our results thus strongly support the existence of a meaningful continuum gravitational regime in TGFT quantum gravity, as well as the closely related spin foam models and lattice quantum gravity. This relies crucially on the completion of such discrete gravity models in terms of a sum over lattices generated by a field theory, thereby allowing us to study their phenomenology by standard field-theory methods in a mean-field approximation. In this sense, the TGFT formulation provides new, powerful tools for tackling the difficult problem of the continuum limit in lattice models of quantum gravity and the quantum dynamics of spin network states.

Mean-field TGFT.—TGFT is a field theory that perturbatively generates a sum over lattices to which group-theoretic data are associated, encoding their discrete geometry [15,16]. The field $\Phi\colon G^r \to \mathbb{R}$ excites r elements $\boldsymbol{g} = (g^1, \ldots, g^r)$ in a Lie group G. From the Feynman diagrams, one obtains cell complexes due to combinatorially nonlocal interactions $\prod_i^n \Phi(\boldsymbol{g}_i)$, i.e., interactions in which the field arguments are convoluted pairwise, one g_i^a and one g_j^b. This is encoded by an r-valent interaction graph γ with $n_\gamma = n$ vertices and $rn/2$ edges $(i, a; j, b)$. A generic action is thus

$$S[\Phi] = \int_{G^r} \mathrm{d}\boldsymbol{g} \Phi(\boldsymbol{g}) \mathcal{K} \Phi(\boldsymbol{g}) + \sum_\gamma \frac{\lambda_\gamma}{n_\gamma} \mathrm{Tr}_\gamma[\Phi], \tag{1}$$

where $\mathcal{K}$ is a kinetic operator, $\sum_\gamma$ is over a given set of interaction graphs γ, and Tr_γ defines the resulting pairwise convolutions of each γ with kernels $\mathcal{V}$,

$$\mathrm{Tr}_\gamma[\Phi] = \int_{G^{r \cdot n_\gamma}} \prod_{i=1}^{n_\gamma} \mathrm{d}\boldsymbol{g}_i \prod_{(i,a;j,b)} \mathcal{V}(g_i^a, g_j^b) \prod_{i=1}^{n_\gamma} \Phi(\boldsymbol{g}_i). \tag{2}$$

If $a = b$ in each convolution, then the integers a label the edges and one obtains an r-coloured graph (related to tensorial symmetry [18,37]). The resulting Feynman diagrams are then dual to r-dimensional simplicial pseudomanifolds.

To associate discrete geometries with connection variables to the simplicial structures, additional geometricity constraints on the group elements called closure and simplicity constraints are necessary [15,16]. With such constraints in place, the TGFT amplitude on a given simplicial lattice rewritten in terms of dual flux variables [27,38,39] takes the form of a lattice gravity path integral [15,28–30] or, equivalently, of a spin foam state sum [10]. There are different models distinguished by the specific implementation of such constraints, i.e., a choice of kinetic operator $\mathcal{K}$ and interaction convolutions $\mathcal{V}$. TGFT thus differs significantly from local QFT in technical details and interpretation. Still, QFT methods can be adapted to this peculiar quantum gravity framework.

Mean-field theory provides an approximation of the full QFT partition function and thus an effective description of the phase structure [40]. One considers Gaussian fluctuations around a nonvanishing vacuum solution, e.g., for a constant mean-field Φ_0. This is a solution to the TGFT equation of motion [33],

$$\mathcal{K}\Phi + \sum_\gamma \frac{\lambda_\gamma}{n_\gamma} \sum_v \mathrm{Tr}_{\gamma\backslash v}(\Phi) = 0, \tag{3}$$

where the trace is over the graph $\gamma\backslash v$ obtained by deleting the vertex v from γ, and $\sum_v$ runs over all its vertices v.

The validity of a mean-field description of phase transitions with diverging correlation length ξ can be checked via the Ginzburg criterion. The ratio

$$Q \coloneqq \frac{\int_{\Omega_\xi} \mathrm{d}\boldsymbol{g} C(\boldsymbol{g})}{\int_{\Omega_\xi} \mathrm{d}\boldsymbol{g} \Phi_0^2} \tag{4}$$

compares correlations $C(\boldsymbol{g})$ of Gaussian fluctuations with the vacuum Φ_0^2, both averaged up to the correlation length scale ξ, that is, integrated over a suitable domain Ω_ξ [33]. If fluctuations remain small towards the phase transition, i.e., $Q \ll 1$ when $\xi \to \infty$, then mean-field theory is valid. In local d-dimensional Φ_d^4 scalar field theory, $Q \sim \mu^{-2}\xi^{-d} \sim \xi^{4-d}$ such that the mean-field description is valid only beyond the critical dimension $d_{\mathrm{crit}} = 4$. In the following, we explain how this is affected by the various features of TGFT models, progressively including all main ingredients for realistic quantum gravity models.

Combinatorial nonlocality.—The effect of combinatorially nonlocal interactions is most transparent for a simplified TGFT model. We choose $\mathcal{K} = \sum_{c=1}^r -\Delta_c + \mu$, with Δ_c being the Laplacian on G and Dirac delta convolutions $\mathcal{V} = \delta$. Then, the equation of motion (3) for the constant field Φ_0 is

$$\left(\mu + \sum_\gamma \lambda_\gamma V_G^{r\frac{n_\gamma-2}{2}} \Phi_0^{n_\gamma-2}\right)\Phi_0 = 0, \tag{5}$$

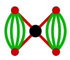
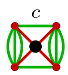
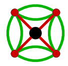
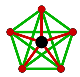
where V_G is the volume of G. Such factors arise due to the combinatorial nonlocality and need regularization if G is not compact. For the example in this section, we take $G = \mathbb{R}^{d_G}$ and regularize it to the d_G-torus $G_L = \mathrm{T}_L^{d_G}$ with radii $L/2\pi$ such that $V_L \equiv V_{G_L} = L^{d_G}$.

For a single interaction γ, the vacuum equation (5) reduces to $(V_L^{r/2}\Phi_0)^{n_\gamma-2} = -\mu/n_\gamma\lambda_\gamma$, which has a real solution for $\mu < 0$. The correlation function of Gaussian fluctuations around such Φ_0 in momentum space is [33]

$$\hat{C}(\boldsymbol{j}) = \frac{1}{\frac{1}{V_L^2}\sum_c \mathrm{Cas}_{j_c} + \mu - \mu\hat{\mathcal{X}}_\gamma(\boldsymbol{j})} = \frac{1}{\frac{1}{V_L^2}\sum_c \mathrm{Cas}_{j_c} + b_{\boldsymbol{j}}}, \tag{6}$$

where j_c labels representations of G, $\boldsymbol{j} \equiv (j_1, \ldots, j_r)$ and Cas_{j_c} denotes the Casimir of j_c. On a compact or compactified group, these are countable; here, $j_c \in \mathbb{Z}^{d_G}$. Moreover, $\hat{\mathcal{X}}_\gamma(\boldsymbol{j})$ is a sum over products of Kronecker deltas specific to γ (see Table I), which gives rise to an effective

TABLE I. Examples of nonlocal interaction graphs for $r = 4$ and the resulting operator $\hat{\mathcal{X}}_\gamma$ [33]. In addition to the usual legs (red) of interacting fields Φ, green half-edges represent the pairwise convolution of group arguments g_i^a [41].

Double trace		$\hat{\mathcal{X}} = 4(2\prod_{c=1}^{4}\delta_{j_c 0} + 1)$
$n = 4$ melonic		$\hat{\mathcal{X}} = 4(\prod_c \delta_{j_c 0} + \prod_{b \neq c}\delta_{j_b 0} + \delta_{j_c 0})$
$n = 4$ necklace		$\hat{\mathcal{X}} = 4(\prod_c \delta_{j_c 0} + \delta_{j_1 0}\delta_{j_2 0} + \delta_{j_3 0}\delta_{j_4 0})$
Simplicial		$\hat{\mathcal{X}} = 5\sum_{i=0}^{4}\prod_{k \neq i}\delta_{j_{(ik)} 0}$

mass $b_{\boldsymbol{j}} = \mu[1 - \hat{\mathcal{X}}_\gamma(\boldsymbol{j})]$. Consequently, correlations expand in various multiplicities of zero modes,

$$C(\boldsymbol{g}) = \frac{1}{V_L^r}\sum_{s=0}^{r}\sum_{(c_1,\ldots,c_s)}\sum_{\substack{j_{c_1},\ldots,j_{c_s}=0 \\ j_{c_{s+1}},\ldots,j_{c_r}\neq 0}} \mathrm{tr}_{\boldsymbol{j}}[\hat{C}(\boldsymbol{j})\bigotimes_{c=1}^{r} D^{j_c}(g_c)], \quad (7)$$

with representation matrices D^{j_c}; here, $D^{j_c}(\mathrm{e}^{i\theta}) = \mathrm{e}^{i\theta j_c}$. Since $D^0(g) = 1$, each s-fold zero-mode $(c_1, \ldots, c_s)$ contribution depends only on the other $r - s$ group variables, and the effective mass reduces to a number $b_{\boldsymbol{j}} = b_{c_1,\ldots,c_s}$.

The correlation length ξ can be obtained from the second moment or asymptotic behavior of $C(\boldsymbol{g})$ [33,34]. It sets the characteristic scale beyond which correlations decay exponentially and diverges as $\xi^2 \sim -1/\mu$ at criticality, i.e., when $\mu \to 0$. For a large cutoff L, integrating $C(\boldsymbol{g})$ up to ξ in each parameter θ_c^a yields [33]

$$\int_{\Omega_\xi} \mathrm{d}\boldsymbol{g}\, C(\boldsymbol{g}) = \sum_{s=s_0}^{r}\left(\frac{\xi}{L}\right)^{d_{\mathrm{G}} s}\sum_{(c_1,\ldots,c_s)}\frac{1}{b_{c_1\ldots c_s}}, \quad (8)$$

where s_0 is the minimal number of zero modes, that is, the deltas in $\hat{\mathcal{X}}$. Interaction graphs γ have multiple edges, in general; it is the maximal multiplicity $s_{\max}$ occurring in γ that determines s_0; in the model here, $s_0 = r - s_{\max}$.

Taking the ratio (4) with integrated Φ_0^2 gives

$$Q_L = \left(\frac{\lambda_\gamma}{-\mu}\right)^{\frac{2}{n_\gamma - 2}}\sum_{s=s_0}^{r}\frac{f_s^\gamma}{-\mu}\left(\frac{\xi}{L}\right)^{-d_{\mathrm{G}}(r-s)}, \quad (9)$$

where we abbreviate the coefficients in the polynomial $f_s^\gamma \coloneqq -\mu\sum_{(c_1,\ldots,c_s)} b_{c_1,\ldots,c_s}^{-1}$. Removing the cutoff L, only the s_0-fold zero modes of the interaction γ survive, yielding large-ξ asymptotics (using $-\mu \sim \xi^{-2}$)

$$Q \underset{\xi\to\infty}{\sim} \bar{\lambda}_\gamma^{\frac{2}{n_\gamma - 2}} f_{s_0}^\gamma \xi^{\frac{2n_\gamma}{n_\gamma - 2} - d_{\mathrm{G}}(r - s_0)}, \quad (10)$$

with $\bar{\lambda}_\gamma = L^{d_{\mathrm{G}}(r-s_0)(n_\gamma-2)/2}\lambda_\gamma$. This is the result $Q \sim \mu^{-(d_{\mathrm{crit}}/2)}\xi^{-d} \sim \xi^{d_{\mathrm{crit}}-d}$ of local QFT with $d_{\mathrm{crit}} = 2n_\gamma/n_\gamma - 2$. The effect of the nonlocal interaction given by the graph γ with s_0 minimal zero modes is a reduction of the configuration space dimension $d_{\mathrm{G}}\, r$ to an effective dimension

$$d = d_{\mathrm{eff}} \coloneqq d_{\mathrm{G}}(r - s_0). \quad (11)$$

A typical model of 4D quantum gravity has $r = 4$, group dimension of at least $d_{\mathrm{G}} = 3$ [e.g., $G = \mathrm{SU}(2)$ models], and quartic, quintic, or higher-order interactions. Though the configuration space is then at least 12 dimensional and $d_{\mathrm{crit}} = 4$ or $10/3$, respectively, the effective dimension d_{eff} might be smaller, e.g., for $r - s_0 = 1$ in the case of γ with no multiple edges, like the simplicial interaction (see Table I). Adding gauge invariance can shift $s_0 \to s_0 + 1$ and thus reduce d_{eff} even further [33]. On the other hand, if there are edges of high enough multiplicities, e.g., melonic interactions with $s_0 = 1$, d_{eff} is larger than d_{crit} such that $Q \ll 1$, and mean-field theory is a valid description of phase transitions. Thus, combinatorial nonlocality of TGFT affects the detailed mean-field behavior but does not spoil the very applicability of mean-field theory.

A special case includes models where G is compact and no thermodynamic limit is applicable. Then, there is a fixed finite volume $L^{d_{\mathrm{G}}}$ (no $L \to \infty$ limit), and the r-fold zero mode dominates in the IR, i.e., at a small momentum scale $k \sim 1/\xi$ [36]. In the current language, this is the $s = r$ mode, and thus $d_{\mathrm{eff}} = 0$. In other words, in TGFT, one also has [35,42] the standard result that a QFT on a compact domain is effectively zero dimensional in the IR and does not allow for phase transitions [43].

Matter degrees of freedom.—Adding matter degrees of freedom in a TGFT model increases the effective dimension [33]. We extend the group field to $\Phi(\boldsymbol{\phi}, \boldsymbol{g})$ with d_ϕ local degrees of freedom $\boldsymbol{\phi} = (\phi_1, \ldots, \phi_{d_\phi}) \in \mathbb{R}^{d_\phi}$ with pointlike interactions $\int \mathrm{d}\boldsymbol{\phi}\,\mathrm{Tr}_\gamma[\Phi](\boldsymbol{\phi})$, while the convolutions of $\boldsymbol{g}_i$ remain as in Eq. (2). This generates free scalar fields minimally coupled to the discrete geometry. Such fields can be used as reference frames, which allows us to retrieve the dynamics of quantum geometry in relational terms [44–47]. Furthermore, this is, of course, a necessary ingredient of realistic models of quantum gravity coupled to elementary particle fields.

In the Landau-Ginzburg analysis, such local degrees of freedom simply add to the dimension. Gaussian correlations (6) extend to

$$\hat{C}(\boldsymbol{p}, \boldsymbol{j}) = \frac{1}{\alpha(\boldsymbol{j})\sum_{a=1}^{d_\phi} p_a^2 + \frac{1}{V_L^2}\sum_c \mathrm{Cas}_{j_c=1}^{d_{\mathrm{G}}} + b_{\boldsymbol{j}}}, \quad (12)$$

where the momenta p_a of ϕ_a can couple to group representations $\boldsymbol{j}$ via a function $\alpha(\boldsymbol{j})$. However, only modes with $p_a \approx 0$ are relevant upon integration over ϕ_a, so the Ginzburg Q in Eq. (10) receives no contribution from $\alpha(\boldsymbol{j})$ and simply gets an additional factor ξ^{-d_ϕ} [33]. Accordingly, the effective dimension is

$$d_{\mathrm{eff}} = d_\phi + d_{\mathrm{G}}(r - s_0). \tag{13}$$

Therefore, even if $d_{\mathrm{G}}(r - s_0) \leq d_{\mathrm{crit}}$, the Ginzburg criterion is still satisfied when there are $d_\phi > d_{\mathrm{crit}} - d_{\mathrm{G}}(r - s_0)$ matter fields. Even in the special case of compact G without the thermodynamic large-volume limit, mean-field phase transitions are possible if $d_\phi > d_{\mathrm{crit}}$. In this way, adding such matter degrees of freedom not only makes these models more realistic from a physical point of view but also improves their mean-field behavior.

Hyperbolic geometry.—Realistic models of quantum gravity involve a Lie group with hyperbolic geometry related to the Lorentz group. The latter is a crucial ingredient to properly implement microcausality.

One such model is the Lorentzian Barrett-Crane TGFT model [48–52], which generates triangulations formed by spacelike tetrahedra. This is a TGFT on the noncompact curved group $G = \mathrm{SL}(2,\mathbb{C})$ subject to closure and simplicity constraints. Because of the restriction to spacelike tetrahedra, the group domain is effectively $r = 4$ copies of the 3-hyperboloid $\mathrm{H}^3 = \mathrm{SL}(2,\mathbb{C})/\mathrm{SU}(2)$. Owing to the interplay with combinatorial nonlocality, infinite volume factors do not cancel, and they need to be regularized [see, e.g., Eq. (5)]. We show this in the Cartan decomposition $\mathrm{SL}(2,\mathbb{C}) \cong \mathrm{SU}(2) \times \mathrm{A}^+ \times \mathrm{SU}(2)$ via a cutoff L on the diagonal Cartan subgroup

$$\mathrm{A}^+ = \{\mathrm{e}^{\frac{\sigma_3 \eta}{2\,a}} | \eta \in \mathbb{R}_+\} \to \mathrm{A}_L^+ \coloneqq \{\mathrm{e}^{\frac{\sigma_3 \eta}{2\,a}} | 0 \leq \eta < L\}. \tag{14}$$

Here, a is the curvature scale, i.e., the skirt radius of the group's hyperbolic part H^3. Using the Haar measure on $\mathrm{SU}(2)$ and $\sinh^2(\eta/a)\mathrm{d}\eta/a$ for A^+, one finds the regularized volume of $\mathrm{SL}(2,\mathbb{C})$ to be

$$V_L = \frac{1}{4}\left(\sinh\left(\frac{2L}{a}\right) - \frac{2L}{a}\right) \underset{L\to\infty}{\sim} \frac{1}{8}\mathrm{e}^{\frac{2L}{a}}. \tag{15}$$

The relative magnitude of Gaussian fluctuations around the mean-field Φ_0 at a large cutoff L is [34]

$$Q_L = \frac{\sum_{s=s_0}^{r} \frac{f_s^\gamma}{-\mu} V_\xi^s V_L^{1-s}}{\left(\frac{-\mu}{\lambda_\gamma}\right)^{\frac{2}{n_\gamma - 2}} V_\xi^r V_L^{1-r-2\frac{n_\gamma-1}{n_\gamma-2}}}. \tag{16}$$

Removing the regularization, the minimal number s_0 of zero modes dominates such that

$$Q_L \underset{L\to\infty}{\sim} \bar{\lambda}_\gamma^{\frac{2}{n_\gamma-2}} (-\mu)^{-\frac{n_\gamma}{n_\gamma-2}} f_{s_0}^\gamma V_\xi^{-(r-s_0)}, \tag{17}$$

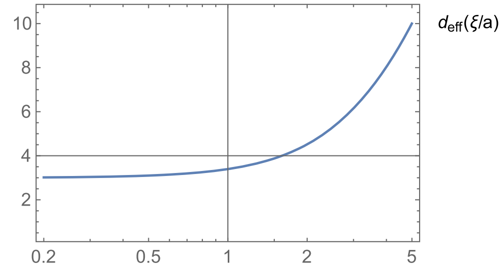


FIG. 1. Effective dimension (20) for $s_0 = r - 1$ flowing from 3 at small ξ to infinity.

with rescaling $\bar{\lambda}_\gamma = V_L^{(n_\gamma - 2/2)(r-s_0)+n_\gamma-1}\lambda_\gamma$. Because of the hyperbolic geometry of the configuration space, the correlation length scales $\xi \underset{\mu\to 0}{\sim} 1/a\mu$ [34]. Thus,

$$Q \underset{\xi\to\infty}{\sim} \bar{\lambda}_\gamma^{\frac{2}{n_\gamma-2}} f_{s_0}^\gamma (a\xi)^{\frac{n_\gamma}{n_\gamma-2}} \mathrm{e}^{-2(r-s_0)\frac{\xi}{a}}. \tag{18}$$

This is an exponential suppression such that the Ginzburg criterion $Q \ll 1$ is satisfied regardless of the order n_γ and zero modes s_0 of the interaction.

A complementary way to understand this case is to use the notion of a scale-dependent effective dimension d_{eff} [36]. One can still write Eq. (17) as a power function but with scale-dependent inverse power,

$$d_{\mathrm{eff}}(\xi) \coloneqq -\frac{\partial \log F(\xi)}{\partial \log \xi}, \text{ here } F(\xi) = f_{s_0}^\gamma V_\xi^{-(r-s_0)}. \tag{19}$$

Then, the general form $Q \propto \mu^{-d_{\mathrm{crit}}/2} \xi^{-d_{\mathrm{eff}}(\xi)}$ is valid on all scales ξ, with the resulting effective dimension

$$d_{\mathrm{eff}}(\xi) = (r - s_0)\frac{\cosh\left(\frac{2\xi}{a}\right) - 1}{\frac{a}{2\xi}\sinh\left(\frac{2\xi}{a}\right) - 1} \tag{20}$$

flowing from $d_{\mathrm{eff}} = 3(r - s_0)$ in the UV (small ξ) [53] to $d_{\mathrm{eff}} \to \infty$ in the IR, see Fig. 1. Thus, mean-field theory can describe the phase transition because the theory's effective dimension blows up towards the IR and thereby supersedes any possible critical dimension.

This result for the Barrett-Crane TGFT model suggests that mean-field theory is valid for any TGFT on hyperbolic spaces irrespective of the dimension $d_{\mathrm{G}} r$ of the field domain or of combinatorics n_γ and s_0. Including d_ϕ matter fields again adds a power ξ^{-d_ϕ} [34], but the exponential suppression still dominates. A hyperbolic sector also arises for models with compact G but Lorentzian integrals implicit in the kernel $\mathcal{K}$ [54]. Similarly, one expects this exponential decay to also remain for TGFT models that contain lightlike and timelike tetrahedra as in the complete Lorentzian Barrett-Crane model [52]. Thus, it seems to be a generic feature of TGFT quantum gravity that phase transitions towards a nonperturbative vacuum state exist, which can be self-consistently described using mean-field theory. Since such a state is typically highly populated by TGFT quanta, this indeed makes a compelling case for an interesting continuum geometric approximation.

A reasonable viewpoint on this general behavior is that of universality of the continuum limit as naturally expected from coarse graining, implemented here via mean-field techniques. This idea resonates with recent results on effective spin foam models, which suggests that models differing in the precise implementation of the above-mentioned geometricity constraints could lie in the same universality class from the perspective of continuum gravitational physics [55–59].

Conclusions.—The key result of this work is that it is possible to understand some part of the phase structure of TGFT with straightforward QFT methods. The full theory space is very involved and largely out of reach for explicit control due to the intricate interplay between the tensorial nature of the field, the Lie group domain and its geometry, combinatorially nonlocal interactions, and geometricity constraints. Still, considering Gaussian fluctuations around a constant background vacuum field, we find that these features of TGFT can be controlled and even work in favor of a mean-field description: (1) The general $\xi^{d_{\mathrm{crit}}-d}$ asymptotic scaling towards the IR remains even with nonlocality but with a modified effective dimension $d = d_{\mathrm{eff}}$. (2) Coupling local degrees of freedom on $\mathbb{R}^{d_\phi}$ adds to the dimension $d_{\mathrm{eff}} \to d_{\mathrm{eff}} + d_\phi$. (3) Most importantly, the holonomies of the Lorentz group produce an exponential suppression of the fluctuation size such that we can always find a transition towards a phase that is self-consistently described in terms of mean-field theory regardless of the critical dimension.

Crucially, this provides evidence for the existence of a condensate phase in quantum geometric TGFTs, the mean-field hydrodynamics of which can be mapped to effective continuum cosmological dynamics [44,51]. In this way, our work gives evidence for the existence of a sensible continuum limit in TGFT quantum gravity as well as in the closely related lattice quantum gravity and spin foam models, where renormalization and the continuum limit are the main outstanding challenges [11,55–61]. Importantly, this is directly rooted in the Lorentz group and thus the causal structure it enforces, which is reminiscent of results of the causal dynamical triangulation approach [5,62–65]. Overall, our results relate to and impact on a wider set of quantum gravity approaches based on discrete structures comprising lattice quantum gravity and loop quantum gravity.

The exponential-suppression effect allows for an educated guess of the phase structure, even beyond the Gaussian approximation. Functional renormalization group calculations of local QFT on H^3 in the local-potential approximation have shown that the flowing effective potential freezes around the curvature scale a [43] such that the entire phase diagram is already described by the two phases of the mean-field regime. Using the floating-point method, one can identify the Wilson-Fisher fixed point, but it is pushed to infinity when $(ka)^{-1} \sim \xi/a \to \infty$, where k denotes the RG scale [43]. This agrees with our result that d_{eff} flows to infinity in this regime such that the nontrivial phase structure at finite dimensions vanishes. Widening the scope from the mean-field perspective applied here towards that of the functional renormalization of TGFT, we can thus also expect [66] that hyperbolic geometry yields a universal phase structure already captured by the two phases around the Gaussian fixed point, though numerical details might still depend on the specific model chosen.

Our results greatly strengthen the evidence for the existence of a continuum regime in TGFT quantum gravity, the phenomenology of which is tractable by explicit computations in a mean-field approximation. This is important given how difficult it is, because of their analytic and combinatorial complexity, to establish the same fact via a complete RG analysis of the relevant quantum geometric TGFT (and spin foam) models.

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