Dissipative Prethermal Discrete Time Crystal

DinhDuy Vu and Sankar Das Sarma
Condensed Matter Theory Center and Joint Quantum Institute, Department of Physics, University of Maryland, College Park, Maryland 20742, USA

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An ergodic system subjected to an external periodic drive will be generically heated to infinite temperature. However, if the applied frequency is larger than the typical energy scale of the local Hamiltonian, this heating stops during a prethermal period that extends exponentially with the frequency. During this prethermal period, the system may manifest an emergent symmetry that, if spontaneously broken, will produce subharmonic oscillation of the discrete time crystal (DTC). We study the role of dissipation on the survival time of the prethermal DTC. On one hand, a bath coupling increases the prethermal period by slowing down the accumulation of errors that eventually destroy prethermalization. On the other hand, the spontaneous symmetry breaking is destabilized by interaction with environment. The result of this competition is a nonmonotonic variation, i.e., the survival time of the prethermal DTC first increases and then decreases as the environment coupling gets stronger.

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Introduction.—For static systems, the spontaneous symmetry breaking (SSB) is paradigm dividing matter into phases, most notably through the Landau-Ginzburg theory. It is important to ask whether SSB can manifest in dynamic systems, in particular those with time translation symmetry. The no-go theorem prohibits the SSB of continuous time translation symmetry [1,2]. Many attempts have been made to circumvent this situation but are still debatable using long-range multispin interaction [3–5] or interacting gauge theory [6–10]. On the other hand, SSB is established to manifest under discrete time translation symmetry (in Floquet systems) [11–14], producing an exotic phase called the discrete time crystal (DTC). The signature of this phase is the many-body collective response exhibiting a longer periodicity than that of the drive, usually an integer multiple. Several experiments have successfully created DTC in atomic systems or quantum simulators [15–22].

One of the main problems in realizing DTC is the heating to infinite temperature by the periodic driving field. Therefore, it is crucial to prevent thermalization or at least delay it by a sufficiently long time. The first discovered strategy for this task is to utilize many-body localization (MBL) by introducing disorder to the Hamiltonian [11–13] or a static electric field [23]. Since MBL violates the eigenstate thermalization hypothesis, the information of the initial state still persists in the longtime dynamics [24,25], thus we can expect the DTC, if protected by MBL, to survive to infinite time. However, disorder-induced MBL requires tuning and might be difficult to engineer, not to mention that the validity of MBL in the thermodynamic limit is still not settled [26–28]. Another approach—prethermalization, on the other hand, only requires the applied frequency to be larger than the smallest energy scale of the Hamiltonian [29–34]. During the prethermal regime, the dynamics manifests an emergent symmetry with exponentially small error even though it is not an exact symmetry of the Hamiltonian. If the symmetry is represented by a $\mathbb{Z}_N$ group and is spontaneously broken, the subharmonic response will emerge in some collective degree of freedom with periodicity NT. In some exotic case, the multiplicity can even be noninteger [35]. Unlike the MBL proposal, the prethermal DTC survives only a finite time before being eventually thermalized, but this time can be exponentially extended by simply increasing the driving frequency. This characteristic has been observed [20–22]. The key point is that prethermal DTC should exist independent of whether MBL exists or not.

The above strategies were initially developed for closed systems which may not reflect the realistic situation as a system is always coupled to the environment not only in terms of heat bath but also noise. In the presence of a bath, MBL is most likely destroyed, while the fate of prethermal DTC is less straightforward. For example, a cold bath can potentially preserve the DTC to infinite time as it absorbs excessive heat generated by the drive [31]. We show that the prethermal formalism extends to open systems, i.e., up to a timescale exponentially long in the applied frequency, the dynamics is approximated by that under a time-independent Lindbladian. However, unlike a constant Hamiltonian that defines a conserved energy, a constant Lindbladian eventually drives the system to a steady state regardless of the initial condition. Therefore, the effect of dissipation on the observability of time crystal is more complicated, involving different timescales.

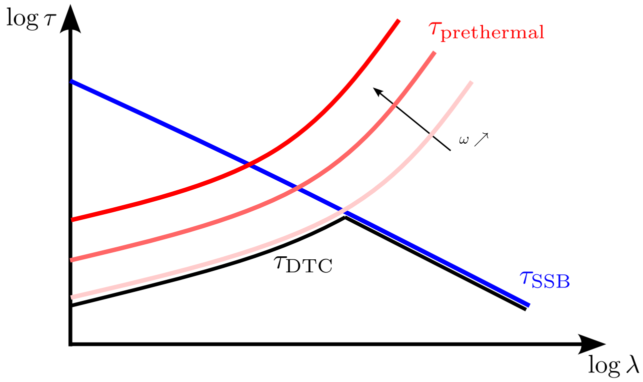

FIG. 1. Variation of the time window of the prethermalization, quasi-SSB, and DTC with respect to the dephasing noise strength. The survival time of DTC is the shorter one between the prethermal and the SSB lifetime.

The emergence of prethermal DTC is based on two conditions: the prethermal regime and the SSB of the emergent symmetry. We note that a true SSB has infinite lifetime by definition, but in a finite system, we can only achieve quasi-SSB whose large fluctuations of the order parameter must eventually vanish to recover the symmetry. The shorter lifetime between the prethermalization and quasi-SSB thus determines the survival time of DTC. Our result is summarized in Fig. 1. We show that as the environment coupling gets stronger, the prethermal regime is extended, while the quasi-SSB lifetime is reduced, leading to a nonmonotonic behavior of the DTC. Additionally, in the early increasing phase, the exponential dependence on the applied frequency is prominent; while in the later decreasing phase, the frequency becomes irrelevant. This statement is demonstrated numerically in the main text and argued analytically in Supplemental Material [36]. We mention that for some specific forms of noise operators, the decreasing trend may start at a very weak noise amplitude, making the first increasing phase almost invisible. Before proceeding to the details, we contrast our study with the dissipative time crystal that does not require emergent symmetry or MBL [42–46]. This proposal, on the other hand, requires the steady states to be degenerate or quasidegenerate, which does not hold generically but can be enforced by tuning the system across a phase transition. As such, DTC only emerges near the critical point of a phase transition and is protected by the dissipative gap between the quasidegenerate manifold and the rest of the spectrum, necessitating a fine-tuned engineering of the Lindbladian.

Prethermalization in open systems.—We first translate the derivation of prethermal DTC from the unitary evolution in closed systems to the Liouvillian evolution of open systems. The dynamics is driven by a time-periodic Liouvillian $\mathcal{L}(t) = \mathcal{L}(t+T)$ with $T = 2\pi/\omega$ being a fixed applied period. We assume that the Liouvillian contains both unitary and dissipative parts and is described by the Lindblad equation

$$\mathcal{L}[\rho] = -i[H,\rho] + \sum_j \lambda_j \left(L_j \rho L_j^\dagger - \frac{1}{2}\{L_j^\dagger L_j, \rho\} \right). \quad (1)$$

In our work, the environmental coupling manifests as local dephasing noise so the channel index j in the dissipative part is also the site index and we choose $\lambda_j = \lambda$. Another approach to access the open system is to solve the stochastic evolution of a pure state, known as Heisenberg-Langevin equation [47].

To make connection with the standard unitary evolution used to derive prethermalization, we promote the density matrix to a supervector $\|\rho\rangle\rangle$ (4^L vector) and the Liouvillian $\mathcal{L}$ to a superoperator $\hat{\mathcal{L}}$ ($4^L \times 4^L$ matrix). As a result, the density matrix at a time t is given by an evolution similar to the unitary case $\|\rho(t+T)\rangle\rangle = \hat{U}_f \|\rho(t)\rangle\rangle = \mathcal{T} e^{\int_0^T \hat{\mathcal{L}}(s) ds} \|\rho(0)\rangle\rangle$ with $\mathcal{T}$ being the time ordering operator. The expectation value of an operator is also brought into a Schrödinger-like form $\langle O(t) \rangle = \langle\langle \mathbb{1} \| O \| \rho(t) \rangle\rangle$ with $\|\mathbb{1}\rangle\rangle$ corresponding to the identity operator. Here, we use the normal definition of vector inner product. The derivation of the slow heating is similar to the unitary case except for $\hat{\mathcal{L}}$ being non-Hermitian. Because of the assumption that the dissipative dynamics is much weaker than the coherent one, the emergent symmetry is given by $X = \mathcal{T} e^{-i \int_0^T H_0(s) ds}$ satisfying $X^N = \mathbb{1}$ so that in the DTC phase, the system repeats itself after N cycles. Here, $H_0(t)$ is the dominant part of the driving Hamiltonian, in particular the energy scale $J_{\rm res}$ of the residue $H - H_0$ is much less than $1/T$. By applying similar iterative optimization as for close systems [30,31,33], we arrive at

$$e^{\hat{A}} \hat{U}_f e^{-\hat{A}} = \hat{X} \mathcal{T} e^{\int_0^T [\hat{D} + \hat{V}(s)] ds}, \quad (2)$$

where $e^{\hat{A}}$ is a time-independent trace-preserving map $[\hat{X}, \hat{D}] = 0$, and $\hat{X}$ is the superoperator promotion of $X[.]X^\dagger$. Importantly, the norm of residue term $\hat{V}$ is exponentially suppressed by the driving frequency. Because $\hat{D}$ is a complete positive trace preserving (CPT) map generator and has a much larger norm than the other terms, the "rotated" $e^{\hat{A}} \hat{U}_f e^{-\hat{A}}$ is also a CPT map. Within a time period $\sim e^{\omega/J_{\rm res}}$ only $\hat{D}$ is relevant to the dynamics and $\hat{X}$ becomes the emergent symmetry of the system. If $\hat{X}$ is spontaneously broken, the prethermal DTC is observable. Up to this point, it appears that DTC in open systems must behave similarly to its closed system counterpart. In the following, we point out two key distinctions in the residual error and the stability of SSB.

Approximating the exact Lindbladian by that only the symmetric time-independent $\hat{D}$ incurs errors which accumulate with time and eventually suppress the prethermal period, thermalizing the system. For a local observable O, the accumulation rate is bounded by the Lieb-Robinson velocity and whether the interaction is short or long ranged [48,49]. Obviously, decreasing Lieb-Robinson velocity necessarily increases $\tau_{\text{prethermal}}$. However, since our noise model is strictly on site, it is not obvious how λ can enter the expression of the Lieb-Robinson velocity v which is responsible for the intersite information propagation, not to mention that the noise magnitude is seemingly irrelevant compared to the other scales. On the other hand, if the bath coupling results in a unique steady state then information of the initial state must be lost eventually. From this longtime limit, Ref. [50] suggests that the velocity may decay exponentially in the presence of environment coupling. In our case, the on site noise induces a deceleration rate so that $\partial_t v(t) \propto -\lambda$. As a result, we can relate the dissipative prethermal period with that in the dissipationless limit

$$\tau^{\lambda}_{\text{prethermal}} = \tau^{0}_{\text{prethermal}}(1 - C\lambda\tau^{0}_{\text{prethermal}})^{-1}, \qquad (3)$$

with C being an $\mathcal{O}(1)$ constant [36]. Therefore, even though λ is the smallest scale by our assumption, it can still result in visible effects on the DTC lifetime when accompanying the exponentially long $\tau^{0}_{\text{prethermal}}$.

The second aspect where the dissipative nature of the system becomes relevant is the survival time of the quasi-SSB. In a closed system, energy is conserved so a single excitation, e.g., spin flip, is not favorable. On the other hand, the creation of multiple excitations so that the ground state is mapped to its degenerate partner requires higher orders of perturbation and hence is suppressed exponentially. Therefore, even for a finite system where the ground state must be symmetric, the energy splitting can be exponentially small. Unsurprisingly, the lifetime of the quasi-SSB scales exponentially with system size and is usually taken to be infinity even for systems with moderate size. In an open system, however, energy can be exchanged with the environment to stabilize the excitation, thus destabilizing the quasi-SSB. In fact, the finite-size effect is much more severe in open systems through the fact that $\tau_{\text{SSB}} \sim L^{1/2}$ in open 1D chains [51]. This necessitates a careful analysis on the survival time of the quasi-SSB. In Supplemental Material [36], we show that the decay rate of the quasi-SSB increases monotonically with the bath coupling strength. Unlike the effect of noise on the prethermalization, different operational forms of noise L_j may lead to vastly different decay rates. Under some form of dephasing noise, the decreasing slope (see Fig. 1) becomes much more prominent so that the increasing slope is most likely unobservable.

Numerical model.—As a demonstration, we study the driven Heisenberg chain subjected to dephasing noise. Referring to Eq. (1)

$$H(t) = \sum_i \frac{\boldsymbol{h}(t)}{2}\boldsymbol{\sigma}^i + \frac{J_{xx}}{4}\sigma_x^i\sigma_x^{i+1} + \sum_{j>i}\frac{J}{4|j-i|^{\alpha}}\sigma_z^i\sigma_z^j, \qquad (4)$$

where $\boldsymbol{\sigma}^i = \{\sigma_x^i, \sigma_y^i, \sigma_z^i\}$ is the collection of Pauli matrices at site i. The periodic Zeemann field $\boldsymbol{h}$ is given by

$$\boldsymbol{h}(t)\boldsymbol{\sigma}^i = \begin{cases} \boldsymbol{h}_s\boldsymbol{\sigma}^i & \text{for } nT < t \le (n+1)T - t_p, \\ \pi t_p^{-1}\sigma_x^i & \text{for } (n+1)T - t_p < t \le (n+1)T, \end{cases} \qquad (5)$$

with finite but small constant $\boldsymbol{h}_s$ and in the limit $t_p \to 0$. The former condition ensures the drive frequency is larger than other energy scales and the latter limit describes an instant spin flip. We note that the physics does not change significantly given a finite-width π pulse. With this setup, the emerging symmetry arises from the dominant π-pulse sequence and is given by $X = \prod_j \sigma_x^j$. Since $X^2 = \mathbb{1}$, the DTC oscillation features doubled periodicity of $2T$. The long-range zz interaction with $1 \le \alpha \le 2$ is vital to drive the transition to the spontaneous Ising symmetry breaking phase [52,53]. Last, the term J_{xx} breaks integrability so that the symmetrized $\hat{D}$ is not trivially diagonal.

For the bath coupling, we use two representative cases of dephasing noise: (i) $L_j = s_z^j$ and (ii) $L_j = s_x^j$ where the quantum channel index j is also the site index. Physically, this quantum channel describes the coupling between the spin and an isolated harmonic oscillator reservoir sitting on the same site. In their respective basis, the dephasing noise keeps the diagonal entries of the density matrix while suppressing off-diagonal ones. In both cases, we set $\lambda_j = \lambda$ thus the system does not have any spatial disorder. Because the entanglement grows much faster under long-range interaction, we limit the system to $L = 12$ and use exact diagonalization to evaluate the longtime behavior [36]. We check that a closed analog of this rather small lattice is already sufficient to demonstrate all the signatures of the DTC.

S_z-dephasing noise.—With the $\mathbb{Z}_2$ Ising symmetry as the emergent symmetry, an observable associated with an extensive degree of freedom in the DTC phase should repeat itself after $2T$. A typical choice is the magnetization or magnetization density. In this Letter, we compute the normalized magnetization, following Ref. [33]

$$C(t) = L^{-1}\sum_j \langle\sigma_z^j(t)\rangle\langle\sigma_z^j(0)\rangle. \qquad (6)$$

By definition, $C(0)$ is always normalized to unity. In Fig. 2(a), we show the stroboscopic $C(t)$ at even cycles [$C(t)$ at odd cycles is the reflection through zero] for weak dephasing noise. Here, we can see that initially the survival time of the DTC increases with the bath coupling strength. It is remarkable that this extension is significant even when λ is much smaller than any energy scales of the

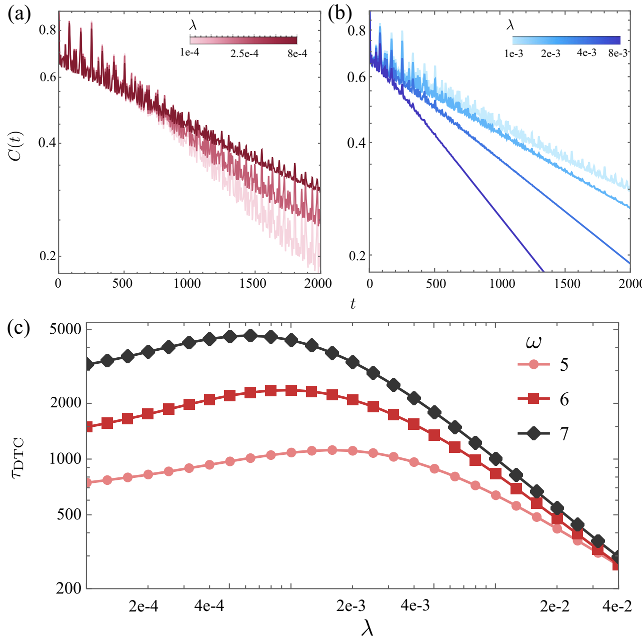

FIG. 2. $2T$-DTC oscillation sampled at $t = 2kT$ at $\omega = 6$ within the gain range (a) and the loss range (b). The color bar is logarithmic in the dephasing strength. (c) Lifetime of the time crystal for different frequency. In the gain regime, the DTC survival time is enhanced exponentially by increasing the applied frequency; while in the loss regime, the applied frequency is almost irrelevant.

Hamiltonian. This is the result of Eq. (3), specifically $\lambda \sim \mathcal{O}(10^{-3})$ but when accompanied by $\tau_{\rm prethermal^0} \sim \mathcal{O}(10^3)$ can yield visible effect on the DTC lifetime. As we further increase the noise amplitude, as shown in Fig. 2(b), the DTC survival time begins to decrease after $\lambda = 0.001$.

To understand these contrasting behaviors, we extract the lifetime $\tau_{\rm DTC}$ by fitting $C(t)$ to an exponential decay and compare among different drive frequencies increases from 5 to 6 and 7, as shown in Fig. 2(c). It is clear that in the increasing phase, the prethermal protection is apparent, i.e., $\tau_{\rm DTC}$ scales exponentially with ω. Unlike the previous increasing phase, in this phase, there is no protection by the drive frequency, showing that $\tau_{\rm DTC}$ is now bounded by a different timescale—the survival time of the quasi-SSB. We also emphases the dephasing strength where the DTC lifetime peaks shifts toward lower λ as ω increases, consistent with the picture we described in Fig. 1.

Beside the magnetization, we also compute the bipartite mutual information to understand which mechanism is responsible for the DTC decay. A subsystem of the spin chain either the rest of the chain (internal coupling) or the thermal bath (external coupling). We define the mutual information between the two halves of chain $\mathcal{I}_{L/2} = S_A + S_B - S_{AB}$ with A and B being the two halves and S_A being the Renyi entropy of the subsystem A. In Fig. 3(a), we demonstrate the prethermal physics in close systems. Under the thermalization generated by internal coupling, the mutual information always increases until saturation with slower rate for larger ω. On the other hand, if bath coupling dominates, the system becomes purely classical with vanished mutual information. In Fig. 3(b), we compare the behavior of $\mathcal{I}$ within $\tau_{\rm DTC}$. For low λ, thermalization is primarily driven by the internal coupling, characterized by the increasing of $\mathcal{I}$ with time. However, the rate decreases, consistent with the prolonged $\tau_{\rm DTC}$. When the dephasing is sufficiently strong, the bath coupling takes over the thermalization, as shown in the decreasing mutual information. For $\omega = 5$, the transition happens around $\lambda \approx 0.002$ which also coincides with the peak in Fig. 2(c). As such, the nonmonotonic behavior reported in our Letter can be associated with the transition from the quantum to classical dynamics.

S_x-dephasing noise.—Compared to the previous case of noise along the z direction, the decreasing trend is much more visible while the increasing one is negligible (see Fig. 4). In Fig. 4(b), we show the scaling with respect to frequency. The invariance against the applied frequency proves that $\tau_{\rm DTC} = \tau_{\rm SSB}$, consistent with our general result. The reason being the decay of SSB under S_x noise is much faster than under S_z noise, making the observed variation of $\tau_{\rm DTC}$ with respect to λ being heavily biased toward the decreasing phase.

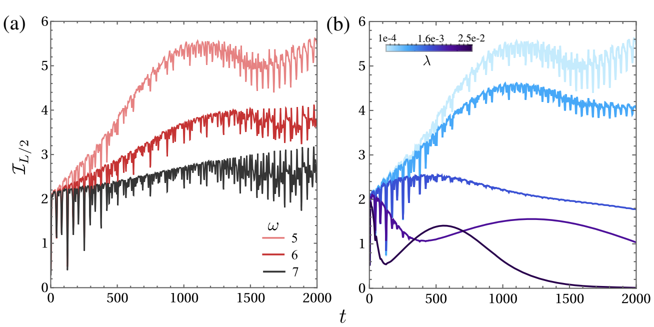

FIG. 3. (a) Bipartite mutual information for close spin chain at different drive frequencies. (b) Mutual information at $\omega = 5$ with different dephasing strengths.

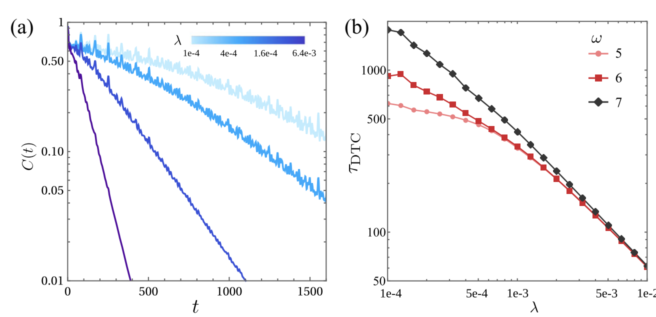

FIG. 4. DTC signal with dephasing strength at fixed $\omega = 6$ (a) and DTC lifetime with applied frequency (b) under s_x-dephasing noise. In this case, the gain regime is almost invisible.

Conclusion.—We establish the general trends of the dissipative prethermal DTC in the presence of an environmental coupling. The physics can be divided into two phases. The first increasing phase where the DTC lifetime increases with the environmental coupling, accompanied by an exponential dependence on the frequency. The second decreasing phase, by contrast, has the DTC lifetime shortened with increasing noise strength and has no frequency dependence. We note that in most of the DTC literature in the presence of bath, the instability from bath coupling is emphasized [54,55]. The stabilizing effect reported in our work is surprising, but can be hard to observe if the form of dissipation is chosen incorrectly. One situation where the environmental coupling is beneficial is the classical DTC where the bath manifests as a damping force and noise [56]. In our case, the mechanism underlying the stable branch is also the damping of nonsymmetric error accumulation manifesting in the decreasing Lieb-Robinson velocity. It is therefore interesting to draw some connection between the classical and quantum DTC.

The physics reported in our Letter can be realized in available quantum devices using stochastic trajectories. In particular, each trajectory is subjected to a randomized Ising field as

$$H_{ST}(t) = H(t) + \sum_i \epsilon_i(t) C^i, \tag{7}$$

where C is either S_z or S_x and ϵ is a random number chosen from a Gaussian distribution so that $\overline{\epsilon_i(t)} = 0$ and $\overline{\epsilon_i(t)\epsilon_j(t')} = \lambda \Delta t \delta_{i,j} \delta_{t,t'}$. Even though each trajectory is entirely Hermitian, but the effect of the dephasing noise shows up in the ensemble over random configurations.

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