Noninvertible Symmetries from Holography and Branes

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We propose a systematic approach to deriving symmetry generators of quantum field theories in holography. Central to this analysis are the Gauss law constraints in the Hamiltonian quantization of symmetry topological field theories (SymTFTs), which are obtained from supergravity. In turn, we realize the symmetry generators from world-volume theories of D-branes in holography. Our main focus is on noninvertible symmetries, which have emerged in the past year as a new type of symmetry in $d \geq 4$ QFTs. We exemplify our proposal in the holographic confinement setup, dual to 4D $\mathcal{N} = 1$ Super-Yang Mills. In the brane picture, the fusion of noninvertible symmetries naturally arises from the Myers effect on D-branes. In turn, their action on line defects is modeled by the Hanany-Witten effect.

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Introduction.—The study of quantum dynamics is at the heart of uncovering any fundamental principles of nature. From various points of view, in condensed matter physics, mathematical physics, and quantum field theory, such explorations have established the study of symmetries as an essential "backbone" of quantum systems. It thus comes as a genuine surprise in the past year where a dramatic extension to symmetries in 4D quantum field theories (QFTs) were uncovered, which unlike ordinary ones that form groups, obey fusionlike composition laws. These *noninvertible symmetries* are well established in $d = 2$, 3, however, they are unexpected in $d \geq 4$. Within the past year various systematic approaches to the construction of noninvertible symmetries have appeared in [1–7]. Physical implications include characterization of deconfining or confining vacua and constraints on pion decays [8,9], and other applications appeared in [8–14].

All constructions thus far rely on field theory methods. Here we provide the holographic perspective from symmetry inflow, supergravity, and branes. Other preliminary aspects of holography and noninvertible symmetries have been recently studied in [14–16]. Fundamental for the holographic construction is the symmetry topological field theory (SymTFT) [17–20], which naturally arises in brane and holographic setups from the anomaly polynomial and inflow [21–25]. The SymTFT on W_{d+1} encodes the full symmetry structure—the background fields for global symmetries and their 't Hooft anomalies—of a QFT on $W_d = \partial W_{d+1}$. When placed on a slab with boundaries W_d and M_d and gapped boundary condition on M_d, the SymTFT reduces to the anomaly theory of the QFT.

In this Letter, we propose a holographic derivation of the SymTFT, as well as the study of the resulting symmetries—including noninvertible ones—that depend on said boundary conditions. We derive the SymTFT by descent from the anomaly polynomial in $d + 2$ dimensions, which is encoded in the supergravity. Motivated by the work on BF-type theories in [26], the Hamiltonian quantization of the SymTFT on W_{d+1} allows us to extract Gauss's law constraints that generate gauge symmetry transformations. Under inflow, the bulk gauge symmetry restricts to the global symmetries of the boundary theory and the bulk generators flow to the desired symmetry operators.

This is complemented by a realization of the symmetry generators in terms of D-branes and their world-volume theories. The bulk supergravity fields, which define the symmetries, pull back on the brane world-volume theories. In addition, the D-branes also contribute topological sectors that dress the symmetry defect, while the kinetic terms of the brane action drop out at the boundary. These defects become noninvertible depending on the boundary conditions for the bulk fields. The brane setup and its dynamics towards the boundary provide a compelling holographic interpretation for the noninvertible fusion via the Myers effect of Dp-branes into a single D$(p+2)$-brane, which in turn implements the noninvertible fusion.

We demonstrate our proposal in the Klebanov-Strassler solution, that is dual to a flow to confining pure $\mathcal{N} = 1$ $SU(M)$ Super-Yang Mills (SYM) [27,28]. Global properties of the gauge group can be identified in holography as in [29] and the study of holographic confinement using the

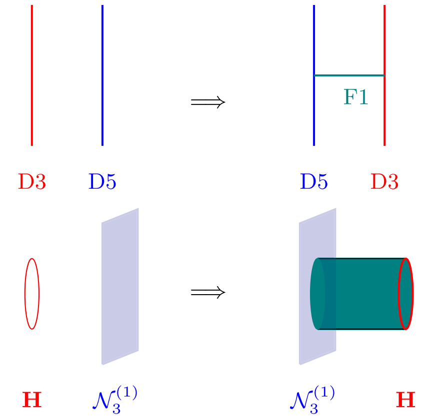

FIG. 1. Top: Hanany-Witten transition, where the (x_0, x_3) plane is displayed to show the equivalence with the field theory transition. Bottom: 't Hooft loop passing through the noninvertible defect $\mathcal{N}_3^{(1)}$ becomes attached to a topological surface operator.

't Hooft anomalies of higher-form symmetries was carried out in [30]. In this Letter, we determine a framework to construct all symmetries in this setup, in particular, the noninvertible symmetries in the $PSU(M) = SU(M)/\mathbb{Z}_M$ theory, which map between deconfining or confining vacua when spontaneously broken. In the brane picture the noninvertible fusion is naturally encoded in the Myers effect on D-branes [31]. Furthermore, the deconfining or confining transition is beautifully modeled by the Hanany-Witten brane transition [32], Fig. 1. Though we focus on holographic confinement, the methods are general and can be used to study the symmetry generators of any QFT from its SymTFT.

Field theory.—Noninvertible symmetries in QFTs in spacetime dimensions $d \geq 4$ have recently been constructed using various approaches. One is based on the presence of global symmetries that enjoy a mixed 't Hooft anomaly [1]. For concreteness we consider a 4D gauge theory (on a spin manifold) with 0-form symmetry $\Gamma^{(0)} = \mathbb{Z}_{2M}$, whose background field is A_1, and 1-form symmetry $\Gamma^{(1)} = \mathbb{Z}_M$ with background field B_2. Consider the anomaly

$$\mathcal{A} = -2\pi \frac{1}{M} \int A_1 \cup \frac{\mathfrak{P}(B_2)}{2}, \qquad (1)$$

where $\mathfrak{P}$ is the Pontryagin square. This anomaly arises in 4D $\mathcal{N} = 1$ supersymmetric Yang-Mills theories, for instance, where $\Gamma^{(0)}$ is the chiral symmetry. There is a nonanomalous $\mathbb{Z}_2^{(0)} \subset \mathbb{Z}_{2M}^{(0)}$.

The generalized 0- and 1-form symmetries [33] are generated by 3D and 2D topological defects $D_3^g(M_3)$ and $D_2^h(M_2)$, respectively, which have group composition $D_p^{g_1}(M_p) \otimes D_p^{g_2}(M_p) = D_p^{g_1 g_2}(M_p)$. Because of the anomaly, the generators for $\Gamma^{(0)}$ transform nontrivially in the presence of background fields for $\Gamma^{(1)}$

$$D_3^g(M_3) \to D_3^g(M_3) \exp\left(\int_{M_4} -\frac{2\pi i}{M} \frac{\mathfrak{P}(B_2)}{2}\right), \qquad (2)$$

where $\partial M_4 = M_3$. Gauging the 1-form symmetry makes this defect inconsistent. The proposal in [1] is to dress the defect $D_3^g(M_3)$ with a minimal TQFT $\mathcal{A}^{M,p}$, which has 1-form symmetry $\mathbb{Z}_M$ and cancels the anomaly [34].

For $\mathbb{Z}_M$ this is the minimal (spin) TQFT $\mathcal{A}^{M,1} = U(1)_M$. The dressed defects are

$$\mathcal{N}_3^{(1)} = D_3^{(1)} \otimes \mathcal{A}^{M,1}, \qquad (3)$$

where the superscript labels the generator of the 0-form symmetry. This defect has noninvertible fusion [4,8]. For M odd the TQFTs obey $\mathcal{A}^{M,1} \otimes \mathcal{A}^{M,1} = \mathcal{A}^{M,2} \otimes \mathcal{A}^{M,2}$. This results in the noninvertible fusion of the 3D defects in the $PSU(M)$ theory

$$\mathcal{N}_3^{(1)} \otimes \mathcal{N}_3^{(1)} = \mathcal{A}^{M,2} \mathcal{N}_3^{(2)}. \qquad (4)$$

Defining the conjugate $\mathcal{N}_3^{(1)\dagger} = D_3^{-1} \otimes \mathcal{A}^{M,-1}$ results in

$$\mathcal{N}_3^{(1)} \otimes \mathcal{N}_3^{(1)\dagger} = \sum_{M_2 \in H_2(M_3, \mathbb{Z}_M)} \frac{(-1)^{Q(M_2)} D_2(M_2)}{|H^0(M_3, \mathbb{Z}_M)|}, \qquad (5)$$

which is the condensation defect of the 1-form symmetry on M_3 with $D_2(M_2) = e^{i2\pi \int_{M_2} b_2/M}$, where b_2 is the gauge field for the 1-form symmetry. We will now turn to supergravity and branes and show how these noninvertible symmetries are naturally implemented in this framework.

Symmetries from holography.—We illustrate the systematic approach by realizing it in the holographic confinement setup in type IIB supergravity introduced by Klebanov-Strassler [28]. It describes the near-horizon geometry of N D3-branes probing the conifold (i.e., the Calabi-Yau cone over the Sasaki-Einstein 5-manifold $T^{1,1} \sim S^3 \times S^2$) with M D5-branes on $S^2 \subset T^{1,1}$. The near-horizon geometry is $W_5 \times T^{1,1}$, where the 4D space-time where the QFT lives is $W_4 = \partial W_5$. We assume integral N/M, so that the duality cascade in field theory ends on 4D $\mathcal{N} = 1$ $SU(M)$ SYM. The 5d effective action is written in terms of p-form field strengths f_1, $\mathcal{F}_2$, g_2, h_3, f_3 with Bianchi identities

$$df_1 = 2M\mathcal{F}_2, \quad dg_2 = Mh_3, \quad d\mathcal{F}_2 = dh_3 = df_3 = 0. \qquad (6)$$

We solve the Bianchi identities (6) in terms of

$$f_1 = f_1^\flat + dc_0 + 2MA_1, \quad \mathcal{F}_2 = \mathcal{F}_2^\flat + dA_1,$$
$$g_2 = g_2^\flat + d\beta_1 + Mb_2, \quad h_3 = h_3^\flat + db_2, \quad f_3 = f_3^\flat + dc_2, \qquad (7)$$

where A_1, c_0, b_2, β_1, c_2 are globally defined p-form gauge potentials and $f_1^{\rm b}$, $\mathcal{F}_2^{\rm b}$, $g_2^{\rm b}$, $h_3^{\rm b}$, $f_3^{\rm b}$ are closed forms with integral periods, representing topologically nontrivial base points. From Eq. (6) it follows that Mh_3 and $2M\mathcal{F}_2$ are cohomologically trivial. Assuming W_5 has no torsion, $h_3^{\rm b} = 0 = \mathcal{F}_2^{\rm b}$. The base-points $f_1^{\rm b}$, $g_2^{\rm b}$ represent integral lifts of classes in $H^1(W_5;\mathbb{Z}_{2M})$, $H^2(W_5;\mathbb{Z}_M)$ describing discrete gauge fields for a $\mathbb{Z}_{2M}$ 0-form symmetry and $\mathbb{Z}_M$ 1-form symmetry.

The relevant terms in the 5d bulk action consist of standard kinetic terms and nontrivial topological terms. The latter can be extracted [30] from the consistent truncation of [35] or via anomaly inflow [24], as reported in the Supplemental Material [36]. In order to construct the symmetry generators, it is convenient to dualize the 0-form potential c_0 into a (globally defined) 3-form gauge potential c_3. Our task, carried out in the Supplemental Material [36], is then to write the 5d bulk action in terms of A_1, b_2, β_1, c_2, c_3 and the base-point fluxes $f_1^{\rm b}$, $g_2^{\rm b}$, $f_3^{\rm b}$. The final action consists of standard kinetic terms and

$$S_{\rm top} = 2\pi \int_{W_5} \left[\frac{1}{2}N(b_2 dc_2 - c_2 db_2) + M(A_1 dc_3 + c_3 dA_1) + N b_2 f_3^{\rm b} + A_1 (g_2^{\rm b})^2\right]. \tag{8}$$

Symmetry generators.—We now analyze the 5d bulk action in the Hamiltonian formalism, treating the radial direction of W_5 as Euclidean time similarly to the AdS$_5$ cases [26,29]. Crucially, the action does not depend on the time derivatives of the time components of the gauge potentials. As a result, the associated canonical momenta are identically zero. Varying the action with respect to the time components of the gauge potentials implements the (classical) Gauss constraints. We denote the variation of the action with respect to the time component of A_1 as $\mathcal{G}_{A_1}$, and so on. We find that $\mathcal{G}_{\beta_1} = \tilde{\mathcal{G}}_{\beta_1}$ and

$$\begin{aligned} &\mathcal{G}_{b_2} = \tilde{\mathcal{G}}_{b_2} - N d_4 c_2 - N f_3^{\rm b}, \quad \mathcal{G}_{c_2} = \tilde{\mathcal{G}}_{c_2} + N d_4 b_2, \\ &\mathcal{G}_{A_1} = \tilde{\mathcal{G}}_{A_1} + 2M d_4 c_3 + (g_2^{\rm b})^2, \quad \mathcal{G}_{c_3} = \tilde{\mathcal{G}}_{c_3} + 2M d_4 A_1. \end{aligned} \tag{9}$$

Here, d_4 denotes external derivative along the spatial slice W_4, tilde the kinetic term contributions, and all fields are understood as restricted to W_4. The contributions $\tilde{\mathcal{G}}$ of the bulk kinetic terms are suppressed near the boundary [26,29].

We provide a detailed derivation of symmetry generators from Gauss's law constraints in the Supplemental Material [36]. For concreteness, let us illustrate the general analysis here by considering $e^{2\pi i \int_{M_4} [2M d_4 c_3 + (g_2^{\rm b})^2]}$ as it is brought to the boundary. Our task is to define a genuine operator on a 3-cycle M_3 such that, when raised to the $2M$th power and with $M_3 = \partial M_4$, it reproduces the operator $e^{2\pi i \int_{M_4} [2M d_4 c_3 + (g_2^{\rm b})^2]}$ from the Gauss constraint. We consider two options. In option (i), we fix $g_2^{\rm b}$ at the boundary as a classical background. This corresponds to 4D $\mathcal{N} = 1$ $SU(M)$ SYM, with a global electric $\mathbb{Z}_M$ 1-form symmetry coupled to a nondynamical discrete 2-form field. The genuine operator on M_3 in this case is simply the standard holonomy $e^{2\pi i \int_{M_3} c_3}$ accompanied by the c-number phase $e^{(2\pi i/2M) \int_{M_4} (g_2^{\rm b})^2}$. This operator obeys grouplike fusion rules. In option (ii) we sum over $g_2^{\rm b}$ at the boundary. In field theory, we gauge the electric 1-form symmetry of 4D $\mathcal{N} = 1$ $SU(M)$ SYM, thereby getting the $PSU(M)$ theory. Casting the phase $e^{(2\pi i/2M) \int_{M_4} (g_2^{\rm b})^2}$ as a genuine operator on M_3 we can rewrite $(1/2M)(g_2^{\rm b})^2$ using a 3D auxiliary theory (this is a type of inflow from the bulk operator on M_4 to M_3), which we detail in the Supplemental Material [36]. The symmetry generator on M_3 is thus

$$\mathcal{N}_3^{(1)}(M_3) = \int \mathcal{D}a e^{2\pi i \int_{M_3} (c_3 + \frac{1}{2} M a da + a g_2^{\rm b})}, \tag{10}$$

which has the noninvertible fusion rule (4).

The far IR for $PSU(M)$.—The $\mathbb{Z}_{2M}^{(0)}$ global symmetry of 4D $\mathcal{N} = 1$ $SU(M)$ SYM is spontaneously broken to $\mathbb{Z}_2^{(0)}$ in the far IR and the theory has M confining vacua. The mixed anomaly (1) is matched by a nontrivial 4D symmetry enhanced topological phase (SET) [33]

$$\mathcal{L}_{\rm 4d} = M\phi dc_3 + \frac{1}{2}\phi db_1 db_1 + \Lambda_2(db_1 + Mb_2), \tag{11}$$

where ϕ is a compact scalar of period 1, c_3, b_2, b_1 are gauge potentials and Λ_2 a Lagrange multiplier. The b_1, b_2 fields are nondynamical. The possible VEVs $\langle e^{2\pi i\phi}\rangle = e^{2\pi i p/M}$ $(p = 0, 1, \ldots, M-1)$ label the M vacua, while $e^{2\pi i \int_{C_3} c_3}$ describes a domain wall between vacua. The action (11) is invariant under the gauge transformations $b_1' = b_1 - M\lambda_1$, $c_3' = c_3 + db_1\lambda_1 - \frac{1}{2}M\lambda_1 d\lambda_1$. Thus $e^{2\pi i \int_{M_3} c_3}$ has a 't Hooft anomaly, consistently with the fact that the domain walls in the $SU(M)$ theory support a 3D TQFT $\mathcal{A}^{N,-1}$ [37].

In [30] it is demonstrated how the SET (11) emerges from the 5d bulk couplings in the IR geometry T^*S^3 (deformed conifold). In contrast to the UV analysis above, the IR analysis receives contributions from both topological and kinetic terms. The Lagrange multiplier Λ_2 is an imprint of the Stückelberg pairing between b_1, b_2 in the 5d action. The scalar ϕ is identified as c_0/M.

Let us now turn to the $PSU(M)$ theory. The far IR is still described by (11), but now b_1, b_2 are local dynamical fields. Using $db_1 = -Mb_2$, we see that the vacuum with $\langle e^{2\pi i\phi}\rangle = e^{2\pi i p/M}$ exhibits a discrete 2-form gauge theory

$\int_{M_4}(pM/2)b_2^2$. The domain walls are no longer realized as $e^{2\pi i\int_{M_3}c_3}$, which is not gauge invariant, but precisely by (10). Indeed, this operator raised to the $2M$th power with $M_3 = \partial M_4$ reduces to the manifestly gauge invariant quantity $e^{2\pi i\int_{M_4}(2Mdc_3+db_1db_1)}$ (where g_2^{b} is locally modeled by db_1). On the domain wall, both a and b_1 are dynamical and summed over. The total 3D theory is then an Abelian CS theory with levels encoded in the matrix $\left(\begin{smallmatrix} M & 1 \\ 1 & 0\end{smallmatrix}\right)$. This is a Dijkgraaf-Witten theory with gauge group $\mathbb{Z}_1$, hence trivial, as anticipated in [34].

D-branes as symmetry generators.—The topological defects also arise as boundary limits of probe branes in the bulk that are parallel to the boundary. Both in AdS in hyperpolar coordinates and in the W_5 geometry of the KS solution, where the boundary sits at $r \to \infty$, the tension $T_{\mathrm{Dp}} \sim r^p$ $(p > 0)$, such that the DBI part of the action decouples and the Wess-Zumino term dominates. In addition, we stress that these D5-branes are not necessarily BPS but are expected to be stable in the sense of [38] in $r \to \infty$. The topological terms for a D5-brane wrapping the S^3 contain the bulk forms c_3 (from C_6 on S^3) and b_1 (from C_4 on S^3), as well as the $U(1)$ gauge field a on the brane. We derive the action on the defect by reducing the D5-brane Wess-Zumino action in the Supplemental Material [36]. The result reads

$$S_{\mathrm{D5}} = 2\pi \int_{M_3} \left(c_3 + \frac{M}{2} ada + adb_1 \right). \qquad (12)$$

Here b_1 is a local gauge field. The cohomology class of db_1 is identified with g_2^{b} and is part of the data of the b_2 configuration in (7). It is interesting to understand what happens as (12) is pushed to the boundary. We always perform a path integral over a, which is a localized mode on the D5-brane. We may or may not integrate over the topologically trivial part of b_1, which is a bulk mode, depending on the boundary conditions. If we do not integrate over it, the holonomy of c_3 is dressed with the nontrivial TQFT $U(1)_M$. If we integrate over it, it becomes a trivial theory just as in the supergravity derivation. The D-branes therefore precisely give rise to the minimal TQFT stacking.

Noninvertible fusion and Myers effect.—To see the noninvertible fusion, we can either repeat the field theory analysis, given the explicit form of (12). There is a much more elegant way to obtain the fusion directly in string theory. The fusion is computed by stacking two D5-branes, which gives rise to a non-Abelian gauge theory. However, a non-Abelian brane configuration with an orthogonal S^2 geometry and a nontrivial B field undergoes the Myers effect [31] in reaching the configuration with minimal energy (see the Supplemental Material [36]). The end point configuration is given by a single D7-brane with two units of world-volume gauge flux on S^2. We then write $f_2 = f_2^{S^2} + da$, with $\int_{S^2} f_2^{S^2} = 2$. From the expansion of the Wess-Zumino action of the D7, integrating on S^2 and S^3, the terms are

$$S_{\mathrm{D7/2D5}} = 2\pi \int_{M_3} (2c_3 + Mada + 2adb_1). \qquad (13)$$

Note that this argument is applicable for any integral value of M. From the brane we thus obtain the following perspective on the fusion. Each single D5-brane results in topological defects that are dressed with $U(1)_M = \mathcal{A}^{M,1}$ CS theories—thus string theory construction automatically results in the minimal TQFT dressing of the defects. The "brane-fusion" predicts the action (13), which is $U(1)_{2M}$ CS theory coupled to b_2. This is obtained also by fusing two $U(1)_M$ theories, [39] and therefore realizes the field theory fusion rule in (4).

It is also tempting to conjecture that the fusion of $\mathcal{N}_3^{(1)}$ with its conjugate $\mathcal{N}_3^{(1)\dagger}$ (5) is the fusion between defects created by brane and antibrane, with a nontrivial field configuration. This result in the condensation defect, which is the lower-dimensional brane that couples to db_1. This is in fact expected from tachyon condensation of the D-Dbar system [40], which needs to preserve the charge under db_1, and thus is expected to give rise to a nontrivial condensate. This will be discussed elsewhere, and shown to correspond to a mesh of D3-branes.

Action on 't Hooft lines and Hanany-Witten.—The brane perspective makes the interaction between the 't Hooft line $\mathbf{H}$ and the noninvertible symmetry defect, $\mathcal{N}_3^{(1)}$, manifest. Field theoretically, when such a line crosses the noninvertible topological defect, a topological surface operator is created, which connects $\mathcal{N}$ and $\mathbf{H}$, see Fig. 1. This is due to gauge transformation $\mathbf{H} \to \mathbf{H}e^{2\pi i \oint \Lambda_1}$, where also $b_2 \to b_2 + d\Lambda_1$ [4,8].

In order to see this effect in supergravity we need to define a surface operator, which extends in the radial direction, r, and ends on the boundary, $\mathcal{O}_2(M_2)$. The 5d bulk EOMs select a natural candidate for $\mathcal{O}_2(M_2)$: the b_2 EOM imply

$$-k_{f_3} d * f_3 = Nf_3 + f_1 g_2 - Mk_{g_2} * g_2 =: M\mathcal{F}_3, \qquad (14)$$

where the k's are constants from the kinetic terms, and Nf_3 and $f_1 g_2 - Mk_{g_2} * g_2$ are separately closed. The latter combination encodes the 3-form field strength of the 2-form potential dual to b_1. On shell, $\mathcal{F}_3 = d\hat{a}_2$ for some globally defined 2-form potential. The operator $\mathcal{O}_2(M_2)$ is identified with a Wilson surface for $\hat{a}_2$,

$$\mathcal{O}_2(M_2) = e^{2\pi i \int_{M_2} \hat{a}_2}, \qquad \hat{a}_2 = a_2 + \kappa c_2, \qquad (15)$$

where $\kappa = N/M$ and a_2 is the dual of b_1 (in type IIB a_2 comes from C_4 on S^2, and the duality is a consequence of the self-dual F_5 flux).

In the frame where we keep b_1, $\mathbf{H}$ is defined by the κth power of the Wilson surface for c_2 and the ‘t Hooft surface $\mathbf{H}_{b_1}$, i.e., the disorder operator defined by $\int_{S^2} db_1 = 1$ on a small S^2 that nontrivially links with M_2 in the 5d spacetime. Under a gauge transformation $b_1 \to b_1 - M\Lambda_1$, $b_2 \to b_2 + d\Lambda_1$, $\mathcal{O}_2(M_2)$ is not gauge invariant (the periods of db_1 are only invariant modulo M). We then see that $\mathcal{O}_2(M_2)$ needs to be dressed by $e^{-2\pi i \oint b_2}$. This dressing is meaningful at the boundary when b_2 is allowed to freely vary, hence matching the field theory picture described in [8] and the fractionalization of the ‘t Hooft line when a noninvertible defect $\mathcal{N}_3^{(1)}$ is crossed.

This bulk picture fits with the D-brane picture, in which $\mathcal{O}_2(M_2)$ is realized by a D1-D3-brane bound state on $S^2 \subset T^{1,1}$, or alternatively D3s with $\kappa = N/M$ units of flux supported on S^2 [30]. In brane engineering, a Hanany-Witten transition [32] can occur when two branes link nontrivially in spacetime and are passed through each other, thereby creating a new extended object stretching between them. In our setup this can happen for D3s wrapping S^2 and extending along the radial direction r and D5s on S^3 localized at the boundary:

Brane	x_0	x_1	x_2	x_3	r	z_1	z_2	w_1	w_2	w_3
D3	X				X	X	X			
D5	X	X	X					X	X	X
F1	X			X						

(16)

Here $z_{1,2}$, $w_{1,2,3}$ are local coordinates on S^2 and S^3, respectively. The relevant brane linking in our system is measured by the following quantity L defined modulo M,

$$L = \int_{M_2 \times S^3} F_5 = -\int_{M_1 \times S^2} F_3 = \int_{M_2} db_1 = -\int_{M_1} dc_0, \quad (17)$$

where $M_2 = \mathbb{R}_{x_1} \times \mathbb{R}_{x_2}$ and $M_1 = \mathbb{R}_r$. On the world volume of $\mathcal{N}_3^{(1)}$, the EOM for a implies $db_1 = -M da$. Thus, db_1 is exact modulo M. As a result, the linking L must be conserved modulo M. When the D3 crosses the D5, this changes to $db_1 = -M da + \delta(\mathrm{pt} \subset M_2)$. The localized source is the effect of a new object (an F1-string) that is created, which intersects both M_2 and M_3 and extends along $t = x_0, x_3$, Fig. 1. The system in (16) is related to the original Hanany-Witten setup NS5-D5-D3 by S and T dualities. The D3- and D5-branes link in the direction x_3, this means that an F1 is created when the D3 crosses the D5 (16). F1 strings are indeed electrically charged under $e^{-2\pi i \oint b_2}$, which was precisely the dressing for $\mathcal{O}_2(M_2)$. This also matches the physics of the action on the ‘t Hooft loop in $PSU(M)$ through a noninvertible domain wall between deconfining or confining vacua, that mimics closely the order or disorder transition in the Ising model.

Outlook.—We provide a bottom-up approach—via Gauss law constraints in supergravity—and top-down one—via branes in string theory—for constructing symmetry operators in holography. Our methods are crucial for a systematic extraction of symmetry defects, whenever SymTFTs are available. It deserves further study. Future applications include theories that have a similar type of mixed anomalies in the SymTFT, such as $\mathcal{N} = 4$ SYM theories holographically dual to $\mathrm{AdS}_5 \times S^5$ with noninvertible duality defects [2,13]. A similar realization of these topological defects in terms of M5-branes at the boundary of conical in G_2-holonomy spaces is also tempting, and show similar features to (16). Finally, we also briefly comment on the holographic realization of the (self-) duality and triality of noninvertible topological defects [4] for $\mathcal{N} = 4$ SYM in $\mathrm{AdS}_5 \times S^5$. Duality and triality are all subgroups of $SL(2,\mathbb{Z})$ symmetries, therefore is very tempting to conjecture that the topological defects in this case are engineered by 7-branes wrapping S^5. These are just examples of possible applications of this approach, which we plan to come back to in the future, but very importantly they show the broader scope of the holographic supergravity and brane approach, which are meant to address questions about symmetries of the QFTs living at the boundary.

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