What Is the Gravitational Field of a Mass in a Spatially Nonlocal Quantum Superposition?

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The study of the gravitational field produced by a spatially nonlocal, superposed quantum state of a massive particle is an interesting and active area of research. One outstanding issue is whether the gravitational field behaves like the classical superposition of the gravitational field of two particles separated by a spatial distance with half the mass at each position. Alternatively, does the gravitational field behave as a quantum superposition with a far more interesting and subtle behavior than a simple classical superposition? Quantum field theory is ideally suited to probe exactly this kind of question. We study the scattering of a massless scalar on a spatially nonlocal quantum superposition of a massive particle. We compute the differential scattering cross section corresponding to one-graviton exchange. We find that the scattering cross section disagrees with the Newton-Schrödinger picture of potential scattering from two localized sources with half the mass at each source. This suggests that experimental observation of gravitational scattering could inform the viability of the semiclassical treatment of the gravitational field, as in the Newton-Schrödinger description, vs the fully quantum mechanical treatment adopted here. We comment on the experimental feasibility of observing such effects in systems with many particles such as Bose-Einstein condensates.

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Introduction.—At the core of modern physics lie two of the most successful theories of physics: general relativity and quantum field theory. Combining these theories in a consistent manner is arguably the most important challenge of modern theoretical physics. The most common approach is to consider the quantum theory to be more fundamental and to attempt to quantize gravity [1]. This choice is justified indirectly; see, for instance, [2]. The opposite point of view, "gravitizing" quantum mechanics, has also been advocated, with some interesting repercussions [3–5].

In this Letter, we will sidestep the challenge of formulating a complete quantum theory of gravity and will investigate quantum gravity as a low-energy effective field theory. This approach, while limited in scope, is perfectly consistent within its domain of validity [6–9]. However, the directly observable predictions of such an effective quantum field theory present significant observational challenges. An intriguing possibility for the detection of noise due to quantum fluctuations in the gravitational field at gravitational wave detectors such as LIGO [10] has been suggested recently [11]. Within the effective field theory point of view, gravitation must be treated as a fully quantum mechanical field, giving rise to the possibility of complex and surprising behavior in scattering processes if the initial state is nonstandard (whereby *standard* we refer to the scattering of two particles in wave packets highly peaked in momentum space, or simply in plane waves). It is in this context that we examine the scattering of a massless particle in a plane wave due to the gravitational field of a massive particle in a spatially nonlocal quantum superposition.

We begin with a brief discussion of the semiclassical Newton-Schrödinger formalism to compare its implications with our fully quantum treatment that follows. Then we give a description of how to analyze scattering from a nonlocal wave packet using the Korvalets-Kotkin *et al.* formalism [12,13]. In the subsequent sections, we analyze the one-graviton exchange scattering amplitude, the differential scattering cross section, and finally the multipole expansion of this cross section. The main, interesting result that we find is that the differential scattering cross section is essentially insensitive to the fact that the massive particle was in a spatially nonlocal quantum superposition. We end with a discussion of our results and conclusion.

Newton-Schrödinger formalism.—The Newton-Schrödinger (NS) equation was first introduced by Ruffini and Bonazzola [14], in the analysis of self-gravitation of boson stars. The NS equation appears as a nonrelativistic limit of the Klein-Gordon equation or the Dirac equation in the context of general relativity [15]. It also describes fuzzy dark matter obeying a Vlasov-Poisson type equation [16] as a model for cold dark matter in the limit of large particle mass. The NS equation appears in the context of the mutual gravitational or electromagnetic interaction of a plasma as proposed by Choquard as cited in [17], where he has proved the uniqueness of the ground state. There have been many other investigations into the use of the NS equation; see, for example [18–21]. Diósi [22] and Penrose [3], who actually coined the name of the NS equation, studied the equation in an effort to understand the collapse of the wave function. More recently, it has been discussed in connection with the suggestion that gravity might not need quantization [5]. The formalism incorporates the effect of gravity on a quantum particle under the assumption that the gravitational potential is treated classically, with the mass probability density of the particle appearing as the source.

Indeed, if the quantum particle is of mass m and is described by the wave function ψ, according to the NS formalism the gravitational potential Φ is determined by Poisson's equation,

$$\nabla^2\Phi = 4\pi G m\psi^\dagger\psi, \tag{1}$$

whose formal solution is well known:

$$\Phi(\boldsymbol{x}) = -Gm\int d^3x' \frac{\psi^\dagger\psi(\boldsymbol{x}')}{|\boldsymbol{x}-\boldsymbol{x}'|}. \tag{2}$$

The potential, multiplied by m, appears as a potential energy term in the Schrödinger equation, giving the so-called NS equation

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + m\Phi\psi. \tag{3}$$

Note that, given Eq. (2), the NS equation is nonlinear and nonlocal. We see clearly that if a particle is in a superposition of two spatially separated components, according to the NS formalism the gravitational potential will be the coherent sum of the contributions from each component. Gravitational scattering off such a particle would simply result in the sum of two contributions, one from each component.

Scattering on a spatially nonlocal wave packet.—The goal of scattering experiments is to infer information about the intrinsic interaction between the scattering particles (the differential cross section) which is independent of the details of the experiment (the particle fluxes, etc.). Typically, the initial particles are presumed to be in plane waves (or in states highly peaked in momentum space), and therefore in spatially extended states. The cross section depends only on the nature of the interaction, the energy of the experiment, and the scattering angle; see, for example, [23] for a detailed discussion including a definition of all the parameters.

The usual formula for the differential cross section is

$$d\sigma = \left(\prod_f \frac{d^3p_f}{(2\pi)^3}\frac{1}{2E_f}\right)\int\frac{d^3k_1}{(2\pi)^3}\int\frac{d^3k_2}{(2\pi)^3}\frac{|\mathcal{M}(k_1,k_2\to\{p_f\})|^2}{2\epsilon_1 2\epsilon_2|v_1-v_2|}$$
$$\times|\phi_1(\boldsymbol{k}_1)|^2|\phi_2(\boldsymbol{k}_2)|^2(2\pi)^4\delta^4\left(k_1+k_2-\sum p_f\right), \tag{4}$$

where $\phi_1(\boldsymbol{k}_1)$, $\phi_2(\boldsymbol{k}_2)$ are the momentum-space wave functions of the initial particles. Here, k_1, etc., are the four momenta; $\boldsymbol{k}_1$, etc., are the three momenta; and $\hat{\boldsymbol{k}}_1$, etc., are the unit three momenta; all the initial and final particles are on shell, ϵ_i is the energy of the ith particle. The highly peaked nature of the initial momentum-space wave functions is an integral part of using Eq. (4) to extract the differential scattering cross section; for a more general initial state, a more careful treatment must be performed.

Here, we wish to consider a process outside the usual paradigm, wherein one of the particles, of mass M, is *not* highly peaked in momentum space. Rather, it is in a superposition of two spatially localized states with negligible overlap between them. As this is *a priori* not a standard textbook calculation of a cross section, it is necessary to use an approach that would enable us to study the scattering of the wave packets in general. The work [12,13] develops exactly such a method, generalizing the customary (plane-wave) S-matrix formalism. In their analysis, the incoming particles are described by their Wigner functions [24].

Let the momentum-space wave function of the incoming massive particle, which is necessarily broad, be $\phi_1(\boldsymbol{k}_1)$. For simplicity, we suppose ϕ_1 is symmetrically distributed around $\boldsymbol{k}_1 = 0$, i.e., $\phi_1(-\boldsymbol{k}_1) = \phi_1(\boldsymbol{k}_1)$. Thus, the calculation is done in the center-of-mass frame of the massive particle. A second particle, presumed massless and of well-defined momentum, scatters off the first. Let the momentum-space state of this second particle be $\phi_2(\boldsymbol{k}_2)$, highly peaked about its central value $\boldsymbol{p}_2$.

We are interested in the inclusive scattering cross section for the massless particle, $\boldsymbol{p}_2\to\boldsymbol{p}_4$, with the massive particle, whose initial momentum is not highly peaked around a single value, scattering into all possible single particle final states as shown in Fig. 1. It is most convenient to consider the wave function of the scattered massive particle, that was initially in the spatially nonlocal wave function, to be scattered to wave functions that are eigenstates of momentum $\boldsymbol{p}_3$ that respect energy-momentum conservation. Then to integrate over this momentum, as the set of momentum eigenstates do correspond to a complete set of final states for the massive particle. In practice, the integration is of course not required as energy-momentum conservation for given on-shell four momenta k_1, p_2, p_4 fixes the value of p_3 (correspondingly for the three-momenta, $\boldsymbol{k}_1$, $\boldsymbol{p}_2$, $\boldsymbol{p}_4$ fix the value of $\boldsymbol{p}_3$). Thus, the scattering will give rise to final momenta $k_1\to p_3$ and

$p_2 \to p_4$ with integration over $\boldsymbol{k}_1$ smeared with wave function $\phi_1(\boldsymbol{k}_1)$ understood.

In order to handle the broad nature of $\phi_1(\boldsymbol{k}_1)$, we will use the formalism of Kotkin *et al.* [12], which employs Wigner functions [24] to describe the incoming particles that are not in momentum eigenstates. We refer the reader to [13] for a detailed discussion of the formalism and its domain of validity. The formula given by Eq. (2.4) in [13] simplifies for the case of one particle in a momentum eigenstate scattering with one particle in a nontrivial wave packet, we find

$$d\sigma = \frac{\sigma^2}{L\pi^{3/2}} \int \frac{d^3k_1}{(2\pi)^3} |T_{PW}(\{k_1, p_2\} \to \{p_3, p_4\})|^2 |\phi_1(\boldsymbol{k}_1)|^2 (2\pi)^4 \delta^{(4)}(k_1 + p_2 - p_3 - p_4) \times \frac{d^3p_3}{(2\pi)^3} \frac{d^3p_4}{(2\pi)^3}, \tag{5}$$

where the plane wave amplitude $T_{PW}(\{k_1, p_2\} \to \{p_3, p_4\})$ is given by

$$T_{PW}(\{k_1, p_2\} \to \{p_3, p_4\}) = \frac{\mathcal{M}(\{k_1, p_2\} \to \{p_3, p_4\})}{\sqrt{2\epsilon_1 2\epsilon_2 2\epsilon_3 2\epsilon_4}} \tag{6}$$

with $\mathcal{M}(\{k_1, p_2\} \to \{p_3, p_4\})$ the invariant matrix element for the momentum space scattering process [23], ϵ_i the energy of particle i, and L the total luminosity is given by [13]

$$L = \frac{\sigma^2}{\pi^{3/2}} \int \frac{d^3k_1}{(2\pi)^3} \left(1 - \frac{\boldsymbol{k}_{1z}}{M} + \frac{|\boldsymbol{k}_1|^2}{2M^2} + \cdots \right) |\phi_1(\boldsymbol{k}_1)|^2 = \frac{\sigma^2}{\pi^{3/2}} \left(1 + \frac{3}{4M^2\sigma^2} + o(e^{-r^2/\sigma^2})\right). \tag{7}$$

We note that L replaces the usual factor of $|v_1 - v_2|$ of the particle flux. The velocity of the massless particle is unity, and the expression simplifies because of the assumed symmetry of the wave function $\phi_1(\boldsymbol{k}_1)$. The normalized wave function for the spatially nonlocal particle in momentum space is taken to have the form

$$|\phi_1(\boldsymbol{k}_1)|^2 = 4(\pi\sigma^2)^{3/2} e^{-\sigma^2|\boldsymbol{k}_1|^2} \frac{2 + e^{2i\boldsymbol{r}\cdot\boldsymbol{k}_1} + e^{-2i\boldsymbol{r}\cdot\boldsymbol{k}_1}}{1 + e^{-|\boldsymbol{r}|^2/(\sigma^2)}}, \tag{8}$$

where σ is the width of a Gaussian wave packet that is superposed at spatial position $\boldsymbol{r}$ and $-\boldsymbol{r}$. We could use a more general form for the wave function of the nonlocal state, however, we find that our main result depends only on the overlap integral of the two spatially nonlocal peaks.

The one-graviton exchange and the scattering cross section.—The amplitude is easily computed using the linearized gravitational theory and corresponding graviton propagator and matter vertices, following Donoghue [7,8] (see also [9,25–29]), as prescribed by the Feynman diagram Fig. (1).

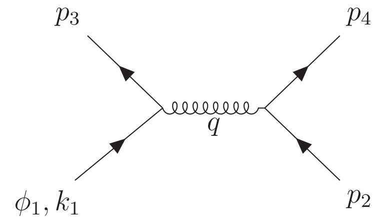

FIG. 1. One-graviton exchange scattering Feynman diagram.

The explicit expression for the amplitude is given by

$$\begin{aligned} \mathcal{M} = \frac{\kappa^2}{1 - \hat{\boldsymbol{p}}_2 \cdot \hat{\boldsymbol{p}}_4} [&2M^2 - 2M\boldsymbol{k}_1 \cdot (\hat{\boldsymbol{p}}_2 + \hat{\boldsymbol{p}}_4) \\ &+ M(\omega_2 - \omega_4)(1 - \hat{\boldsymbol{p}}_2 \cdot \hat{\boldsymbol{p}}_4) + 2|\boldsymbol{k}_1|^2 \\ &+ 2(\boldsymbol{k}_1 \cdot \hat{\boldsymbol{p}}_2)(\boldsymbol{k}_1 \cdot \hat{\boldsymbol{p}}_4) - \boldsymbol{k}_1 \cdot (\omega_2 \hat{\boldsymbol{p}}_2 - \omega_4 \hat{\boldsymbol{p}}_4)(1 - \hat{\boldsymbol{p}}_2 \cdot \hat{\boldsymbol{p}}_4) \\ &- \omega_2\omega_4(1 - \hat{\boldsymbol{p}}_2 \cdot \hat{\boldsymbol{p}}_4)^2]. \end{aligned} \tag{9}$$

where κ is the gravitational coupling constant and ω_2 and ω_4 are shorthand notation for p_2^0 and p_4^0. The momentum transfer is $q = p_3 - k_1 = p_2 - p_4$ with $q^2 = -2p_2 \cdot p_4 = -2\omega_2\omega_4(1 - \hat{\boldsymbol{p}}_2 \cdot \hat{\boldsymbol{p}}_4)$ as p_2 and p_4 are massless and on shell. The amplitude must now be folded in with the wave function and various kinematical and numerical factors as in Eq. (5) and then integrated over d^3p_3 (which removes the spatial delta functions) and $d\omega_4$ (which removes the temporal delta function), where ω_i is simply compact notation for p_i^0 which is also the energy of the massless particles, yielding the differential scattering cross section

$$\frac{d\sigma}{d\Omega_4} = \frac{1}{[1 + \frac{3}{4M^2\sigma^2} + O(e^{-r^2/\sigma^2})]} \int \frac{d^3k_1}{2^9\pi^5} |\phi_1(\boldsymbol{k}_1)|^2 \times \frac{|\mathcal{M}|^2}{\epsilon_1\omega_2\epsilon_3\omega_4} \frac{\omega_4^2}{|f'_\delta(\omega_4)|}. \tag{10}$$

The energy conserving delta function is given by $\delta[f_\delta(\omega_4)] = \delta(\epsilon_1 + \omega_2 - \epsilon_3 - \omega_4)$ where the complications arise because $\epsilon_3 = \sqrt{M^2 + (\boldsymbol{k}_1 + \boldsymbol{p}_2 - \omega_4\hat{\boldsymbol{p}}_4)^2}$ where $\hat{\boldsymbol{p}}_4$ corresponds to the unit three vector in the $\boldsymbol{p}_4$ direction. $\epsilon_1 = \sqrt{M^2 + \boldsymbol{k}_1^2}$ is the energy of the Fourier component corresponding to momentum $\boldsymbol{k}_1$ of the spatially nonlocal particle wave function and ω_2 is the energy of the incoming massless particle. The full expression for the cross section is complicated and unenlightening; however, its multipole expansion does shed some light on the gravitational interaction.

Multipole expansion of the scattering cross section.—We can write the cross section from Eq. (10) as

$$\frac{d\sigma}{d\Omega_4} = \frac{1}{[1+\frac{3}{4M^2\sigma^2}+o(e^{-r^2/\sigma^2})]}\int\frac{d^3k_1}{2^9\pi^5}|\phi_1(\boldsymbol{k}_1)|^2 g(\boldsymbol{k}_1)|\mathcal{M}|^2 \tag{11}$$

(we will drop the exponentially small terms in the prefactor in the following) and then $g(\boldsymbol{k}_1)|\mathcal{M}|^2$ admits an expansion in powers of $\boldsymbol{k}_1$. It is important to always remember that the true small parameter that we take is $|\vec{k}_1|/M$. The Taylor expansion is

$$g(\boldsymbol{k}_1)|\mathcal{M}|^2 = .g(\boldsymbol{k}_1)|\mathcal{M}|^2|_{\boldsymbol{k}_1=0} + \frac{1}{2}\partial_{\boldsymbol{k}_1^i}\partial_{\boldsymbol{k}_1^j}(g(\boldsymbol{k}_1)|\mathcal{M}|^2)\big|_{\boldsymbol{k}_1=0}\boldsymbol{k}_1^i\boldsymbol{k}_1^j + \cdots, \tag{12}$$

where the terms odd in $\boldsymbol{k}_1$ are absent due to symmetry and due to the fact that there is no physical, negative mass and correspondingly the dipole term can always be removed by a judicious choice of the origin of coordinates. Then the scattering cross section admits the following multipole expansion:

$$\frac{d\sigma}{d\Omega_4} = \alpha\int|\phi_1(\boldsymbol{k}_1)|^2\frac{d^3k_1}{(2\pi)^3} + \beta^{ij}\int\boldsymbol{k}_{1i}\boldsymbol{k}_{1j}|\phi_1(\boldsymbol{k}_1)|^2\frac{d^3k_1}{(2\pi)^3}+\cdots \tag{13}$$

$$= \alpha + \frac{\beta^{ij}\delta_{ij}}{3}\int|\boldsymbol{k}_1|^2|\phi_1(\boldsymbol{k}_1)|^2\frac{d^3k_1}{(2\pi)^3} + \beta^{ij}\int\left(\boldsymbol{k}_{1i}\boldsymbol{k}_{1j}-\frac{|\boldsymbol{k}_1|^2}{3}\delta_{ij}\right)|\phi_1(\boldsymbol{k}_1)|^2\frac{d^3k_1}{(2\pi)^3} + \cdots \tag{14}$$

$$= \alpha + \left\{\frac{\beta^{ij}\delta_{ij}}{3}\left(\frac{3}{2\sigma^2}-\frac{|\boldsymbol{r}|^2}{\sigma^4}\frac{1}{1+e^{|\boldsymbol{r}|^2/\sigma^2}}\right)\right\} + \left[\beta^{ij}\left(\frac{|\boldsymbol{r}|^2}{3}\delta_{ij}-\boldsymbol{r}_i\boldsymbol{r}_j\right)\frac{1}{\sigma^4(1+e^{|\boldsymbol{r}|^2/\sigma^2})}\right]+\cdots \tag{15}$$

where evidently $\alpha = \{1/[1+(3/4M^2\sigma^2)]\}(1/2^6\pi^2)g(\boldsymbol{k}_1)\times|\mathcal{M}|^2|_{\boldsymbol{k}_1=0}$ and $\beta^{ij} = (1/2^7\pi^2)\{1/[1+(3/4M^2\sigma^2)]\}\partial_{\boldsymbol{k}_1^i}\partial_{\boldsymbol{k}_1^j}\times(g(\boldsymbol{k}_1)|\mathcal{M}|^2)|_{\boldsymbol{k}_1=0}$ and where we have used the integral

$$\int\boldsymbol{k}_{1i}\boldsymbol{k}_{1j}|\phi_1(\boldsymbol{k}_1)|^2\frac{d^3k_1}{(2\pi)^3} = \frac{\delta_{ij}}{2\sigma^2}-\frac{\boldsymbol{r}_i\boldsymbol{r}_j}{\sigma^4}\frac{1}{1+e^{|\boldsymbol{r}|^2/\sigma^2}}. \tag{16}$$

These leading terms in the expansion about $\boldsymbol{k}_1 = 0$ are found after a somewhat long calculation. For the second derivative we will use

$$\partial_{\boldsymbol{k}_1^i}\partial_{\boldsymbol{k}_1^j}(g(\boldsymbol{k}_1)|\mathcal{M}|^2)\big|_{\boldsymbol{k}_1=0} = \mathcal{M}^2\partial_{\boldsymbol{k}_1^i}\partial_{\boldsymbol{k}_1^j}g + 2\mathcal{M}(\partial_{\boldsymbol{k}_1^i}g\partial_{\boldsymbol{k}_1^j}\mathcal{M} + \partial_{\boldsymbol{k}_1^j}g\partial_{\boldsymbol{k}_1^i}\mathcal{M}) + 2g(\partial_{\boldsymbol{k}_1^i}\mathcal{M}\partial_{\boldsymbol{k}_1^j}\mathcal{M} + \mathcal{M}\partial_{\boldsymbol{k}_1^i}\partial_{\boldsymbol{k}_1^j}\mathcal{M})\big|_{\boldsymbol{k}_1=0}. \tag{17}$$

Putting everything together, we find

$$\alpha = \frac{1}{(1+\frac{3}{4M^2\sigma^2})}\left(\frac{\kappa^4M^2}{16\pi^2(1-\hat{\boldsymbol{p}}_2\cdot\hat{\boldsymbol{p}}_4)^2}\right), \tag{18}$$

$$\beta^{ij} = \frac{1}{(1+\frac{3}{4M^2\sigma^2})}\left(\frac{\kappa^4}{16\pi^2(1-\hat{\boldsymbol{p}}_2\cdot\hat{\boldsymbol{p}}_4)^2}\right)(\delta_{ij}+3\hat{\boldsymbol{p}}_{2i}\hat{\boldsymbol{p}}_{2j}). \tag{19}$$

The term α gives exactly the limiting gravitational deflection of a massless particles from a massive particle, at small momentum transfer [29].

Discussion and conclusions.—The term proportional to α, the lowest order monopole term, corresponds to the scattering cross section from a single point particle of mass M, the analog of the Rutherford-Thompson cross section of a massless particle scattering from a pointlike Newtonian potential. We see that the higher-order terms coming from the $\beta^{ij}\delta_{ij} = \{6/[1+(3/4M^2\sigma^2)]\}[\kappa^4/16\pi^2(1-\hat{\boldsymbol{p}}_2\cdot\hat{\boldsymbol{p}}_4)^2]$ term in Eq. (15) contribute to the monopole, these contributions are not exponentially suppressed. This does, however, mean that the scattering cross section is able to probe the nonpointlike nature of the gravitating source. This addition to the monopole contribution is given by the term in Eq. (15) in the curly brackets.

Our most surprising result is that the actual quadrupole-type contribution is nothing like what would be expected if the gravitational field behaved according to the NS formalism [3,22]. In the NS formalism we would expect to have the gravitational field as if one-half the mass were concentrated at each of the two spatially nonlocal points [30], a configuration which has a quadrupole moment $M[(|\boldsymbol{r}|^2/3)\delta_{ij}-\boldsymbol{r}_i\boldsymbol{r}_j]$. The corresponding gravitational field yields a contribution to the (gravitational) scattering cross section

$$\frac{d\sigma^{NS}}{d\Omega_{4,\mathrm{quad}}} = \frac{\kappa^4M^2\omega_2^2}{16\pi^2(1-\hat{\boldsymbol{p}}_2\cdot\hat{\boldsymbol{p}}_4)^2}(\hat{\boldsymbol{p}}_{4i}-\hat{\boldsymbol{p}}_{2i})(\hat{\boldsymbol{p}}_{4j}-\hat{\boldsymbol{p}}_{2j}) \times\left(\frac{|\boldsymbol{r}|^2}{3}\delta_{ij}-\boldsymbol{r}_i\boldsymbol{r}_j\right) \tag{20}$$

obtained from a presumed Newtonian potential scattering from two point sources of mass $M/2$ located at the two peaks of the spatially nonlocal wave function where $\kappa = \sqrt{8\pi G}$ which gives a coefficient $4G^2$. This result is not at all what we find, the quadrupole contribution is given by the term in Eq. (15) in the square brackets

$$\frac{d\sigma^{1\mathrm{GE}}}{d\Omega_{4,\mathrm{quad}}} = \frac{1}{(1+\frac{3}{4M^2\sigma^2})}\frac{\kappa^4}{16\pi^2\sigma^4(1+e^{|\boldsymbol{r}|^2/\sigma^2})} \times \frac{3\hat{\boldsymbol{p}}_{2i}\hat{\boldsymbol{p}}_{2j}}{(1-\hat{\boldsymbol{p}}_2\cdot\hat{\boldsymbol{p}}_4)^2}\left(\frac{|\boldsymbol{r}|^2}{3}\delta_{ij} - \boldsymbol{r}_i\boldsymbol{r}_j\right), \quad (21)$$

where 1 GE stands for one graviton exchange. Indeed, the quadrupole term from one-graviton exchange scattering is exponentially small as $|\boldsymbol{r}| \gg \sigma$ compared to the result expected from scattering from a potential where the mass is split between two positions as prescribed by the NS formalism. The exponential suppression occurs from the integration over the momenta of the incoming spatially nonlocal wave function. In momentum space this wave function has the form given in Eq. (8). Performing a multipole expansion of the amplitude and the subsequent momentum integral will only give multipole moments in $\boldsymbol{r}$ that are exponentially suppressed. For a generic spatially nonlocal wave function with two peaks, the suppression would be proportional to the overlap integral of the two peaks.

This result casts doubt on the general applicability of the NS formalism; the calculation corresponding to one-graviton exchange is clearly justified in the context of scattering of a massless particle, where the relativistic formalism is indispensable. Furthermore, one-graviton exchange is perfectly understood in an effective quantum field theory of gravitation that is a valid quantum description of gravitation in all processes that do not involve the ultraviolet limit [9]. Our result Eq. (21) shows that the wave function is only probed by the incoming massless particle's direction relative to the direction of separation $\boldsymbol{r}$, a behavior that may prove to be experimentally verifiable. One should not conclude that the differential cross section is independent of p_4 as it does appear in the factor $[1/(1-\hat{\boldsymbol{p}}_2\cdot\hat{\boldsymbol{p}}_4)^2]$ in the front of the expression for the quadrupole contribution Eq. (21). That the p_4 dependence is not analogous to that for the NS result in Eq. (20) is a further manifestation of the apparent fact that the differential scattering cross section is rather insensitive to the nonlocal superposition. Further quantum corrections can of course be computed via higher-order loop corrections to the scattering cross section.

A useful further calculation would be to compute the gravitational contribution to the self-energy of a massive particle in a spatially nonlocal wave function. One would look for the amplitude and behavior of the self-energy as a function of the spatial separation r of the nonlocal wave function. A $1/r$ behavior of the self-energy would correspond to the Newtonian potential and the $1/r^2$ law of attraction of the two nonlocal lumps. A calculation of the amplitude would indicate how the two nonlocal lumps behave gravitationally with respect to each other.

If the NS formalism is assumed derived from a fully quantum fundamental theory, it should admit improvements from relativistic quantum mechanics as worked out here. But the NS formalism in the more limited sense is considered complete in itself [21,31,32] and it predicts deviation from linear superposition. It will face tests through proposals such as [33,34]. Experimental proposals include measurement of Aharonov-Bohm type phases [35] and inference of the gravitational field superposition through quantum measurements [33]. A more direct check that the NS needs to be modified along the lines of this investigation, would require an experimentally obtained spatially nonlocal superposition of about 10^9 atoms. A Bose condensate, which typically comprises this number of atoms could in principle be launched in an atom interferometer to give rise to a spatially nonlocal superposition [33,36] and might be experimentally realizable [37]. Further possibilities include molecular interferometry [38] and its possible deployment in space [31].

Another very interesting aspect would be to investigate the decoherence of a spatially nonlocal superposition. However, it is clear from our calculations that the differential scattering cross section is unable to do so. A finer-toothed comb would be necessary to uncover any relationship between the gravitational field and decoherence. Coupling to an external heat bath and entanglement with other external systems would be critical for the analysis of decoherence [39,40] which is a clearly identified direction for future work.

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[1] Claus Kiefer, *Quantum Gravity*, 2nd ed. (Oxford University Press, New York, 2007).
[2] Don N. Page and C. D. Geilker, Indirect Evidence for Quantum Gravity, Phys. Rev. Lett. **47**, 979 (1981).
[3] Roger Penrose, On gravity's role in quantum state reduction, Gen. Relativ. Gravit. **28**, 581 (1996).
[4] Roger Penrose, On the gravitization of quantum mechanics 1: Quantum state reduction, Found. Phys. **44**, 557 (2014).
[5] Steve Carlip, Is quantum gravity necessary?, Classical Quantum Gravity **25**, 154010 (2008).
[6] John F. Donoghue, Leading Quantum Correction to the Newtonian Potential, Phys. Rev. Lett. **72**, 2996 (1994).

[7] John F. Donoghue, General relativity as an effective field theory: The leading quantum corrections, Phys. Rev. D **50**, 3874 (1994).
[8] John F. Donoghue, Introduction to the effective field theory description of gravity, in *Advanced School on Effective Theories*, edited by F. Cornet and M. J. Herrero (World Scientific Press, Singapore, 1995), p. 27610.1142/3326.
[9] C. P. Burgess, Quantum gravity in everyday life: General relativity as an effective field theory, Living Rev. Relativity **7**, 5 (2004).
[10] Gregory M. Harry *et al.* (LIGO Scientific Collaboration), Advanced LIGO: The next generation of gravitational wave detectors, Classical Quantum Gravity **27**, 084006 (2010).
[11] Maulik Parikh, Frank Wilczek, and George Zahariade, Quantum Mechanics of Gravitational Waves, Phys. Rev. Lett. **127**, 081602 (2021).
[12] G. L. Kotkin, V. G. Serbo, and A. Schiller, Processes with large impact parameters at colliding beams, Int. J. Mod. Phys. A **07**, 4707 (1992).
[13] Dmitry V. Karlovets, Scattering of wave packets with phases, J. High Energy Phys. 03 (2017) 049.
[14] Remo Ruffini and Silvano Bonazzola, Systems of self-gravitating particles in general relativity and the concept of an equation of state, Phys. Rev. **187**, 1767 (1969).
[15] Domenico Giulini and André Großardt, The Schrödinger-Newton equation as a non-relativistic limit of self-gravitating Klein-Gordon and Dirac fields, Classical Quantum Gravity **29**, 215010 (2012).
[16] Philip Mocz, Lachlan Lancaster, Anastasia Fialkov, Fernando Becerra, and Pierre-Henri Chavanis, Schrödinger-Poisson–Vlasov-Poisson correspondence, Phys. Rev. D **97**, 083519 (2018).
[17] Elliott H. Lieb, Existence and uniqueness of the minimizing solution of Choquard's nonlinear equation, Stud. Appl. Math. **57**, 93 (1977).
[18] Oliver Robertshaw and Paul Tod, Lie point symmetries and an approximate solution for the Schrödinger-Newton equations, Nonlinearity **19**, 1507 (2006).
[19] Paul Tod and Irene M. Moroz, An analytical approach to the Schrödinger-Newton equations, Nonlinearity **12**, 201 (1999).
[20] Mohammad Bahrami, André Großardt, Sandro Donadi, and Angelo Bassi, The Schrödinger-Newton equation and its foundations, New J. Phys. **16**, 115007 (2014).
[21] Huan Yang, Haixing Miao, Da-Shin Lee, Bassam Helou, and Yanbei Chen, Macroscopic Quantum Mechanics in a Classical Spacetime, Phys. Rev. Lett. **110**, 170401 (2013).
[22] L. Diosi, Gravitation and quantum-mechanical localization of macro-objects, Phys. Lett. **105A**, 199 (1984).
[23] Michael E. Peskin and Daniel V. Schroeder, *An Introduction to Quantum Field Theory* (Addison-Wesley, Reading, USA, 1995).
[24] E. Wigner, On the quantum correction for thermodynamic equilibrium, Phys. Rev. **40**, 749 (1932).
[25] Bryce S DeWitt, Quantum theory of gravity. I. The canonical theory, Phys. Rev. **160**, 1113 (1967).
[26] Bryce S. DeWitt, Quantum theory of gravity. II. The manifestly covariant theory, Phys. Rev. **162**, 1195 (1967).
[27] Bryce S. DeWitt, Quantum theory of gravity. III. Applications of the covariant theory, Phys. Rev. **162**, 1239 (1967).
[28] Martin J. G. Veltman, *Quantum theory of gravitation, Méthodes en Théorie des Champs* (North-Holland, Amsterdam, 1976), pp. 266–328.
[29] Michael D. Scadron, *Advanced Quantum Theory* (World Scientific Publishing Company, Singapore, 2006).
[30] Lajos Diosi, Nonlinear Schrödinger equation in foundations: Summary of 4 catches, in *Journal of Physics: Conference Series* (IOP Publishing, 2016), Vol. 701, p. 012019, 10.1088/1742-6596/701/1/012019.
[31] Angelo Bassi, André Großardt, and Hendrik Ulbricht, Gravitational decoherence, Classical Quantum Gravity **34**, 193002 (2017).
[32] Stephen L. Adler, Comments on proposed gravitational modifications of Schrödinger dynamics and their experimental implications, J. Phys. A **40**, 755 (2007).
[33] Chris Overstreet, Joseph Curti, Minjeong Kim, Peter Asenbaum, Mark A. Kasevich, and Flaminia Giacomini, Inference of gravitational field superposition from quantum measurements, arXiv:2209.02214.
[34] Sourav Kesharee Sahoo, Ashutosh Dash, Radhika Vathsan, and Tabish Qureshi, Testing gravitational self-interaction via matter-wave interferometry, Phys. Rev. A **106**, 012215 (2022).
[35] Chris Overstreet, Peter Asenbaum, Joseph Curti, Minjeong Kim, and Mark A. Kasevich, Observation of a gravitational Aharonov-Bohm effect, Science **375**, 226 (2022).
[36] Simon A. Haine, Searching for signatures of quantum gravity in quantum gases, New J. Phys. **23**, 033020 (2021).
[37] Marc Kasevich, Public Communication, Testing Gravity 2019, Simon Fraser University, Vancouver, 2019.
[38] Stefan Gerlich, Lucia Hackermüller, Klaus Hornberger, Alexander Stibor, Hendrik Ulbricht, Michael Gring, Fabienne Goldfarb, Tim Savas, Marcel Müri, Marcel Mayor, and Markus Arndt, A Kapitza-Dirac-Talbot-Lau interferometer for highly polarizable molecules, Nat. Phys. **3**, 711 (2007).
[39] Maximilian Schlosshauer, Decoherence, the measurement problem, and interpretations of quantum mechanics, Rev. Mod. Phys. **76**, 1267 (2005).
[40] Wojciech Hubert Zurek, Decoherence, einselection, and the quantum origins of the classical, Rev. Mod. Phys. **75**, 715 (2003).