Locally Mediated Entanglement in Linearized Quantum Gravity

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The current interest in laboratory detection of entanglement mediated by gravity was sparked by an information-theoretic argument: entanglement mediated by a local field certifies that the field is not classical. Previous derivations of the effect modeled gravity as instantaneous; here we derive it from linearized quantum general relativity while keeping Lorentz invariance explicit, using the path-integral formalism. In this framework, entanglement is clearly mediated by a quantum feature of the field. We also point out the possibility of observing "retarded" entanglement, which cannot be explained by an instantaneous interaction. This is a difficult experiment for gravity, but is plausible for the analogous electromagnetic case.

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It is often assumed that quantum gravitational effects only show up at high-energy or short length scale regimes, out of reach of current technology. Recent proposals for low-energy tabletop experiments could be game changers [1–7]. Rapid technological progress in quantum manipulation of solid-state matter at larger microscopic mass scales [8,9] and in gravitational measurements at smaller mesoscopic mass scales [10] have raised expectations that probing gravitational phenomena of quantum source masses may be within reach [11]. In particular, it might be possible to detect entanglement between two masses generated by their gravitational interaction, or gravity induced entanglement (GIE) [2,3].

Verifying GIE would spectacularly support what is expected from most tentative quantum gravity theories: spacetime has quantum properties. It would also falsify—or put limits on—the alternatives that have been considered in the absence of empirical evidence for quantum gravity: for example, that gravity is a classical field obeying semi-classical Einstein equations [12–14] or that quantum mechanics breaks down at a scale before measurable quantum gravity effects appear [15–17]. Specifically, a general quantum information argument has been invoked to argue that GIE would rule out the possibility that the gravitational field is a local, classical field [2,3,18–21]. The argument is based on the fact that local operations and classical communication cannot produce entanglement according to quantum theory [22], as well as to more general approaches [19,21]. Then, the argument goes, observing GIE certifies that gravity cannot be described by classical physics: either the interaction is nonlocal, or it is nonclassical.

However, the implications of GIE detection are being debated even when assuming linearized quantum gravity. In this context, some claims have been made that the experiment does not detect a quantum property of the gravitational field [23,24]. The disagreement partially stems from the fact that the effect has generally been computed within the approximation of an instantaneous interaction. Indeed, since the imagined experiment involves masses with nonrelativistic motion placed close to each other, in this regime gravity can effectively be described without the need of a dynamical field. But this approach hides a core ingredient of the theory: relativistic locality. There is strong independent experimental evidence that the gravitational interaction is not instantaneous.

We provide a derivation of the effect within linearized quantum gravity, using the path-integral formalism, which keeps the symmetries explicit. In particular, spacetime locality is kept manifest. Starting from two established paradigms of physics [25]—general relativity and quantum field theory—we show here that the quantum phases responsible for gravity mediated entanglement production are on-shell actions [cf. Eq. (5)], which we compute below [cf. Eq. (8)]. This provides an explicitly Lorentz invariant, hence spacetime local, and gauge invariant description of GIE. This is our main result.

In our analysis, GIE turns out to be due to the fact that the overall path integral reduces to a finite sum, in each term of which the functional integral can be estimated by a "semiclassical" saddle point approximation. In other words, in this formalism the effect is due to a genuinely quantum feature of the gravitational field: the possibility to be in a quantum superposition of distinct semiclassical configurations.

This implies that, in the context of linearized quantum gravity, GIE arises due to a quantum superposition of spacetimes [27], each propagating information causally. Thus, information travels in a quantum superposition of wave fronts in the field and entanglement starts being generated only *after* a light crossing time has elapsed.

We consider a consequence of this local propagation in linearized quantum gravity: the existence of an experiment where both entanglement and relativistic locality can be observed, thus, incompatible with an instantaneous interaction description. For gravity, this is currently out of reach, but we find the analogous experiment in electromagnetism to be feasible. Since our analysis proceeds completely analogously for the electromagnetic case, this would inform the outcome of the gravitational experiment.

Locally mediated entanglement from the path integral of the quantum field.—Consider the experimental setup in [2] that comprises two [28] masses m_a ($a = 1, 2$), each with an embedded spin-$1/2$ degree of freedom. At time t^i, the particles are at initial positions x_a^i and are then put in a spin-dependent planar motion $x_a^{s_a}(t)$, by being passed through inhomogeneous and possibly time varying magnetic fields B_z oriented along the axis z, perpendicular to the plane of motion. We denote $|\sigma\rangle = \otimes_a |s_a\rangle$ the spin configurations, where $s_a \in \{\uparrow, \downarrow\}$.

The spacetime curvature is assumed to be small, and so the linear approximation of general relativity holds. We denote the gravitational perturbation sourced by the particles as $\mathcal{F}$ [29]. Preparing each of the particles in a spin superposition state, the magnetic field B_z drives the particles into a path superposition by coupling to the spins s_a. The field $\mathcal{F}$ couples to the masses m_a of the moving particles. After recombining the interferometer paths at time t_2, see Fig. 1, a spin measurement is performed on each particle at time t^f. The spins can become entangled due to the gravitational interaction between the masses m_a. The coupling of B_z with $\mathcal{F}$, the backreaction of s_a on B_z, and the backreaction of $\mathcal{F}$ on the particle trajectories $x_a^{s_a}(t)$ are taken to be negligible.

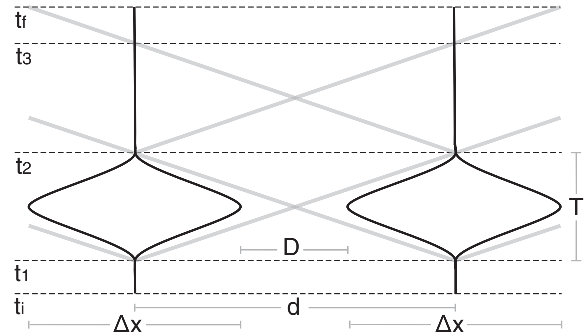

FIG. 1. The light cone structure forbids entanglement when the superposition happens at spacelike separation. This occurs when $d/T \geq c$.

The transition amplitudes are computed using the path integral

$$\int \mathcal{D}\mathcal{F}'\mathcal{D}x' \exp\left(\frac{iS}{\hbar}\right), \tag{1}$$

where $S = [x_a'(t), \mathcal{F}'(x,t); m_a, B_z, \sigma]$; $\mathcal{D}x' = \prod_a \mathcal{D}x_a'$, and the integration is over field configurations $\mathcal{F}'(x,t)$ and paths of the particles $x_a'(t)$ [30]. The quantities m_a, B_z, and σ are not affected by the dynamics in (1).

The path that each particle takes is determined by the spin, which does not change along the path. Thus, the joint evolution is of the form

$$U_{i\to f} = \sum_\sigma |\sigma\rangle\langle\sigma| \otimes U^\sigma_{i\to f}, \tag{2}$$

with $U^\sigma_{i\to f}$ defined by folding (1) with initial and final states $|\psi^{i,f}\rangle = |\mathcal{F}^{i,f}[x_a^{i,f}]\rangle \otimes |x_a^{i,f}\rangle$, where the paths and field states are assumed pure and separable at $t^{i,f}$. The boundary conditions are taken the same for all spin configurations σ. The time t^f is far enough in the future for the field to have time to relax in the vicinity of the spin measurement. It is sufficient to take the boundary conditions as given by the static Newtonian field $\mathcal{F}^{i,f}[x_a^{i,f}]$ of masses sitting at the initial and final particle positions $x_a^{i,f}$.

The task is to calculate $U^\sigma_{i\to f}$ up to normalization. The field integration can be heuristically performed by a stationary phase approximation, keeping the contribution of the field configurations $\mathcal{F}[x_a(t)]$ that solve the classical field equations sourced by particles of mass m_a with classical paths $x_a(t)$ and boundary conditions $|\psi^{i,f}\rangle$. Then,

$$U^\sigma_{i\to f} \propto \int_i^f \mathcal{D}x' \exp\left(\frac{iS[x_a', \mathcal{F}[x_a']]}{\hbar}\right)|\psi^f\rangle\langle\psi^i|. \tag{3}$$

This approximation allows us to sidestep the rigorous definition of the path integral [32,33] and neglects quantum fluctuations.

Between times t^i and t^f, for each spin configuration σ there is a classical path $x_a^{s_a}$ determined by the magnetic field B_z coupled to the spin s_a of each particle. These paths can be taken as orthogonal states and the remaining integral approximated by a second stationary phase approximation, keeping only the contribution on these paths

$$U^{\sigma}_{i\to f} \propto \exp\left(\frac{iS^{\sigma}[x_a^{s_a}, \mathcal{F}[x_a^{s_a}]]}{\hbar}\right)|\psi^f\rangle\langle\psi^i|. \tag{4}$$

Here, for a given spin configuration σ, S^σ is the on-shell action for the joint system of spins, paths, and field.

The action S splits as $S = S_0 + S_{\mathcal{F}}$. S_0 does not depend on $\mathcal{F}$; it contains the matter kinetic terms and the coupling of B_z with the spins s_a. S_0 can be calculated, or measured, separately. For simplicity, we assume the setup to be chosen so that S_0 is the same for all σ and becomes a global phase. $S_{\mathcal{F}}$ contains the on-shell contributions of the kinetic terms for the field $\mathcal{F}$ and of the coupling of $\mathcal{F}$ with the masses m_a along their motion x_a: $S_{\mathcal{F}}$ contains the "field mediation." We define

$$\phi_\sigma = \frac{S^{\sigma}_{\mathcal{F}}[x_a^{s_a}, \mathcal{F}[x_a^{s_a}]]}{\hbar}. \tag{5}$$

Given an initially separable state $|\Psi^i\rangle \propto |\psi^i\rangle \otimes \sum_\sigma A_\sigma|\sigma\rangle$ of field, paths, and spins, with A_σ complex amplitudes, the final state is given by

$$|\Psi^f\rangle = U_{i\to f}|\Psi^i\rangle \propto |\psi^f\rangle \otimes \sum_\sigma A_\sigma e^{i\phi_\sigma}|\sigma\rangle. \tag{6}$$

Note that boundary states are not entangled with the spin configurations at initial and final times. However, depending on the values of $S^\sigma_{\mathcal{F}}$, entanglement can be produced among the spin degrees of freedom. The phases ϕ_σ are the result of the entanglement production mediated through $\mathcal{F}$ [2,3,27]. We have shown that the phases ϕ_σ are on-shell actions, therefore they are manifestly local and gauge invariant. Differences of ϕ_σ for different σ, the relative phases among branches, are the observables measured by the experiment. We now compute ϕ_σ.

Covariant phases for the gravitational field of moving particles.—The action of linearized gravity coupled to matter is gauge invariant. On shell, it reads

$$S_{\mathcal{F}} = \frac{1}{4}\int d^4x\, h_{\mu\nu}T^{\mu\nu}, \tag{7}$$

where $h_{\mu\nu}$ is the metric perturbation and $T_{\mu\nu}$ is the energy-momentum tensor. Modeling the masses as point particles with arbitrary timelike trajectories, their gravitational field is the gravitational analog of the Liénard-Wiechert potentials of electromagnetism [31,34,35]. The on-shell action (7) is then given by

$$S_{\mathcal{F}} = \frac{G}{c^4}\sum_{a,b}^{a\neq b}\int dt\frac{m_a m_b \bar{V}_a^{\mu\nu}(t_{ab})V_{b\mu\nu}(t)}{d_{ab}(t) - \boldsymbol{d}_{ab}(t)\cdot\boldsymbol{v}_a(t_{ab})/c}, \tag{8}$$

where $V_a^{\mu\nu} = \gamma_a v_a^\mu v_a^\nu$, $v_a^\mu = (c, \boldsymbol{v}_a)$, $\boldsymbol{v}_a$ is the three velocity, and γ_a is the Lorentz factor.

The analogous formula in electromagnetism is obtained by replacing $\bar{V}^{a\mu\nu}V_{b\mu\nu} \to v_a^\mu v_{b\mu}$, $m \to q$ and $G/c^4 \to \kappa_e/2c^2$, where q is the charge and κ_e is Coulomb's constant. For the notation and a detailed derivation of (8), see the Appendix and the Supplemental Material [31]. The crucial point is that the distance d_{ab} and the time t_{ab} are retarded quantities.

The action (8) is a sum of two terms per pair of particles. Each term is the contribution from one particle at coordinate time t interacting with the other causally, that is, with retardation. The causal interaction between matter and the gravitational field is thus entirely encoded in $S_{\mathcal{F}}$. This manifestly Lorentz and gauge invariant quantity gives the observables measured in the experiment.

Slow-motion approximation versus Newtonian limit.—When the source is "slow moving," meaning moving at nonrelativistic speeds $|\boldsymbol{v}_a| \ll c$, we have $\bar{V}_a^{\mu\nu}(t_{ab})V_{b\mu\nu}(t) = c^4 + \mathcal{O}(c^3|\boldsymbol{v}_a|)$ and (8) approximates

$$S^\sigma_{\mathcal{F}} \approx \frac{1}{2}G\sum_{a,b}^{a\neq b}\int dt\frac{m_a m_b}{d^\sigma_{ab}(t)}. \tag{9}$$

In this regime the interaction is still local. The distance $d^\sigma_{ab}(t) = |\boldsymbol{x}_b^{s_b}(t) - \boldsymbol{x}_a^{s_a}(t^\sigma_{ab})|$ depends on the retarded time function $t^\sigma_{ab}(t)$. While the speed of light c has canceled out in the prefactor of (9), it is still present implicitly in the definition of $t^\sigma_{ab}(t)$. Equation (9) can be regarded as the causal version of Newton's law for gravitation.

A different approximation for (8) can be taken when the source's characteristic scale of time variation divided by c is much larger than the distance of the source. Then, retardation in the field can be neglected in the vicinity of the source. This is a near-field approximation; it amounts to replacing the retarded time functions t_{ab} in (8) with the coordinate time t, hence modeling the gravitational interaction as an instantaneous interaction. The slow-moving and near-field approximations do not imply each other; there are physical regimes when one is applicable and the other is not, and vice versa. When both approximations are applied, they yield the "Newtonian limit." Taking in addition d to be constant during a relevant time T, corresponding to considering a static approximation, we recover the formula used in the literature

$$\phi_\sigma \approx \frac{Gm_1m_2T}{\hbar d}. \tag{10}$$

This expression for the phases ϕ_σ naively models an instantaneous interaction, but it is just an approximation to the manifestly local on-shell action (8) of the joint system of paths, spins, and field.

Observable effect of retardation.—The effect of retardation can be quantified as the correction to the Newtonian limit (10) by the slow-moving approximation (9). Importantly, a *qualitatively* different behavior can be observed when the spatial superposition of the particles happens entirely within spacelike separated regions.

Take the particles at rest at a distance d for all times $t < t_1$ and $t > t_2$. Between t_1 and t_2, the particles undergo a spin-dependent motion. The setup is such that $c(t_2 - t_1) < d$, so that the nonstationary parts of the worldlines are spacelike separated. From time $t_3 = t_2 + d/c$, the retarded position of each particle with respect to the other is again constant (see Fig. 1). With this setup, no entanglement can be generated.

Let $x_a^{s_a}(t)$ be the displacement of particle a from its initial position due to the coupling of the external magnetic field B_z with its spin s_a in the spin configuration σ. We remind the reader that $|\sigma\rangle = \otimes_a |s_a\rangle$. Using (9), ϕ_σ is a sum of integrals that can be done by splitting the domain of integration in four. Then,

$$\int_{t^i}^{t^f} \frac{dt}{d_{21}^{\sigma}(t)} = \int_{t^i}^{t_1} \frac{dt}{d} + \int_{t_1}^{t_2} \frac{dt}{d - x_1^{s_1}(t)} + \int_{t_2}^{t_3} \frac{dt}{d + x_2^{s_2}[t_{21}(t)]} + \int_{t_3}^{t^f} \frac{dt}{d}. \qquad (11)$$

Then, the phases are of the form $\phi_\sigma = C + \phi_{s_a} + \phi_{s_b} + C'$ with terms that depend on, at most, one spin [36]. Thus, if the initial states of the spins is separable, so will be the final state. If, on the other hand, one calculates the phase in the Newtonian limit with instantaneous interaction, the spins result in an entangled state.

Experimental considerations.—The effect described above can, in principle, be observed experimentally, even though the parameters may be challenging. One possible way to achieve spacelike separation between the two interferometer loops of [2] is to increase the velocity $\mathbf{v}$ at which the particles traverse the apparatus. We denote d as the initial distance of the particles, Δx as the maximum separation of the path superposition, and $D = d - \Delta x$ as the minimum distance of the branches at closest approach, see Fig. 1.

Using the Newtonian limit (10) and assuming $\Delta x \ll d$, the entanglement is maximal when [2] $\Delta\phi \approx (A/A_P)^2(\Delta x/d)^2(cT/D) = \pi$, where $T = t_2 - t_1$, A is the mass m, and A_P is the Planck mass $m_P = \sqrt{\hbar c/G}$. For the Coulomb case, A is the charge q and A_P is the Planck charge $q_P = \sqrt{4\pi\epsilon_0 \hbar c}$.

As we showed above, when $d \geq cT$, no entangling interaction can take place between the particles. In other words, one can create a situation in which the Newtonian limit yields $\Delta\phi \sim 1$, while the actual value predicted by (8) and (9) is $\Delta\phi \sim 0$. Fixing the speed v so that $d/T = c$ and assuming $\Delta x \ll d$ at these timescales, achieving such maximum discrepancy would require $A \gg A_P$. For the gravity case, this results in magnetic fields and coherence requirements that are not realistic for the foreseeable future (for comparison, current proposals operate in a regime $A \approx 10^{-10} A_P$ at much larger timescales, while the magnetic field requirements for coherent splitting scale with both mass and time).

Smaller effects of retardation are more easily measured. Let us assume, for the sake of the argument, that one can detect a one part in a thousand deviation from the Newtonian approximation. One can estimate the retarded phases by replacing T with $T - d/c$, which implies a correction $\delta(\Delta\phi) \approx (A/A_P)^2(\Delta x/d)^2$. This will still require fairly large A/A_P, which is unlikely reachable for the gravity case. For the electromagnetic case, however, ion and electron interferometry offers a promising path [37]. For a single electron, $A/A_P \approx 0.1$. Assuming $d \approx 1$ cm, $T \approx 50$ ns, and that the superposition is produced by diffraction with a grating of periodicity 10 nm, it is possible to produce $\Delta x/d \approx 0.3$ and thus the desired $\delta(\Delta\phi) \approx 10^{-3}$. In an interferometer of length 10 cm, this can be achieved with electron velocity of $v = 10^{-2} c$, which is reachable in current electron microscopes.

One possible way of testing for entanglement generation in such a scenario could be indirectly via controllable decoherence and recoherence of the single-electron interference signals [38]: if no entanglement is generated, both interferometers will show full (single-electron) coherence, while any generation of entanglement would decohere the single-electron interference signals. Both scenarios are accessible by changing the velocity of both beams. While this is not an easy experiment to perform, it is plausible for the near future.

Discussion.—We considered experimental proposals aiming at observing the entanglement between two masses (or charges) due to the mediation of their gravitational (or electromagnetic) interaction. Entanglement happens because different quantum branches accumulate different phases. The phases were previously computed using an instantaneous interaction, which in part obscured the relevance of the experiment.

We computed the phases from first principles and showed there are differences in on-shell actions. They are manifestly Lorentz invariant, hence causal, and gauge invariant. We considered the approximation where the particles' motion is nonrelativistic and showed that this is still causal as it includes the corrections for retardation. As expected, retardation has an observable effect in the production of mediated entanglement.

The physical picture arising from our analysis is that the mechanism giving rise to entanglement is a quantum superposition of macroscopically distinct dynamical field configurations. Per Eq. (5), it is this superposition that gives rise to different phases for each quantum branch.

Our analysis gives a complementary point of view to the work in [18,39–41], where it is concluded that the mediation of quantum information takes place through the exchange of virtual gravitons (or virtual photons). Indeed, at the level of perturbation theory, scattering potentials can be understood as the result of exchanging virtual particles. We have seen here that setting the field on shell (that is, neglecting quantum fluctuations) on each quantum branch is sufficient to recover the causal propagation of signals. A physical interpretation of our analysis is that quantum information propagates casually due to the field wave fronts being in a quantum superposition.

As an application of our results, we considered an experiment to detect retardedly induced entanglement. This is for the moment a gedanken experiment for gravity. Because of the theoretical and physical analogies, it is interesting to consider performing the analogous experiment in electromagnetism. We estimate this task to be challenging but plausible.

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Appendix: Derivation of (8).—Below we summarize the derivation of the on-shell action (8) and explain the notation. A more pedagogical derivation is provided in the Supplemental Material [31], also for the electromagnetic case.

The gauge invariance of $S_{\mathcal{F}}$ can be used to simplify computations by writing the Lagrangian in the Lorenz gauge $\partial^\nu \bar{h}_{\mu\nu} = 0$. The action for linearized gravity coupled to matter then simplifies to [42–45]

$$S_{\mathcal{F}} = \frac{c^4}{64\pi G}\int d^4x\left(-\partial_\rho h_{\mu\nu}\partial^\rho h^{\mu\nu} + \frac{1}{2}\partial^\mu h\partial_\mu h\right) + \frac{1}{2}\int d^4x h_{\mu\nu}T^{\mu\nu}. \tag{A1}$$

where $d^4x = dtd^3x$ and $T^{\mu\nu}$ is the energy-momentum tensor. Greek indices denote four-vectors and bold latin letters denote three-vectors. The metric perturbation satisfies $|h_{\mu\nu}| \ll 1$ and $\eta_{\mu\nu}$ is the Minkowski metric. We use the notation $X = \eta_{\mu\nu}X^{\mu\nu}$ for the trace and $\bar{X}^{\mu\nu} = X^{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}X$ the trace reversed of a two-tensor.

The Euler-Lagrange equations for the field are

$$\Box h_{\mu\nu} = -\frac{16\pi G}{c^4}\bar{T}_{\mu\nu}. \tag{A2}$$

When the field is taken on shell, we can integrate by parts the terms with two derivatives of $h_{\mu\nu}$ in (A1) to obtain two terms of the form $h_{..}\Box h^{..}$ and use (A2) to get (7).

Next, we consider the gravitational interaction of N point particles of masses m_a. The use of point particles is an approximation that allows one to use an explicit solution of the field equations. So long as the size of the two matter distributions is much smaller than their separation, so that finite size effects can be neglected, the use of point charges will be a good approximation.

The solution obtained here is the gravitational analog of the well-known Liénard-Wiechert potential of electromagnetism [34,35].

The stress-energy tensor for N point masses is

$$T^{\mu\nu}(t,\boldsymbol{x}) = \sum_{a=1}^{N} m_a\delta^{(3)}[\boldsymbol{x} - \boldsymbol{x}_a(t)]V_a^{\mu\nu}(t), \tag{A3}$$

where $V_a^{\mu\nu}(t) = \gamma_a(t)v_a^\mu(t)v_a^\nu(t)$ with $v_a^\mu(t) = (c, \boldsymbol{v}_a)$, where $\boldsymbol{v}_a = d\boldsymbol{x}_a/dt$ is the velocity of particle a and $\gamma_a(t) = [1 - |\boldsymbol{v}_a(t)|^2/c^2]^{-1/2}$ is the corresponding Lorentz factor. The retarded solution of the wave equation (A2) for all times is

$$h_{\mu\nu}(t,\boldsymbol{x}) = \frac{4G}{c^4}\int dx'^3\,\frac{\bar{T}_{\mu\nu}(\boldsymbol{x}', t_r)}{|\boldsymbol{x} - \boldsymbol{x}'|}, \tag{A4}$$

with the retarded time $t_r = t_r(t,\boldsymbol{x},\boldsymbol{x}')$ defined by $ct_r = ct - |\boldsymbol{x}' - \boldsymbol{x}|$. Plugging in the expression for the energy-momentum tensor, we obtain

$$h_{\mu\nu}(t,\boldsymbol{x}) = \frac{4G}{c^4}\sum_a m_a\int d^3x'\,\frac{\delta^{(3)}[\boldsymbol{x}' - \boldsymbol{x}_a(t_r)]\bar{V}_a^{\mu\nu}(t_r)}{|\boldsymbol{x} - \boldsymbol{x}'|}. \tag{A5}$$

To deal with the awkward dependence of the retarded time t_r on $\boldsymbol{x}'$ we introduce an integration in a dummy time variable t' over a delta function $\delta(t' - \tilde{t})$. We can then do the $\boldsymbol{x}'$ integration to get

$$h_{\mu\nu}(t,\boldsymbol{x}) = \frac{4G}{c^4}\sum_a m_a\int dt'\,\frac{\bar{V}_{\mu\nu}(t')\delta(t' - \tilde{t}(\boldsymbol{x}, t, t'))}{|\boldsymbol{x} - \boldsymbol{x}_a(t')|}. \tag{A6}$$

For the remaining integration in t', we make use of the identity $\delta[f(y)] = \sum_i[\delta(y - y_i)/|\partial_y f(y_i)|]$, where y_i are zeros of $f(y)$. It follows that

$$\delta(t' - \tilde{t}(\boldsymbol{x}, t, t')) = \frac{\delta(t' - t_a)}{1 - \boldsymbol{d}_a\cdot\boldsymbol{v}_a(t_a)/(d_a c)}, \tag{A7}$$

where the retarded time t_a is implicitly defined as a function of t and $\boldsymbol{x}$ as satisfying $c(t-t_a) = |\boldsymbol{x} - \boldsymbol{x}_a(t_a)|$. Here, t_a is the time at which the past light cone of the event $(t, \boldsymbol{x})$ intersects the worldline of particle a. We also defined the retarded displacement $\boldsymbol{d}_a = \boldsymbol{d}_a(t, \boldsymbol{x}) = \boldsymbol{x} - \boldsymbol{x}_a(t_a)$ and its magnitude $d_a = |\boldsymbol{d}_a|$. One then obtains the following field:

$$h^{\mu\nu}(t, \boldsymbol{x}) = \frac{4G}{c^4} \sum_a \frac{m_a \bar{V}_a^{\mu\nu}(t_a)}{d_a - \boldsymbol{d}_a \cdot \boldsymbol{v}_a(t_a)/c}, \qquad \text{(A8)}$$

where the values of the field at any given spacetime point $(t, \boldsymbol{x})$ depend exclusively on the behavior of the particles on the past light cone of $(t, \boldsymbol{x})$.

Next we calculate the on-shell action of N interacting point masses. We plug in the energy-momentum tensor (A3) for N point particles into (7) and perform the space integration

$$S_{\mathcal{F}} = \frac{1}{4} \sum_{b=1}^{N} \int dt \, m_b V_b^{\mu\nu}(t) h_{\mu\nu}(t, x_b(t)). \qquad \text{(A9)}$$

Next, we use (A8) to obtain (8)

$$S_{\mathcal{F}} = \frac{G}{c^4} \sum_{a,b} \int dt \frac{m_a m_b \bar{V}_a^{\mu\nu}(t_{ab}) V_{b\mu\nu}(t)}{d_{ab} - \boldsymbol{d}_{ab} \cdot \boldsymbol{v}_a(t_{ab})/c}. \qquad \text{(A10)}$$

We denote as $t_{ab} = t_{ab}(t)$ the retarded time, at which the past light cone of the event $(t, \boldsymbol{x}_b(t))$ intersects the timelike worldline of particle a. This is defined implicitly by

$$c(t - t_{ab}) = |\boldsymbol{x}_b(t) - \boldsymbol{x}_a(t_{ab})|. \qquad \text{(A11)}$$

We also defined the retarded displacement

$$\boldsymbol{d}_{ab} = \boldsymbol{d}_{ab}(t) = \boldsymbol{x}_b(t) - \boldsymbol{x}_a(t_{ab}), \qquad \text{(A12)}$$

and its magnitude $d_{ab} = |\boldsymbol{d}_{ab}|$.

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