## Interaction between an Optically Levitated Nanoparticle and Its Thermal Image: Internal Thermometry via Displacement Sensing

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We propose and theoretically analyze an experiment where displacement sensing of an optically levitated nanoparticle in front of a surface can be used to measure the induced dipole-dipole interaction between the nanoparticle and its thermal image. This is achieved by using a surface that is transparent to the trapping light but reflective to infrared radiation, with a reflectivity that can be time modulated. This dipoledipole interaction relies on the thermal radiation emitted by a silica nanoparticle having sufficient temporal coherence to correlate the reflected radiation with the thermal fluctuations of the dipole. The resulting force is orders of magnitude stronger than the thermal gradient force, and it strongly depends on the internal temperature of the nanoparticle for a particle-to-surface distance greater than two micrometers. We argue that it is experimentally feasible to use displacement sensing of a levitated nanoparticle in front of a surface as an internal thermometer in ultrahigh vacuum. Experimental access to the internal physics of a levitated nanoparticle in vacuum is crucial to understanding the limitations that decoherence poses to current efforts devoted to preparing a nanoparticle in a macroscopic quantum superposition state.

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Today, it is experimentally possible to optically levitate a nanoparticle in vacuum [[1](#page-4-0)] and (i) feedback cool its centerof-mass motion to the ground state [\[2](#page-4-1)–[6\]](#page-4-2), (ii) place it near a surface  $[7-13]$  $[7-13]$  $[7-13]$  $[7-13]$ , (iii) measure the induced dipole-dipole interaction with another nanoparticle levitated in a second optical tweezer [[14](#page-4-5)], and (iv) use displacement sensing to detect forces in the zeptonewton regime [[15](#page-5-0)–[21](#page-5-1)]. In this Letter, we propose to combine these experimental capabilities to measure the dipole-dipole interaction of an optically levitated nanoparticle with its thermal image [see Fig.  $1(a)$ ]. This interaction depends on the internal temperature of the nanoparticle for particle-to-surface distances comparable to the thermal wavelength. Hence, we propose to leverage displacement sensing for internal thermometry of a levitated nanoparticle in vacuum [[22](#page-5-2)–[26](#page-5-3)].

Our proposal aims not only at experimentally measuring an out-of-equilibrium Casimir force, which has been the object of intense research [\[30](#page-5-4)–[37](#page-5-5)], but also at giving experimental access to the internal physics of a levitated nanoparticle in vacuum which is very relevant for the field of levitodynamics [\[1](#page-4-0)[,38](#page-5-6)]. Knowledge of the internal temperature T of a nanoparticle and the imaginary part of its polarizability Im $\{\alpha\}$  in the infrared regime is essential to quantify one of the most limiting sources of decoherence for experiments aiming to prepare a macroscopic quantum superposition of a nanoparticle [[28](#page-5-7)[,39](#page-5-8)–[47](#page-5-9)]: decoherence due to thermal emission of photons [[28](#page-5-7),[40](#page-5-10)[,48,](#page-5-11)[49\]](#page-6-0). The associated decoherence rate approximately scales with  $T^6$  and critically depends on Im $\{\alpha\}$ . Furthermore, the assignment of an internal temperature to the nanoparticle in out-of-equilibrium situations assumes that the nanoparticle internally equilibrates faster than any other relevant timescale. This assumption (known as the local equilibrium assumption) underlies the current understanding of the internal physics of levitated nanoparticles and the associated sources of decoherence, but whether it holds for a nanoparticle in ultrahigh vacuum or not is a question that needs to be answered experimentally [\[50\]](#page-6-1). Our experimental proposal would test this assumption and its consequences.

More specifically, we propose to optically trap a silica glass nanoparticle of radius  $r$  and mass  $m$  at a distance  $\zeta$ from a surface which is transparent at optical wavelengths but reflective at thermal (infrared) wavelengths, as illustrated in Fig. [1\(a\).](#page-1-0) We assume that the interaction between the nanoparticle and the electromagnetic field of wavelengths  $\lambda$  can be treated in the dipole approximation ( $r \ll \lambda$ ) with isotropic polarizability  $\alpha(\lambda)$ , which we calculate from bulk electric permittivity data  $\epsilon(\lambda)$  for silica glass [\[27\]](#page-5-12) using  $\alpha(\lambda) = 3V\epsilon_0[\epsilon(\lambda) - 1][\epsilon(\lambda) + 2]$ <br>vacuum permittivity and V the volume  $]^{-1}$ , where  $\epsilon_0$  is the vacuum permittivity and V the volume of the nanoparticle. We display Im $\{\alpha(\lambda)\}\$ for infrared and optical wavelengths in Fig. [1\(c\)](#page-1-0). To enable displacement sensing of the particlesurface interaction force, we propose to use a surface whose reflection coefficient can be modulated in time [[51](#page-6-2)–[55\]](#page-6-3). As illustrated in Fig. [1\(b\)](#page-1-0), we assume that the reflection coefficient [[56](#page-6-4)] can be modulated around a high value in the infrared, while simultaneously being practically zero

<span id="page-1-0"></span>

FIG. 1. (a) Illustration of the proposed experiment. A nanoparticle is optically trapped at a distance  $\zeta$  from a surface with time- and wavelength-dependent reflection coefficient  $R(t, \lambda)$ . The effect of the surface can be approximated by the mirror image nanoparticle on the left. (b) The surface is transparent at the trapping wavelength, but the reflection coefficient can be modulated in time between high and low values at the peak wavelength of the nanoparticle's thermal emission. The modulation is sinusoidal with frequency  $\Omega_d$  and amplitude η. (c) The imaginary part of the polarizability of an  $SiO<sub>2</sub>$  nanoparticle in the optical and infrared region of the spectrum using the model of  $\epsilon(\lambda)$  from Ref. [\[27\]](#page-5-12) (solid line). For  $\lambda \in [1, 7]$  µm, the imaginary part of the nolarizability is smaller than the lower limit of the graph [27]. At polarizability is smaller than the lower limit of the graph [\[27\]](#page-5-12). At  $\lambda = 1.550$  μm we use Im $\{\alpha\}/(3\epsilon_0V) = 1.5 \times 10^{-9}$  based on data reported in Refs. [\[28](#page-5-7)[,29\]](#page-5-15). The properties of the nanoparticle's infrared radiation is dominated by the peak at 8.9 μm, to which we fit a Lorentzian function (dashed line).

and without modulation at the optical wavelength. As we discuss quantitatively later, high and unmodulated transparency at the optical wavelength is necessary to avoid the influence of an interaction force between the nanoparticle and its optical image in the force sensing experiment [\[14](#page-4-5)[,57](#page-6-5),[58](#page-6-6)]. Similarly, since the charge on optically levitated nanoparticles can be controlled [\[66\]](#page-6-7), we propose to use electrically neutral particles to avoid interactions with the electrostatic mirror image [\[10\]](#page-4-6). We note that all-optical cold damping schemes for electrically uncharged silica nanoparticles have recently been demonstrated [[5](#page-4-7),[67](#page-6-8)].

In order to evaluate the force acting on the particle in this scenario, we use the theory of fluctuational electrodynamics [[31](#page-5-13)–[35](#page-5-14)[,37,](#page-5-5)[68](#page-6-9)]. The steps we perform are summarized as follows (more details are given in the Supplemental Material [[58](#page-6-6)]): we treat the nanoparticle as an electric dipole  $d$  which has a part induced by the electric field  $d_{ind}$ and a thermally fluctuating part  $\mathbf{d}_{th}$ . The total electric field E is composed of a fluctuating field from the radiation of the nanoparticle  $\mathbf{E}_{ind}$ , a second fluctuating field from thermal radiation from the environment  $E_{th}$ , and a nonfluctuating field due to the presence of the optical tweezer  $E_{\text{tw}}$ . The force acting on the nanoparticle is the expectation value of the standard force on an electric dipole in an electric field:  $\mathbf{F} = \sum_{i \in \{x, y, z\}} \langle d_i(t) \nabla E_i(\mathbf{r}_0, t) \rangle$ , where  $\mathbf{r}_0$  is the equilibrium position of the dipole. To evaluate this expression, we diagonalize the total dipole moment  $\mathbf{d}(t)$ and field  $\mathbf{E}(\mathbf{r}_0,t)$  in terms of the input quantities  $\mathbf{d}_{th}$  and  $E_{th}$ , whose correlations we assume to be given by the fluctuation-dissipation theorem, and  $E_{tw}$ . This is possible by transforming to the frequency domain where  $\mathbf{d}_{ind}(\omega) =$  $\alpha(\omega)\mathbf{E}(\mathbf{r}_0,\omega)$  and  $\mathbf{E}_{ind}(\mathbf{r},\omega)=\mathbb{G}(\mathbf{r},\mathbf{r}_0,\omega)\mathbf{d}(\omega)$ , with  $\omega = 2\pi c/\lambda$ , c the speed of light in vacuum, and  $\alpha(\omega)$ defined in terms of  $\alpha(\lambda = 2\pi c/\omega)$  [see top axis of Fig. [1\(c\)](#page-1-0)]. Here  $\mathbb{G}(\mathbf{r}, \mathbf{r}_0, \omega)$  is the electromagnetic Green's tensor. The presence of the surface modifies the Green's tensor, adding a scattering part  $\mathbb{G}_1$  to the freespace Green's tensor  $\mathbb{G}_0$  so that  $\mathbb{G} = \mathbb{G}_0 + \mathbb{G}_1$ . After diagonalization, one finds  $\mathbf{d}(\omega) = \mathbb{T}(\mathbf{r}_0, \omega) \{ \mathbf{d}_{\text{th}}(\omega) + \mathbf{d}_{\text{th}}(\omega) \}$  $\alpha(\omega)[\mathbf{E}_{\text{tw}}(\mathbf{r}_0,\omega)+\mathbf{E}_{\text{th}}(\mathbf{r}_0,\omega)]\},$  and  $\mathbf{E}(\mathbf{r},\omega)=\mathbf{E}_{\text{tw}}(\mathbf{r},\omega)+\mathbf{E}_{\text{tw}}(\mathbf{r},\omega)+\mathbf{E}(\mathbf{r},\mathbf{r},\omega)d(\omega)$ . The tensor  $\mathbb{E}(\mathbf{r},\omega)=\mathbb{I}-\mathbb{I}$  $\mathbf{E}_{\text{th}}(\mathbf{r},\omega) + \mathbb{G}(\mathbf{r},\mathbf{r}_0,\omega)\mathbf{d}(\omega)$ . The tensor  $\mathbb{T}(\mathbf{r}_0,\omega) \equiv \mathbb{I} \alpha(\omega) \mathbb{G}_1(\mathbf{r}_0, \mathbf{r}_0, \omega)$ <sup>-1</sup> accounts for multiple reflections<br>between the surface and the dipole Since the subbetween the surface and the dipole. Since the subwavelength nanoparticle scatters radiation only weakly, reflections beyond the first order turn out to have a negligible impact on the forces, and one can approximate  $\mathbb{T}(\mathbf{r}_0,\omega)\approx 1$ . F can now be evaluated with the fluctuationdissipation relations  $\langle d_{j,th}(\omega) d_{k,th}^*(\omega') \rangle = 2\pi \hbar \delta(\omega - \omega')$  $[2n(\omega,T)+1] \text{Im} \{\alpha(\omega)\}\delta_{jk} \text{ and } \langle E_{j,\text{th}}(\mathbf{r},\omega)E_{k,\text{th}}^*(\mathbf{r}_0,\omega')\rangle =$ <br>  $2\pi\hbar \delta(\omega-\omega')2n(\omega,T) + 1 \text{Im} \{G_{\omega}(\mathbf{r},\mathbf{r},\omega)\}\$ . Here  $\frac{2\pi\hbar\delta(\omega-\omega')[2n(\omega, T_{\text{env}}) + 1]\text{Im}\{G_{jk}(\mathbf{r}, \mathbf{r}_0, \omega)\}}{P(\omega, T)}$ . Here  $n(\omega,T)$  is the Bose–Einstein distribution,  $\hbar$  is the reduced Planck's constant, and  $T$  and  $T_{env}$  are the temperatures of the nanoparticle and the electromagnetic environment, respectively. The quantities  $E_{th}$ ,  $d_{th}$ , and  $E_{tw}$  are assumed to have vanishing cross-correlations.

With this method, the total force  $F$  on the nanoparticle is found to have five contributions (see the Supplemental Material [[58\]](#page-6-6) for details): (1) the optical force from the optical tweezer; (2) an interaction force between the nanoparticle and its optical mirror image [\[57](#page-6-5)], which we neglect in accord with our assumption that the surface is sufficiently transparent at the optical wavelength [\[58](#page-6-6)]; (3) the zerotemperature Casimir force between the surface and the nanoparticle [\[69](#page-6-10)]; (4) a force due to interaction with environmental thermal radiation which depends on  $T_{env}$ , and (5) a force due to interaction between the nanoparticle and its reflected thermal radiation [[31](#page-5-13)[,35\]](#page-5-14), or equivalently, its thermal mirror image. The last contribution depends on the nanoparticle internal temperature  $T$ , and we will denote it as

<span id="page-1-1"></span>
$$
\mathbf{F}_{\rm rad}(T,z) = \frac{\mathbf{e}_z \hbar c}{4\pi \epsilon_0 z^4} \int_0^\infty \frac{d\lambda}{\lambda^2} \frac{\mathrm{Im}\{\alpha(\lambda)\} f(2\pi z/\lambda)}{\exp\left[2\pi \hbar c/(k_B T \lambda)\right] - 1}.
$$
 (1)

Here,  $e_z$  is the surface normal vector,  $k_B$  is Boltzmann's constant, and  $f(x)$  is the oscillatory dimensionless function  $f(x) = \text{Re}\{\exp(2ix)(-3+6ix+6x^2-4ix^3)\}\.$  The expressions of the other four contributions to  $\bf{F}$  are given in the Supplemental Material [[58\]](#page-6-6).

In Fig. [2](#page-2-0), we plot  $e_z \cdot F$  as a function of z and T in the range 300 to 1500 K with increments of 100 K. We assume  $T_{\text{env}} = 300$  K. Note that therefore the black line  $(T = 300 \text{ K})$  corresponds to the equilibrium case. We find that as  $T$  is increased, the total force becomes dominated by the T-dependent fifth contribution  $\mathbf{F}_{rad}$ . Only at the smallest distances  $z \leq 2$  μm is the temperature dependence lost. This is due to the dominance of the zero-temperature Casimir force component over all the other force contributions at small separations [\[35,](#page-5-14)[36](#page-5-16),[68](#page-6-9)]. At distances  $z > 2$  μm, the force scales more slowly with distance than the  $z^{-5}$  scaling characteristic of zero-temperature Casimir– Polder forces [\[35,](#page-5-14)[69\]](#page-6-10). Additionally, we observe that the force oscillates in sign along  $\zeta$  with a temperatureindependent period. The oscillations arise due to the peak at  $\lambda_{\text{peak}} = 8.9 \text{ }\mu\text{m}$  of Im $\{\alpha(\lambda)\}\$  (which is attributed to vibrations of the Si─O bond in silica glass [\[27\]](#page-5-12)) dominating the integral in Eq. [\(1\)](#page-1-1) for  $T \gtrsim 400$  K. To confirm this explanation, we fit a Lorentzian function centered at  $\lambda_{peak}$ to Im $\{\alpha(\lambda)\}\$ , finding that the fit has full width at half maximum (FWHM) of 0.4 μm [dashed curve in Fig.  $1(c)$ ]. In the inset of Fig. [2](#page-2-0), we compare the total force calculated using the fitted and full  $\text{Im}\{\alpha(\lambda)\}\$  [the two curves in Fig. [1\(c\)\]](#page-1-0), finding excellent agreement when the temperature of the nanoparticle is above 400 K. We remark

<span id="page-2-0"></span>

FIG. 2. The total dipole force on the nanoparticle along the direction normal to the surface when  $R(t, \lambda) = 1$  for  $\lambda \ge 7$  µm, using the full spectral dependence of  $\alpha(\omega)$  [solid line in Fig. [1\(c\)\]](#page-1-0). Solid (dashed) lines indicate that the force is repulsive (attractive). The  $T = 300$  K curve (black) is the force on the particle in equilibrium since we assume that  $T_{env} = 300$  K. Inset: Comparison of the total dipole force on the nanoparticle computed using the full spectral dependence (lines) versus the Lorentzian fit (dots) of  $\alpha(\omega)$  as shown in Fig. [1\(b\)](#page-1-0).

that the oscillatory nature of the force could be utilized to perform a measurement of the distance from the nanoparticle to the surface.

We emphasize that the interaction giving rise to  $\mathbf{F}_{rad}$  is of a temporally coherent nature [\[14\]](#page-4-5). That is, the thermal dipole moment  $\mathbf{d}_{\text{th}}(t)$  remains correlated with itself during the time it takes for the thermally emitted radiation to be reflected by the surface and return to the particle: in the time domain,  $\mathbf{E}_{ind}(t) = \int_{-\infty}^{\infty} dt' \mathbf{G}(t-t') \mathbf{d}(t')$ , and<br>therefore  $\mathbf{F}_{ind}(t) \propto (d_{ind}(t))$ . The temporal coherence therefore  $\mathbf{F}_{\text{rad}} \propto \langle d_{\text{th},j}(t) d_{\text{th},k}(t') \rangle$ . The temporal coherence of the nanoparticle's thermal radiation is endowed by the narrow frequency spectrum of Im $\{\alpha\}$ . A hypothetical increasingly broadband emitter, with accordingly shorter coherence time, would produce a force  $\mathbf{F}_{rad}$  with weakening oscillations which in the white-noise limit  $\langle d_{\text{th},j}(t) d_{\text{th},k}(t') \rangle \rightarrow \delta(t-t')$  tends to zero since the time domain Green's tensor vanishes for zero time argutime-domain Green's tensor vanishes for zero time argument. Additionally, we point out that  $\mathbf{F}_{rad}$  cannot be written as the gradient of an electromagnetic field intensity [[70,](#page-6-11)[71\]](#page-6-12). The gradient force that the nanoparticle experiences due to its radiated field intensity is a higher-order term in the tensor  $\mathbb{T}(\mathbf{r}_0,\omega)$  which is not included in Eq.  $(1)$  and the contribution of which to **F** is negligible.

Figure [2](#page-2-0) shows that **F** reaches values above  $10^{-21}$  N, the currently demonstrated sensitivity in dynamic force sensing experiments with optically levitated nanoparticles in vacuum [\[2](#page-4-1)–[4](#page-4-8)[,15](#page-5-0)–[17](#page-5-17)[,19](#page-5-18)[,21](#page-5-1)[,72](#page-6-13),[73](#page-6-14)], for a wide range of temperatures  $T$  and particle-surface separations  $z$ . This suggests that  $\mathbf{F}_{rad}(T, z)$  can be measured with current laboratory capabilities. The thermally limited force sensitivity  $S$  is defined as  $S = \sqrt{S_{noise}}$ , where  $S_{noise}$  is the power spectral density (PSD) of the thermal forces acting on the nanodensity (PSD) of the thermal forces acting on the nanoparticle. Under cold damping of the nanoparticle motion, the dominant sources of thermal noise are collisions with residual gas molecules and photon shot noise [[73](#page-6-14)], and it can be shown that [[58](#page-6-6)]

$$
S = \sqrt{\frac{2\hbar P_{\text{scatt}}}{5c\lambda_0} + \frac{m\gamma_{\text{gas}}k_B T_{\text{env}}}{\pi}}.
$$
 (2)

Here  $P_{\text{scatt}}$  is the power that the nanoparticle scatters from the laser beam of wavelength  $\lambda_0$  [[73](#page-6-14)], and  $\gamma_{\text{gas}}$  is the damping rate due to collisions with gas molecules, directly proportional to the vacuum chamber pressure  $p$  [\[74\]](#page-6-15). We assume that the contributions from other sources of noise (e.g., surface-induced noise on neutral particles [\[75\]](#page-6-16)) are negligible compared with the photon shot noise. In Fig. [3\(a\)](#page-3-0) we show that for the choice of experimental parameters presented in Table [I](#page-3-1), zeptonewton force sensitivity is achieved for  $p \leq 10^{-9}$  mbar with saturation to the photon shot noise limit  $4 \times 10^{-22}$  N/ $\sqrt{Hz}$  at  $p_{sat} = 10^{-10}$  mbar.<br>We assume this value of the pressure for the remainder of We assume this value of the pressure for the remainder of the discussion.

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FIG. 3. (a) The force sensitivity as a function of the ambient gas pressure. The vertical dashed line identifies  $p_{\text{sat}}$ , the pressure below which the sensitivity is photon shot-noise limited. We use  $p_{\text{sat}}$  for plotting the other panels. (b) The signal-to-noise ratio (SNR) as a function of the nanoparticle internal temperature for three selected values of  $z$ . The gray area indicates the limit of measurability (SNR  $<$  1). (c) The force (right y axis) and SNR (left y axis) as a function of z for  $T = 300$  K, 400 K, ..., 1500 K. The grid lines follow the force axis. The vertical lines indicate the selected values of  $z$  for which the full  $T$  dependence is displayed in panel (b).

In order to generate a time-oscillating T-dependent force that can be dynamically sensed [[1\]](#page-4-0), the reflection coefficient of the surface around the wavelength  $\lambda_{peak}$  is modulated as  $R(t, \lambda) = 1 - \eta [1 - \cos(\Omega_d t)]/2$  [Fig. [1\(b\)](#page-1-0)].<br>By assuming that  $\partial_t R(t, \lambda) = 0$  across the 0.4 um EWHM By assuming that  $\partial_{\lambda}R(t,\lambda) = 0$  across the 0.4 μm FWHM of the peak in Im{ $\alpha(\lambda)$ } at  $\lambda_{\text{peak}}$ ,  $\mathbf{F}_{\text{rad}}$  splits into an average force and a time-dependent force  $\eta$ | $\mathbf{F}_{rad}(T, z)$ | cos $(\Omega_d t)/2$ [\[58\]](#page-6-6). The result is the appearance of a peak in the

<span id="page-3-1"></span>TABLE I. Table of proposed experimental parameters.

Parameter	Value
Nanoparticle radius $r$	$100 \text{ nm}$
Nanoparticle density $\rho$	$2200 \text{ kg} \text{m}^{-3}$
Environment temperature $T_{env}$	300 K
Gas pressure $p_{\text{sat}}$	$10^{-10}$ mbars
Tweezer wavelength $\lambda_0$	$1.550 \mu m$
Numerical aperture N	0.75
Laser power $P_{\text{tw}}$	$10 \text{ mW}$
Reflection modulation amplitude $\eta$	0.5
Driving frequency $\Omega_d$	$2\pi \times 12$ kHz
Driving quality factor $Q$	10 <sup>6</sup>

nanoparticle's motional PSD at the frequency  $\Omega_d$  whose maximum is proportional to  $|\mathbf{F}_{rad}(T,z)|^2$ . The  $\Omega_d$  frequency component of the reflection coefficient at the optical wavelength  $R(t, \lambda_0)$ , which we define as  $\eta_0$ , must simultaneously be kept small to avoid a similar and competing contribution to the motional PSD from the optical mirror image interaction force. More precisely, we require  $\eta_0/\eta < |F_{rad}|/|F_{cs}|$ , where  $F_{cs}$  is the force the nanoparticle would experience due to its interaction with its optical mirror image in a perfectly reflecting surface. As we show in the Supplemental Material [\[58\]](#page-6-6), the ratio  $|\mathbf{F}_{rad}|/|\mathbf{F}_{cs}| \sim 10^{-4}$ , and we therefore require  $\eta_0/\eta < 10^{-4}$ . An experimentally feasible method to achieve this is to use metasurface optical filters [[51](#page-6-2),[52](#page-6-17),[55](#page-6-3),[76](#page-6-18)–[79](#page-6-19)]; see the Supplemental Material [[58](#page-6-6)] for further details. When this requirement is fulfilled, the ratio of the  $\Omega_d$  peak to the thermally driven motional PSD defines the signal-to-noise ratio (SNR), which for  $p \le p_{\text{sat}}$  becomes [\[58\]](#page-6-6)

$$
SNR(\Omega_d) = \frac{15c\epsilon_0^2}{256\pi^4\hbar} \frac{\lambda_0^7 Q \eta^2 |\mathbf{F}_{\text{rad}}(T, z)|^2}{\Omega_d |\alpha(\lambda_0)|^2 N^2 P_{\text{tw}}}.
$$
 (3)

Here  $Q$  is the quality factor of the reflection coefficient modulation, and  $P_{tw}$  is the optical tweezer power. We plot the SNR as a function of T in Fig.  $3(b)$  and as a function of  $\zeta$  in Fig. [3\(c\)](#page-3-0). Figure [3\(b\)](#page-3-0) shows that the force is measurable  $(SNR > 1)$  in the entire temperature range 300–1500 K for  $z = 3.25$  μm, and at least in the range 400–1500 K for  $z = 7.50$  μm. In Fig. [3\(c\)](#page-3-0), we observe that with the chosen parameters, the force remains measurable for temperatures above 800 K even at separations of 19 μm and above. Additionally, we see in Fig. [3\(b\)](#page-3-0) that the SNR depends strongly on the internal temperature  $T$ , changing by several orders of magnitude (depending on  $z$ ) as the temperature is increased by a factor of 5.

<span id="page-3-2"></span>These results show that measurement of the driven  $\mathbf{F}_{rad}$  provides a way to perform thermometry of the nanoparticle's internal temperature in ultrahigh vacuum [[22](#page-5-2)–[26\]](#page-5-3). Because of the absence of internal cooling by residual gas at  $p_{\text{sat}}$ , the main heat dissipation channel of the nanoparticle will be radiative cooling. This opens up the prospect of investigating the radiative thermalization of the nanoparticle, similar to what was done for a silica nanofiber in Ref. [[80\]](#page-6-20), but here for an isolated (i.e., unclamped) nanosized object. It was recently argued that the radiative thermalization of a highly isolated nanoparticle might differ from the predictions of fluctuational electrodynamics due to failure of the local equilibrium assumption during thermalization [[50](#page-6-1)]. Qualitatively, the internal temperature of a subwavelength silica nanoparticle with polarizability as in Fig. [1\(c\)](#page-1-0) and which is being heated by laser absorption and cooled radiatively is predicted by fluctuational electrodynamics to obey [[58\]](#page-6-6)

$$
T(t) = \begin{cases} T_{\infty} \tanh\left[t/\tau + \operatorname{artanh}(T_0/T_{\infty})\right] & (T_{\infty} > T_0) \\ T_{\infty} \coth\left[t/\tau + \operatorname{artanh}(T_{\infty}/T_0)\right] & (T_{\infty} < T_0) \end{cases} (4)
$$

Here,  $T_0$  and  $T_{\infty}$  are the temperatures at  $t = 0$  and  $t = \infty$ , respectively, and  $\tau$  is a time constant which depends only on the properties of silica glass and the optical tweezer parameters but is independent of the subwavelength particle size. With the polarizability of Fig. [1\(c\)](#page-1-0) and parameters as in Table [I,](#page-3-1) we find  $T_{\infty} \approx 500 \text{ K}$  and  $\tau \approx 0.2$  s. In experiments,  $T_0$  can be independently controlled by using a dedicated heating laser [[25\]](#page-5-19). By combining our proposed thermometry scheme with such a setup, departures from the radiative cooling described by Eq. [\(4\)](#page-3-2), as predicted in Ref. [\[50\]](#page-6-1), could be experimentally tested.

To conclude, we have proposed an experiment to measure an out-of-equilibrium Casimir force using an optically levitated nanoparticle in ultrahigh vacuum. We believe this is interesting *per se* given the challenge to measure these forces [[36,](#page-5-16)[37](#page-5-5),[81\]](#page-7-0). In addition, we have discussed how dynamic sensing of this force can be used to measure the internal temperature of the nanoparticle, both in the steady state as well as in a dynamical setting with radiative cooling taking place. This complements previous methods for measuring the internal temperatures of levitated nanoparticles, requiring either higher pressures [[22](#page-5-2),[25\]](#page-5-19) or nanoparticles with embedded quantum emitters [[23](#page-5-20)[,26,](#page-5-3)[82](#page-7-1)]. Despite operating in the classical regime, we view the proposed experiment as highly relevant for current efforts to prepare large quantum superposition states of a levitated nanoparticle [[28](#page-5-7),[39](#page-5-8)–[47\]](#page-5-9). The design of these protocols is constrained by decoherence due to thermal radiation. One can show that this decoherence rate is proportional to  $\int_{0}^{\infty} d\lambda \lambda^{-7} n(T, \lambda) \text{Im} \{\alpha(\lambda)\}\$  [\[49](#page-6-0),[83\]](#page-7-2), and hence critically<br>depends on both the nanoparticle's internal temperature depends on both the nanoparticle's internal temperature and polarizability at infrared frequencies. Our proposed experiment gives information about both properties; the latter since the decoherence rate shows a similar dependence on  $\alpha(\lambda)$  as Eq. [\(1\).](#page-1-1) In addition, the observation that the thermal radiation of the nanoparticle is dominated by a narrow wavelength range would enable new strategies for the management of center-of-mass decoherence. We are currently investigating the effect on the decoherence rate of suppressing the radiation at the peak thermal wavelength. We hope this Letter will trigger the realization of classical experiments with levitated nanoparticles that provide key information about the physics related to sources of decoherence, which we consider pivotal in enabling the ambitious goal of preparing a nanoparticle in a large quantum superposition state.

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