Entropy Functions for Accelerating Black Holes

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We introduce an entropy function for supersymmetric accelerating black holes in four-dimensional anti–de Sitter space that uplift on general Sasaki-Einstein manifolds X_7 to solutions of M theory. This allows one to compute the black hole entropy without knowing the explicit solutions. A dual holographic microstate counting would follow from computing certain supersymmetric partition functions of Chern-Simons-matter theories compactified on a spindle. We make a general prediction for a class of such partition functions in terms of "blocks," with each block being constructed from the partition function on a three-sphere.

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Introduction.—Obtaining a precise microstate counting interpretation of black hole entropy is one of the major achievements of string theory. This was first studied in the context of supersymmetric and asymptotically flat black holes in [1] and this led to an enormous literature of further work. More recently, starting with [2,3], there has been a growing body of similar work in the context of supersymmetric and asymptotically anti–de Sitter (AdS) black holes. In contrast to the earlier work, where it is the Cardy formula that underlies the microstate counting, instead the results for AdS black holes use holography and exact localization results for supersymmetric partition functions.

The present Letter builds on [4–7], which introduced an entropy function for a large class of supersymmetric AdS_4 black holes in M theory. The seven-dimensional internal space X_7 is taken to be an arbitrary Sasaki-Einstein manifold, with the three-dimensional holographic duals being Chern-Simons-matter theories. The entropy function, similar in spirit to that of Sen [8], allows one to compute the entropy without knowing the explicit supergravity solutions: the inputs are only X_7, the topology of the black hole horizon Σ, taken to be a Riemann surface, and the magnetic charges. This entropy was shown to match a dual computation using the localization results of [9,10], for infinite families of black holes.

In this Letter, we extend [4–6] to *accelerating* black holes in AdS_4. This leads to a number of novel features [11]: the black holes have different horizon topologies, with conical deficit angles; supersymmetry is preserved in a novel way; and when the deficit angles are appropriately quantized, so that the horizon Σ is an orbifold known as a *spindle*, remarkably, the uplifted $D = 11$ solutions are completely smooth on and outside the horizon.

We will explain how to compute the entropy function for a general class of accelerating AdS_4 black holes, which takes a simple "gravitational block" form, vastly extending [12–14]. This leads to a general prediction for the partition functions of supersymmetric Chern-Simons-matter theories compactified on a spindle, with magnetic fluxes switched on for flavor and baryonic global symmetries, in the large N limit [15]. We also point out a striking relation of our entropy function to the on-shell action of the black holes in various examples and comment on including angular momentum and electric charges in this formalism.

Supersymmetric AdS_2 *solutions.*—Our starting point is the following general class of supersymmetric AdS_2 solutions to $D = 11$ supergravity introduced in [16] and clarified in [17]:

$$\begin{aligned} ds^2_{11} &= e^{-2B/3}(ds^2_{\mathrm{AdS}_2} + ds^2_{Y_9}),\\ G &= \mathrm{vol}_{\mathrm{AdS}_2} \wedge [J - d(e^{-B}\eta)]. \end{aligned} \tag{1}$$

Here $ds^2_{\mathrm{AdS}_2}$ is a unit radius metric on AdS_2, with volume form $\mathrm{vol}_{\mathrm{AdS}_2}$ and G is the $D = 11$ four-form. The Gauntlett-Kim (GK) space Y_9 has a canonically defined Killing vector field ξ, called the R-symmetry vector, and it plays a central role. The metric on Y_9 takes the form

$$ds^2_{Y_9} = \eta^2 + e^B ds^2_T, \tag{2}$$

where the one-form η is dual to ξ, ds^2_T is a Kähler metric transverse to η, with associated Kähler two-form J and Ricci two-form $d\eta = \rho$. The function $e^B = \frac{1}{2}R$, where $R > 0$, is the scalar curvature of the Kähler metric.

The metric and four-form in (1) give supersymmetric solutions to $D = 11$ supergravity provided also

$$\Box R = \frac{1}{2}R^2 - R_{ab}R^{ab}, \tag{3}$$

where R_{ab} denotes the Ricci tensor for the Kähler metric, and $\Box$ is the Laplacian operator. However, to define our entropy function, we wish to go *off shell* [4], and in particular, we will not directly impose (3) in what follows.

Near horizon limits of black holes.—For the solutions of interest, the $D = 11$ vacuum solution, without the black hole, is $\mathrm{AdS}_4 \times X_7$, where X_7 is a Sasaki-Einstein manifold. A putative black hole may then carry conserved charges associated with various massless U(1) gauge fields in AdS_4. The latter arise from Kaluza-Klein reduction on X_7, either from isometries of X_7 ("flavor symmetries") or from homology cycles ("baryonic symmetries").

Sasaki-Einstein manifolds X_7 have a $\mathrm{U}(1)^s$ isometry, where necessarily $s \geq 1$, and we may choose an associated normalized basis of Killing vector fields ∂_{φ_i}, $i = 1, \ldots, s$. X_7 is equipped with a Killing spinor, and without loss of generality, we choose the basis so that this spinor has charge $\frac{1}{2}$ under ∂_{φ_1} and is uncharged under the remaining vector fields. Via the Kaluza-Klein mechanism, massless U(1) gauge fields A_i in AdS_4 are obtained by gauging $d\varphi_i \to d\varphi_i + A_i$ in the metric on X_7, together with adding a corresponding term to the $D = 11$ six-form potential C_6, given in (4) below, where $dC_6 = *_{11}G$. On the other hand, if $\Sigma_I \subset X_7$ form a basis of 5-cycles, $I = 1, \ldots, b_5(X_7) = \dim H_5(X_7, \mathbb{R})$, then reducing C_6 on each 5-cycle Σ_I also leads to massless U(1) gauge fields A_I in AdS_4. Altogether, we have the linear perturbation

$$\delta C_6 = \sum_{i=1}^{s} A_i \wedge \omega_i + \sum_{I=1}^{b_5(X_7)} A_I \wedge \omega_I. \tag{4}$$

Here both ω_i and ω_I are coclosed five-forms on X_7, but ω_I is closed, while $d\omega_i = \partial_{\varphi_i} \lrcorner \mathrm{vol}_{X_7}$ [18], for a suitably normalized volume form on X_7. Notice that in (4) we are free to shift $\omega_i \to \omega_i + \sum_I c_i^I \omega_I$, for arbitrary constants c_i^I, which is precisely the freedom to mix baryonic symmetries into flavor symmetries in field theory. This correspondingly shifts $A_I \to A_I - \sum_i c_i^I A_i$ and hence the notion of baryonic fluxes in the reduced theory on AdS_4.

Consider introducing a supersymmetric extremal black hole into this AdS_4 vacuum. The near horizon limit should be $\mathrm{AdS}_2 \times \Sigma$, where the two-dimensional surface Σ is the black hole horizon [19]. For an accelerating black hole, we take Σ to be a spindle [11]. This is topologically a two-sphere, but with conical deficit angles $2\pi(1 - 1/m_\pm)$ at the poles, specified by two coprime positive integers $m_\pm$. The nonaccelerating case is recovered simply by setting $m_\pm = 1$, so $\Sigma = S^2$.

Now consider turning on flavor magnetic charges, for the gauge fields originating from isometries of X_7, with

$$\frac{1}{2\pi}\int_\Sigma dA_i = \frac{p_i}{m_- m_+}, \tag{5}$$

the magnetic flux through the horizon. This precisely fibers X_7 over Σ to give a GK geometry of the form

$$X_7 \hookrightarrow Y_9 \to \Sigma. \tag{6}$$

The fibration is well defined [20] when the flavor magnetic charges p_i are integers [21]. Imposing supersymmetry requires [21] that

$$p_1 = -\sigma m_+ - m_-, \tag{7}$$

where recall that the first copy of U(1) is singled out by the Killing spinor being charged under this symmetry. Here $\sigma = \pm 1$ are called "twist" and "antitwist," respectively.

On the other hand, the $D = 11$ seven-form flux satisfies the Dirac quantization condition

$$\frac{1}{(2\pi\ell_p)^6}\int_\Upsilon dC_6 = N_\Upsilon \in \mathbb{Z}, \tag{8}$$

where ℓ_p is the $D = 11$ Planck length, and $\Upsilon \subset Y_9$ is any 7-cycle. When Y_9 takes the fibered form (6) there is a distinguished such cycle, namely, a copy $\Upsilon = X_7$ of the fiber, and we identify $N \equiv N_{X_7}$ with the number of M2-branes generating the original $\mathrm{AdS}_4 \times X_7$ vacuum [22]. If we pick representatives of the 5-cycles $\Sigma_I \subset X_7$ that are invariant under the $\mathrm{U}(1)^s$ action, then via (6) these will fiber over the spindle Σ to give associated 7-cycles $\Upsilon_I \subset Y_9$. We denote the corresponding flux numbers in (8) as $N_I \equiv N_{\Upsilon_I}$, and analogously to (5) define *flux* magnetic charges $P_I \equiv N_I/N$ [24]. Notice that via (4) these fluxes will in general include contributions from the flavor magnetic charges p_i in (5), and also *baryonic* magnetic charges $(1/2\pi)\int_\Sigma dA_I$. However, as explained above, defining the latter, in general, requires (arbitrary) choices, and so we instead parametrize the baryonic magnetic charges of the black hole via the P_I. This accounts for all quantized fluxes on Y_9.

Entropy function.—We have seen how fixing the magnetic charges p_i, P_I of the black hole encodes the twisting of the fibration (6) and also quantized flux numbers (8) for the corresponding near horizon $\mathrm{AdS}_2 \times Y_9$ solution. This can be related to geometric data on Y_9 as follows [4]. First, evaluating the left-hand side of (8) on the background (1) gives

$$\frac{1}{(2\pi\ell_p)^6}\int_\Upsilon \eta\wedge\rho\wedge\frac{1}{2}J^2 = N_\Upsilon, \tag{9}$$

while imposing that the *integral* of (3) over Y_9 holds gives

$$\int_{Y_9}\eta\wedge\rho^2\wedge J^2 = 0. \tag{10}$$

The integrals (9) and (10) are functions of Kähler class parameters $[J]$ and the R-symmetry vector, which we may write as

$$\xi = \sum_{\mu=0}^{s} b_\mu \partial_{\varphi_\mu}. \tag{11}$$

Here ∂_{φ_0} is a Killing vector field rotating the spindle Σ, fixing its poles. The Kähler class parameters lie in the basic cohomology $[J]\in H^2_B(\mathcal{F}_\xi)$ associated with the foliation $\mathcal{F}_\xi$ defined by ξ. One can show that the total number of such parameters is $\dim H_5(X_7,\mathbb{R})+2$. On the other hand, fixing the flux magnetic charges P_I, together with N and imposing (10), imposes the same number of constraints. Although we do not have a general argument, in all examples fixing p_i, which determines the topology of Y_9, together with P_I and N fixes all the Kähler class parameters. This leaves the R-symmetry vector (11) still unspecified, apart from the constraint $b_1 = 1$, which corresponds to the Killing spinor necessarily having charge $\frac{1}{2}$.

The main result of [4] is that solutions to the partial differential equation (3) extremize the "entropy function,"

$$\mathcal{S} \equiv \frac{4\pi}{(2\pi)^8\ell_p^9}\int_{Y_9}\eta\wedge\rho\wedge\frac{1}{3!}J^3. \tag{12}$$

For fixed X_7, spindle data $m_\pm$, magnetic charges p_i, P_I, and N, we have $\mathcal{S}=\mathcal{S}(b_\mu)$ is a function only of the R-symmetry vector. The near horizon AdS_2 solution necessarily extremizes this, as a function of $(b_0, b_1 = 1, b_2, \ldots, b_s)$, with the black hole entropy $S_{\mathrm{BH}} = \mathcal{S}(b^*_\mu)$ being the entropy function evaluated at the critical point.

Gravitational blocks.—Using Stokes's theorem, one can show that the entropy function (12) can be written in the block form

$$\mathcal{S} = \frac{4\pi}{(2\pi)^7\ell_p^9}\frac{b_1}{b_0}[\mathrm{Vol}(X_7^+) - \mathrm{Vol}(X_7^-)] \tag{13}$$

(see Ref. [23] for details). Here $X_7^\pm$ are the copies of X_7 over the two poles of the spindle [25], and $\mathrm{Vol}(X_7^\pm) = \int_{X_7^\pm}\eta\wedge(1/3!)J^3$ is the volume induced by the choice of Kähler class. For toric X_7, which by definition have $s=4$ and so at least $U(1)^4$ isometry, this was called the "master volume" in [5], and Refs. [5,23] describe in detail how to compute this master volume in terms of toric data.

In practice, (9) and (10) are quadratic in the Kähler class parameters, and for more than one such parameter it is typically difficult to solve for $[J]$ in closed form and thus obtain the entropy function $\mathcal{S}$ as described after Eq. (12). In the remainder of this Letter [26], we will hence focus on a restricted, but still very rich, class of examples that we refer to as "flavor twists." This generalizes a similar class studied in [6], where by definition we impose that $[J|_{X_7^\pm}] \propto [\rho|_{X_7^\pm}]$. It can be shown that (13) leads to the result

$$\mathcal{S} = \frac{8\pi^3 N^{3/2}}{3\sqrt{6}b_0}\left(\frac{1}{\sqrt{\mathrm{Vol}_S(X_7)|_{\vec{b}^+}}} - \frac{\sigma}{\sqrt{\mathrm{Vol}_S(X_7)|_{\vec{b}^-}}}\right). \tag{14}$$

Here $\sigma=\pm1$ as in (7), and $\mathrm{Vol}_S(X_7)$ is the Sasakian volume of X_7, introduced in [27]. This is a function only of the R-symmetry vector $\vec{b} = (b_1 = 1, b_2, \ldots, b_s)$ (i.e., excluding the b_0 spindle direction), with

$$\vec{b}^+ \equiv \vec{b} - \frac{b_0}{m_+}(1, -a_+p_2, \ldots, -a_+p_s),$$
$$\vec{b}^- \equiv \vec{b} + \frac{b_0}{m_-}(\sigma, -a_-p_2, \ldots, -a_-p_s), \tag{15}$$

where $a_\pm$ are integers satisfying $a_-m_+ + a_+m_- = 1$. Such $a_\pm$ exist by Bézout's lemma, as $m_\pm$ are coprime. They are not unique, but different choices amount to a different choice of basis for the $U(1)^{s+1}$ action on Y_9, with generators ∂_{φ_μ}, $\mu = 0, 1, \ldots, s$, and the black hole entropy that extremizes (14) is independent of this choice.

For fixed X_7, the entropy function (14) is manifestly a function of only $m_\pm$, N the flavor magnetic charges p_i, and the R-symmetry vector $(b_0, b_1 = 1, b_2, \ldots, b_s)$. The flavor charges $p_2, \ldots, p_s$ are here arbitrary, but in this class of examples the flux charges P_I are determined by the remaining data. Specifically, one can show [23]

$$P_I = \frac{\pi}{3b_0}\left(\frac{\mathrm{Vol}_S(\Sigma_I)}{\mathrm{Vol}_S(X_7)}\bigg|_{\vec{b}^+} - \frac{\mathrm{Vol}_S(\Sigma_I)}{\mathrm{Vol}_S(X_7)}\bigg|_{\vec{b}^-}\right), \tag{16}$$

where these are again Sasakian volumes. Various methods for computing these volumes, for different classes of X_7, were given in [27], including using toric geometry, a fixed point theorem, and a limit of an equivariant index.

The entropy function (14) may thus be written down for infinite families of accelerating AdS_4 black holes in M theory, with general flavor magnetic charges p_i, and extremized over the R-symmetry vector to obtain the entropy. We present some examples in the next section.

The $\mathrm{AdS}_4/\mathrm{CFT}_3$ correspondence relates the free energy of the dual field theory (here typically Chern-Simons-matter theories) on the three-sphere S^3 [28–30] to a gravitational quantity via $\mathcal{F}_{S^3}(\vec{b}) = 2^{1/2}3^{-3/2}\pi^3/\sqrt{\mathrm{Vol}_S(X_7)|_{\vec{b}}}$. This has been shown in many examples, although we are not aware of a general proof. We may then write (14) as

$$\mathcal{S} = \frac{4}{b_0}[\mathcal{F}_{S^3}(\vec{b}^+) - \sigma \mathcal{F}_{S^3}(\vec{b}^-)]. \tag{17}$$

Here the free energy blocks are functions of the shifted R-symmetry vectors $\vec{b}^\pm$, which in field theory correspond to certain shifted trial R-symmetry assignments for the fields.

Finally, consider setting $m_\pm = 1$, so that the horizon $\Sigma = S^2$ and there is no acceleration, and also taking the limit $b_0 \to 0$ so that the R-symmetry vector is purely tangent to X_7. From (14) [or (17)], we then obtain

$$\mathcal{S} = 4\sum_{i=1}^{s} p_i \frac{\partial}{\partial b_i}\sqrt{\frac{2\pi^6}{27\mathrm{Vol}_S(X_7)|_{\vec{b}}}} N^{3/2}, \tag{18}$$

and here we should take $b_1 = 1$ after taking the derivative. This recovers the results of [5,6], where the derivative operator precisely acts on $\mathcal{F}_{S^3}(\vec{b})$.

Examples.—There are two particularly interesting classes of examples of the flavor twist construction described above, where in particular cases we may also make contact with various explicit solutions. The first is when there are no baryonic symmetries, i.e., $H_5(X_7, \mathbb{R}) = 0$. In this case, J is necessarily exact on X_7, and the condition (16) is vacuous. A simple example is $X_7 = S^7$, for which $s = 4$ and in a natural choice of basis for $\mathrm{U}(1)^4$ we have

$$\mathrm{Vol}_S(S^7)|_{\vec{b}} = \frac{\pi^4}{3b_2b_3b_4(b_1 - b_2 - b_3 - b_4)}. \tag{19}$$

One can check that the entropy function (14) agrees with the entropy function in [14] (after a simple linear change of variable) and, moreover, extremizing the function to obtain the entropy one finds a result that agrees with the explicit near horizon supergravity solutions in [31]. Instead, the nonaccelerating result (18) was in this case already known [32] to reproduce the entropy of the family of STU supergravity black hole solutions in [3]. Another example in this class, treated in [23], is $X_7 = V_{5,2}$, for which no explicit supergravity solutions are known.

The second class of examples are referred to as the "universal antitwist." These correspond to the explicit accelerating black hole solutions constructed in [11]. They are universal in the sense that the solutions exist for arbitrary choice of Sasaki-Einstein X_7 with rational R-symmetry vector [see (20) below]. Moreover, the solutions exist only in the antitwist case, with $\sigma = -1$, as we shall see momentarily [33]. The universal antitwist may be characterized geometrically by saying that the flavor twisting is only along the R-symmetry direction of the Sasaki-Einstein metric. This is equivalent to imposing

$$\vec{p} = \frac{p_1}{b_1^+}\vec{b}^+ = \frac{p_1}{b_1^-}\vec{b}^-. \tag{20}$$

Using a homogeneity property of the Sasakian volume, one can show [23] that (14) leads to the simple result

$$\mathcal{S} = \frac{1}{4b_0}[(b_1^+)^2 - \sigma(b_1^-)^2]\mathcal{F}_{S^3}. \tag{21}$$

Here the free energy $\mathcal{F}_{S^3} = \mathcal{F}_{S^3}(\vec{b}^*)$ is computed using the (extremal) Sasaki-Einstein metric. In (21) one should set $b_1 = 1$ and extremize over b_0 to obtain the entropy. If $\sigma = +1$, there are no extrema, forcing $\sigma = -1$. One can check that the (positive) extremal value $S_{\mathrm{BH}} = \mathcal{S}^*$ is given by

$$S_{\mathrm{BH}} = \frac{(2m_-^2 + 2m_+^2)^{1/2} - m_- - m_+}{2m_-m_+}\mathcal{F}_{S^3}, \tag{22}$$

which precisely agrees with the entropy of the explicit family of supersymmetric accelerating black holes in [11].

On-shell action.—One might wonder how the entropy function we have introduced is related to other approaches to computing black hole entropy and the associated thermodynamics. An immediate issue for extremal black holes in AdS_4 is that the infinite AdS_2 throat leads to an (IR) divergence in the holographically renormalized on-shell action I, which is thus ill defined without some form of regularization.

In [34], a *complex* locus of supersymmetric but nonextremal accelerating black holes was considered that, in addition, have nonzero rotation and electric charge. This complex locus has well-defined but complex action,

$$I = \pm\frac{1}{i\pi}\left[\frac{\varphi^2}{\omega} + \left(\frac{m_- - m_+}{4m_-m_+}\right)^2\omega\right]\mathcal{F}_{S^3}. \tag{23}$$

Here the two signs correspond to two different complex branches, and φ, ω are chemical potentials associated with electric charge and rotation. These satisfy the constraint $\varphi = (\chi/4)\omega \pm i\pi$, where $\chi = (m_+ + m_-)/m_-m_+$ is the orbifold Euler characteristic of the spindle horizon. The entropy is obtained in a standard way from this, as minus the Legendre transform of I, passing from grand canonical to microcanonical ensemble. Remarkably (21) and (23) satisfy $I = -\mathcal{S}$, via the change of variable

$$\omega = \mp 2\pi i b_0. \tag{24}$$

The Legendre transform of I thus extremizes $\mathcal{S}$, and since ω is a chemical potential for rotation of the horizon, and b_0 is the component of the R-symmetry vector rotating the horizon, (24) is a natural identification.

On the other hand, I is an on-shell quantity for the AdS_4 black holes, while $\mathcal{S}$ is an off-shell quantity for the associated near horizon AdS_2 solutions. It is therefore hard to see how these might be related physically, although by construction both are entropy functions, in the sense that extremizing both gives the (same) black hole entropy. There is a similar relation between the entropy function (18) in the case of $X_7 = S^7$ and the on-shell action of the STU black holes computed in [35] (see also [14,36]), suggesting this relation is not accidental.

Angular momentum and electric charge.—It should be possible to generalize the analysis of this Letter and also turn on both angular momentum J and electric charges q_i, Q_I for the AdS_4 black holes. The fact that these are zero here is simply due to (1): adding rotation and electric charge modifies this ansatz [37].

In [23], some additional observables in gravity are also introduced, namely, the geometric R charges [38],

$$R_a^+ \equiv \frac{4\pi m_+}{(2\pi \ell_p)^6 N} \int_{S_a^+} \eta \wedge \frac{J^2}{2!},$$
$$R_a^- \equiv \frac{4\pi \sigma m_-}{(2\pi \ell_p)^6 N} \int_{S_a^-} \eta \wedge \frac{J^2}{2!}. \tag{25}$$

Here $S_a^\pm$ are a set of $\mathrm{U}(1)^s$-invariant supersymmetric 5-submanifolds of the fibers $X_7^\pm$, and note that these exist even when $\dim H_5(X_7, \mathbb{R}) = 0$. These geometric R charges are dual to R charges of baryonic operators associated with M5-branes wrapping the submanifolds. When X_7 is toric, the cones over these are precisely the toric divisors in the Calabi-Yau cone $C(X_7)$, labeled by $a = 1, \ldots, d$. In this toric case we have the identity [23]

$$\frac{1}{2}\sum_{a=1}^{d} (R_a^+ + R_a^-) = 2 - \frac{m_- - \sigma m_+}{m_+ m_-} b_0. \tag{26}$$

In the universal antitwist case, with the identification (24), we then note we may identify the chemical potential $\varphi = \pm(i\pi/4)\sum_{a=1}^{4}(R_a^+ + R_a^-)$.

For the special case of $X_7 = S^7$, where the index a may be identified with the flavor index i, we can define the "master entropy function"

$$S \equiv \mathcal{S} - i\left[4b_0 J - \frac{1}{4}\sum_{a=1}^{d}(R_a^+ + R_a^-)q_a\right]\mathcal{F}_{S^3}. \tag{27}$$

Here $\mathcal{S}$ is the entropy function already introduced, depending on spindle data, magnetic charges, and R-symmetry vector b_0, b_i, $i = 1, \ldots, 4$. In (27), we have further introduced angular momentum J, conjugate to b_0, and electric charges q_a, conjugate to the R charges (25). This is a natural generalization of the entropy function conjectured for the nonaccelerating STU black holes in [3], and moreover, we have checked that extremizing S and imposing that S and the conserved charges are real, precisely leads to the entropy of the family of near horizon solutions constructed in [36]. These were conjectured to be the near horizon limits of general dyonically charged, rotating, and accelerating black holes in AdS_4 in STU gauged supergravity, which uplift on $X_7 = S^7$ to solutions of M theory. In the case with only a single dyonic pair of charges turned on for the graviphoton, these are precisely the black hole solutions in [34].

More generally, the flavor and baryonic charges are naturally combined in toric geometry via the index $a = 1, \ldots, d$, and it is natural to conjecture that (27) is the correct entropy function with general charges, not just for $X_7 = S^7$ but for more general classes of X_7.

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[1] A. Strominger and C. Vafa, Phys. Lett. B **379**, 99 (1996).
[2] F. Benini, K. Hristov, and A. Zaffaroni, J. High Energy Phys. 05 (2016) 054.
[3] F. Benini, K. Hristov, and A. Zaffaroni, Phys. Lett. B **771**, 462 (2017).
[4] C. Couzens, J. P. Gauntlett, D. Martelli, and J. Sparks, J. High Energy Phys. 01 (2019) 212.
[5] J. P. Gauntlett, D. Martelli, and J. Sparks, J. High Energy Phys. 06 (2019) 140.
[6] S. M. Hosseini and A. Zaffaroni, J. High Energy Phys. 07 (2019) 174.
[7] H. Kim and N. Kim, J. High Energy Phys. 11 (2019) 050.
[8] A. Sen, J. High Energy Phys. 09 (2005) 038.
[9] S. M. Hosseini and A. Zaffaroni, J. High Energy Phys. 08 (2016) 064.
[10] S. M. Hosseini and N. Mekareeya, J. High Energy Phys. 08 (2016) 089.
[11] P. Ferrero, J. P. Gauntlett, J. M. Perez Ipiña, D. Martelli, and J. Sparks, Phys. Rev. D **104**, 046007 (2021).
[12] S. M. Hosseini, K. Hristov, and A. Zaffaroni, J. High Energy Phys. 12 (2019) 168.
[13] S. M. Hosseini, K. Hristov, and A. Zaffaroni, J. High Energy Phys. 07 (2021) 182.
[14] F. Faedo and D. Martelli, J. High Energy Phys. 02 (2022) 101.
[15] It would be of much interest to generalize the entropy functions of this Letter to incorporate higher-derivative corrections and hence go beyond the large N limit.
[16] N. Kim and J.-D. Park, J. High Energy Phys. 09 (2006) 041.
[17] J. P. Gauntlett and N. Kim, Commun. Math. Phys. **284**, 897 (2008).
[18] S. Benvenuti, L. A. Pando Zayas, and Y. Tachikawa, Adv. Theor. Math. Phys. **10**, 395 (2006).
[19] While this is generically expected to be true, it is important to understand the necessary and sufficient conditions.
[20] Y_9 is free of orbifold singularities provided the p_i are coprime to both of $m_\pm$.

[21] P. Ferrero, J. P. Gauntlett, and J. Sparks, J. High Energy Phys. 01 (2022) 102.

[22] We must have $N = m_+ N^{X_+} = m_- N^{X_-}$ with $N^{X_\pm} \in \mathbb{Z}$ and hence $N = m_+ m_- \mathcal{N}_0$ with $\mathcal{N}_0 \in \mathbb{Z}$ [23].

[23] A. Boido, J. P. Gauntlett, D. Martelli, and J. Sparks, arXiv:2211.02662.

[24] The normalization factor of N is due to the fact that baryonic operators, dual to M5-branes wrapped on Σ_I, arise as $N \times N$ determinants in Chern-Simons-matter duals, and P_I is then the charge of the associated field.

[25] More precisely, these fibers are $X_7/\mathbb{Z}_{m_\pm}$, so $X_7^\pm$ are really covering spaces of the fibers. The orientations of $X_7^\pm$ are discussed in more detail in [23].

[26] Tools for analyzing toric examples are developed in [23].

[27] D. Martelli, J. Sparks, and S.-T. Yau, Commun. Math. Phys. **280**, 611 (2008).

[28] D. Martelli and J. Sparks, Phys. Rev. D **84**, 046008 (2011).

[29] S. Cheon, H. Kim, and N. Kim, J. High Energy Phys. 05 (2011) 134.

[30] D. L. Jafferis, I. R. Klebanov, S. S. Pufu, and B. R. Safdi, J. High Energy Phys. 06 (2011) 102.

[31] C. Couzens, J. High Energy Phys. 03 (2022) 078.

[32] S. M. Hosseini and A. Zaffaroni, J. High Energy Phys. 03 (2019) 108.

[33] Switching off rotation and electric charge gives solutions where the spindle degenerates at the AdS_4 boundary [11].

[34] D. Cassani, J. P. Gauntlett, D. Martelli, and J. Sparks, Phys. Rev. D **104**, 086005 (2021).

[35] D. Cassani and L. Papini, J. High Energy Phys. 09 (2019) 079.

[36] P. Ferrero, M. Inglese, D. Martelli, and J. Sparks, Phys. Rev. D **105**, 126001 (2022).

[37] C. Couzens, E. Marcus, K. Stemerdink, and D. van de Heisteeg, J. High Energy Phys. 05 (2021) 194.

[38] Ref. [23] also shows that associated with $U(1)^s$-invariant nontrivial 5-cycles Σ_I on X_7, (16) can be recast in the form $P_I = (1/2b_0)(R_I^+ - R_I^-)$.