Time- and Space-Varying Neutrino Mass Matrix from Soft Topological Defects

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We study the formation and evolution of topological defects that arise in the postrecombination phase transition predicted by the gravitational neutrino mass model in Dvali and Funcke [Phys. Rev. D **93,** 113002 (2016)]. In the transition, global skyrmions, monopoles, strings, and domain walls form due to the spontaneous breaking of the neutrino flavor symmetry. These defects are unique in their softness and origin; as they appear at a very low energy scale, they only require standard model particle content, and they differ fundamentally depending on the Majorana or Dirac nature of the neutrinos. One of the observational signatures is the time dependence and space dependence of the neutrino mass matrix, which could be observable in future neutrino experiments. Already existing data rule out parts of the parameter space in the Majorana case. The detection of this effect could shed light onto the open question of the Dirac versus Majorana neutrino nature.

Introduction.—The origin of the observed small neutrino masses is one of the greatest puzzles of the standard model (SM). The most popular directions of model building beyond the SM usually focus on new physics at high-energy scales. Many high-energy origins of neutrino masses have been proposed, which require either hypothetical new particles [1–4] or extra dimensions [5–7] (see Ref. [8] for a review). These scenarios all depend on electroweak symmetry breaking, and the resulting neutrino masses are proportional to (some power of) the SM Higgs vacuum expectation value.

As an alternative direction, a new low-energy solution to the neutrino mass problem was proposed in Ref. [9], which is independent of the SM Higgs vacuum expectation value and does not require any additional dimensions or particles. Using the SM particle content coupled minimally to gravity, it only requires the assumption that pure gravity contains a nonzero topological vacuum susceptibility,

$$\langle R\tilde{R}, R\tilde{R}\rangle_{q\to 0} \equiv \lim_{q\to 0} \int d^4 x e^{iqx} \langle T[R\tilde{R}(x)R\tilde{R}(0)]\rangle$$
$$= \text{const} \neq 0, \tag{1}$$

in the limit of vanishing momentum q. Here, T is the time ordering operator, R is the Riemann tensor, and $\tilde{R}$ is its dual. The condition [Eq. (1)] is equivalent to the statement that the gravitational analog of the QCD θ-term,

$$\mathcal{L}_G \supset \theta_G R\tilde{R}, \tag{2}$$

is physical [10–13], meaning that the free parameter $\theta_G \in [0, 2\pi]$ is a physically measurable quantity.

If the condition [Eq. (1)] is fulfilled, it was shown that fermion condensation $\langle \bar{f}f\rangle$ [14,15] and effective fermion masses m_f [9] emerge at a new fundamental energy scale. Note that this is true for arbitrary numbers of massless fermion flavors, as it is enforced by gravitational anomaly matching conditions [16]. Thus, we get

$$\Lambda_G = \langle R\tilde{R}, R\tilde{R}\rangle_{q\to 0}^{1/8} \sim |\langle \bar{f}f\rangle|^{1/3} \sim m_f. \tag{3}$$

Phenomenological constraints push this scale into the range $\Lambda_G \sim 0.1$ meV-eV [9,17,18] (see Ref. [19] for more details). This opens up the possibility that neutrino masses m_{ν_i} are generated through a Higgs-like composite field $\langle \Phi \rangle \equiv \langle \bar{\nu}_i \nu_i \rangle$ at low energies [9] instead of high-energy extensions of the SM.

In this Letter, we will study the topological defects that form in the late Universe due to this neutrino condensation. The model [9] predicts a phase transition in the late Universe after recombination, in which neutrino flavor symmetry gets spontaneously broken, small effective

neutrino masses $m_\nu \sim \Lambda_G$ arise, and pseudoscalar Nambu-Goldstone bosons ϕ appear [9,15,17]. Moreover, topological defects form due to the Kibble mechanism [22], including skyrmions, global monopoles, strings, and domain walls (DWs). These defects are very soft, only require SM particle content, and depend on the Majorana or Dirac nature of the neutrinos. In the current work, we will provide a detailed study of the formation, evolution, and observational consequences of these defects. Beyond these observational consequences presented here, two more experimental signatures of this model are postrecombination relic neutrino annihilation and astrophysical neutrino decay [9,17,18]. The former prediction can be experimentally verified or falsified with the next-generation cosmological surveys, while the latter can be tested with future astrophysical neutrino observatories.

Relic neutrino phase transition.—The low-energy neutrino mass model proposed in Ref. [9] predicts that the cosmological neutrinos remain massless until the very late Universe, $T \lesssim T_{\mathrm{CMB}} \sim 0.3$ eV. At that time, a neutrino vacuum condensate forms in a cosmological phase transition triggered by the gravitational vacuum susceptibility. The condensation generates small effective neutrino masses and (pseudo)Goldstone bosons, $\phi \equiv \{\phi_k, \eta_\nu\}$, which are called "gravi-majorons" in Ref. [18]. These neutrino-composite bosons arise from the spontaneous breaking of the neutrino flavor symmetry, which reads as $U(3)_L$ in the case of left-handed Majorana neutrinos (for which $k = 1, \ldots, 8$) and $U(3)_L \times U(3)_R$ in the case of Dirac neutrinos (for which $k = 1, \ldots, 14$); see Eqs. (6) and (13).

The η_ν boson is analogous to the heavy η' meson of QCD and acquires a small mass through the chiral gravitational anomaly, $m_{\eta_\nu} \sim \Lambda_G$ [15]. At first sight, one might expect the ϕ_k bosons to be massless, since the neutrinos in the model [9] have no hard masses but only effective ones. However, loop diagrams involving W-boson and charged-lepton exchange give rise to tiny mass contributions for some of the ϕ_k bosons [see Eq. (12) in the Supplemental Material [23]],

$$m_{\phi_k} = \frac{G_F}{4\pi} m_l \Lambda_G^2 = 2 \times 10^{-21} \ \mathrm{eV} \left(\frac{m_l}{m_\tau}\right)\left(\frac{\Lambda_G}{1 \ \mathrm{meV}}\right)^2, \quad (4)$$

thus explicitly breaking the original $U(3)_L$ neutrino flavor symmetry. The largest (smallest) contributions to the ϕ_k-boson masses are given for $m_l = m_\tau$ ($m_l = m_\mu$).

After the phase transition, the massive relic neutrinos rapidly decay into the lightest neutrino mass eigenstate, $\nu_i \to \nu_j + \phi_k$, and bind up or annihilate into the (almost) massless ϕ_k bosons, $\nu_j + \bar{\nu}_j \to \phi_k + \phi_k$ [9]. Note that these bosons only form in the very late Universe and thus do not alter any early-Universe physics. Almost all neutrinos convert into this dark radiation in the late Universe, apart from a negligibly small freeze-out density. This late "neutrinoless Universe" scenario can only be evaded in

the hypothetical presence of neutrino asymmetries [24]. The absolute neutrino mass scale is constrained by $m_\nu \lesssim T_{\mathrm{CMB}} \sim 0.3$ eV if the phase transition takes place instantaneously at a temperature $T_{\Lambda_G} \sim \Lambda_G \sim m_\nu$ [9,25] so as not to conflict with cosmic microwave background (CMB) observations. However, the phase transition can also be supercooled and thus can give rise to relatively large neutrino masses even at a low apparent transition temperature, $T_{\Lambda_G} < \Lambda_G \sim m_\nu \lesssim 1.5$ eV at 95% C.L. [24,26]. Such a supercooling mechanism would allow for substantial energy densities of the ϕ_k bosons and the topological defects after the transition.

In the following, we will discuss the neutrino flavor symmetry breaking patterns from neutrino condensation. We will first focus on the case that neutrinos are Majorana particles, followed by a discussion of the Dirac case. We will restrict our analysis to the minimal SM scenario of three active neutrino species. However, we wish to emphasize that there could be additional sterile neutrino species, which could enhance the neutrino flavor symmetry. Because of the universality of the gravitational neutrino mass mechanism [9], the topological structure in the neutrino sector would be sensitive to the structure of such hidden-sector sterile neutrinos. Interestingly, because of the unique role of neutrinos as the lightest and weakest interacting particles of the SM, this offers an exciting new opportunity to probe hidden-sector physics.

Symmetry breaking in the Majorana case.—The minimal version of the SM only contains three left-handed neutrinos $\nu_{L\alpha}$ (where $\alpha = e, \mu, \tau$ is the flavor index) and no right-handed neutrinos. Thus, the minimal version of the gravitational neutrino mass model [9] generates effective Majorana masses for these three left-handed species.

When neglecting SM interactions, the Lagrangian of the three massless neutrinos only contains a kinetic term,

$$\mathcal{L}_\nu = i\bar{\nu}_{L\alpha}\slashed{\partial}\nu_{L\alpha} + \mathrm{H.c.,} \quad (5)$$

where the implicit sum over the flavor index $\alpha = e, \mu, \tau$ could be equivalently written as a sum over the mass index $i = 1, 2, 3$ [just as in Eq. (12) below]. The Lagrangian in Eq. (5) has the flavor symmetry

$$G = U(3)_L = \frac{SU(3)_L \times U(1)_L}{Z_3}. \quad (6)$$

This neutrino flavor symmetry is a quotient group because the SU(3) and U(1) symmetries have a common center, $U(3) = [SU(3) \times U(1)]/Z_3$. On top of the small explicit breaking through weak effects [Eq. (4)], the flavor symmetry in Eq. (6) gets explicitly broken by the chiral gravitational anomaly [10–13] and spontaneously by the neutrino condensate [9]. The explicit breaking of the $U(1)_L$ part in Eq. (6) is similar to invisible axion scenarios [27–33], where the $U(1)_{\mathrm{PQ}}$ symmetry is explicitly broken

by the chiral anomalies of gravity and QCD. For the spontaneous breaking, the order parameter reads

$$\Phi_{\alpha\beta} \equiv \nu_{L\alpha}^T C \nu_{L\beta}, \tag{7}$$

where $C = i\gamma^2\gamma^0$ is the charge conjugation matrix. Note that the order parameter is symmetric, $\Phi_{\alpha\beta} = \Phi_{\beta\alpha}$.

The effective potential of the symmetry breaking reads

$$V(\Phi) = a\mathrm{Tr}(\Phi^\dagger\Phi) + b\mathrm{Tr}(\Phi^\dagger\Phi\Phi^\dagger\Phi) + \cdots$$
$$+ c \det\Phi + \mathrm{H.c.} + \cdots, \tag{8}$$

where a, b, and c are unknown numerical coefficients and $\det\Phi$ is a 't Hooft-like determinant [34] which breaks the $U(1)_L$ symmetry in Eq. (6) down to Z_6. Note that the potential in Eq. (8) does not necessarily need to respect the full $U(3)$ symmetry but could minimally be invariant under $O(3)$ symmetry only, because the maximal flavor symmetry of flavor-invariant Majorana masses is $O(3)$.

Gravitational anomaly matching conditions require all neutrinos to become massive [16], and these masses need to be different due to the observed neutrino flavor oscillations. Thus, in the neutrino mass eigenbasis, the vacuum expectation value reads

$$\langle\Phi\rangle = \begin{pmatrix} \langle\Phi_{11}\rangle & 0 & 0 \\ 0 & \langle\Phi_{22}\rangle & 0 \\ 0 & 0 & \langle\Phi_{33}\rangle \end{pmatrix}, \tag{9}$$

where $\langle\Phi_{11}\rangle \neq \langle\Phi_{22}\rangle \neq \langle\Phi_{33}\rangle$ for different masses.

In order to determine the resulting flavor symmetry breaking pattern, we need to find all elements of the initial group G in Eq. (6) that leave $\langle\Phi_{11}\rangle$, $\langle\Phi_{22}\rangle$, and $\langle\Phi_{33}\rangle$ invariant, which gives us the unbroken symmetry group H. Among the elements of $SU(3)_L \times U(1)_L$, only the Z_2 elements $\{1, e^{i\pi 1}\}$ leave the different condensates $\langle\Phi_{ii}\rangle$ invariant (for each $i = 1, 2, 3$, with no sum over repeated indices). Therefore, we finally get the symmetry group $H = Z_2 \times Z_2 \times Z_2$ from the spontaneous breaking

$$SU(3)_L \to Z_2 \times Z_2, U(1)_L \to Z_2, \tag{10}$$

where the elements of $Z_2 \times Z_2$ are given by

$$g_1 = \mathrm{diag}(1, -1, -1), \qquad g_2 = \mathrm{diag}(-1, 1, -1),$$
$$g_3 = \mathrm{diag}(-1, -1, 1), \qquad g_4 = 1. \tag{11}$$

Note that this $Z_2 \times Z_2$ symmetry is explicitly broken by weak effects at very low energy scales; see Eq. (4).

Symmetry breaking in Dirac case.—As a minimal extension of the SM, the three left-handed neutrinos $\nu_{L\alpha}$ could be accompanied by two or more right-handed partners. In the following, we will focus on the three-flavor Dirac neutrino case of the neutrino mass model [9],

noting that the model also allows for mixed active-sterile neutrinos with both Dirac and Majorana masses.

When neglecting SM interactions, the Lagrangian of the three massless neutrinos only contains a kinetic term,

$$\mathcal{L}_\nu = i\bar\nu_{L\alpha}\slashed\partial\nu_{L\alpha} + i\bar\nu_{R\alpha}\slashed\partial\nu_{R\alpha} = i\bar\nu_{D\alpha}\slashed\partial\nu_{D\alpha}. \tag{12}$$

In the limit in which all interactions except gravity are switched off, the Lagrangian has a $U(6)$ flavor symmetry. When taking into account the small explicit breaking by the weak interaction [Eq. (4)], the left- and right-handed sectors cannot mix, and we can write the symmetry as

$$G = U(3)_L \times U(3)_R$$
$$= \frac{U(3)_V \times U(3)_A}{Z_2}$$
$$= \frac{SU(3)_V \times SU(3)_A \times U(1)_V \times U(1)_A}{Z_{V3} \times Z_{A3} \times Z_2}, \tag{13}$$

where L, R, $V = L + R$, and $A = L - R$ denote the left, right, vector, and axial symmetries, respectively. The L and R group elements act on ν_D and take the form $\exp[i(1 \pm \gamma_5)\alpha_a T_a]$, where the T_a are group generators. The flavor symmetry G in Eq. (13) is a quotient group because (i) the $SU(3)$ and $U(1)$ symmetries have a common Z_3 center and (ii) the π rotations of the $U(1)_V$ and $U(1)_A$ symmetries are the same element, $\exp(i\pi) = \exp(i\pi\gamma_5) = -1$, which yields $U(1)_L \times U(1)_R = [U(1)_V \times U(1)_A]/Z_2$.

Similar to the Majorana case [Eq. (6)], the initial flavor symmetry G in Eq. (13) is explicitly broken by the gravitational anomaly and spontaneously by the neutrino condensate [9]. The order parameter in the Dirac case is

$$\Phi_{ij}^D \equiv \bar\nu_i\nu_j \tag{14}$$

in the mass eigenbasis, and its expectation value $\langle\bar\nu_i\nu_j\rangle$ is bifundamental and obeys $\langle\bar\nu_i\nu_i\rangle \neq \langle\bar\nu_j\nu_j\rangle$ for $i \neq j$. The effective potential is similar to the one in Eq. (8).

Among the elements of $SU(3)_A \times U(1)_A$, only the Z_2 elements $\{1, e^{i\pi\gamma_5 1}\}$ leave $\langle\bar\nu_i\nu_i\rangle$ invariant (for each $i = 1, 2, 3$). Therefore, we get

$$SU(3)_A \to 1, U(1)_A \to Z_2. \tag{15}$$

Here, we neglected the explicit symmetry breaking of $U(1)_A \to Z_6$ [just as we neglected the explicit breaking of $U(1)_L$ in Eq. (10)] due to the 't Hooft-like determinant [Eq. (8)], as discussed in detail in Ref. [23].

Regarding the $U(3)_V$ symmetry, any phase transformation of ν_i leaves $\langle\bar\nu_i\nu_i\rangle$ invariant. Therefore, we get three unbroken $U(1)$ symmetries,

$$U(3)_V \to U(1)_{V3} \times \frac{U(1)_{V1} \times U(1)_{V8}}{Z_{V3}}, \tag{16}$$

where the elements of $\mathrm{U}(1)_{V1}$ are $e^{i\alpha\mathbb{1}}$, of $\mathrm{U}(1)_{V3}$ are $e^{i\alpha\lambda_3}$, and of $\mathrm{U}(1)_{V8}$ are $e^{i\alpha\lambda_8}$, with $\lambda_3 = \mathrm{diag}(1,-1,0)$ and $\lambda_8 = \mathrm{diag}(1,1,-2)/\sqrt{3}$. The Z_{V3} denominator is because the center of $\mathrm{SU}(3)_V$ is given by $\{\mathbb{1}, e^{i2\pi/3}\mathbb{1}, e^{i4\pi/3}\mathbb{1}\}$ and is contained in both $\mathrm{U}(1)_{V1}$ and $\mathrm{U}(1)_{V8}$. Thus, the symmetry breaking is effectively

$$\mathrm{SU}(3)_V \rightarrow \mathrm{U}(1)_{V3} \times \mathrm{U}(1)_{V8} \qquad (17)$$

and the unbroken symmetry group reads as $H = \mathrm{U}(1)_{V3} \times [\mathrm{U}(1)_{V1} \times \mathrm{U}(1)_{V8}]/Z_{V3}$ because the Z_2 denominator in Eq. (13) cancels the numerator in Eq. (15).

From a broader perspective, we observe that the symmetry breaking patterns substantially differ for the Majorana and Dirac cases. Thus, the resulting topological defects crucially depend on the yet unknown neutrino nature and offer the exciting possibility of shedding new light onto this nature. In the following, we show that only the Majorana neutrino case yields topological defects that can induce a time- and space-varying neutrino mass matrix. In the Dirac case, the defects are quickly annihilating and thus are not expected to have any observational consequences, as demonstrated in Ref. [23].

Cosmic string network.—In the Majorana case, the original $\mathrm{SU}(3)_L$ flavor symmetry of the massless left-handed Majorana neutrinos is spontaneously broken down to $Z_2 \times Z_2$; see Eq. (10). This symmetry breaking gives rise to global strings,

$$\pi_1[\mathrm{SU}(3)/(Z_2 \times Z_2)] = Z_2 \times Z_2, \qquad (18)$$

which are analogous to the cosmic strings of broken flavor symmetry first discussed in Refs. [35–37]. The order parameters in the current and original flavor string scenarios are similar, except that in the present case it is composite instead of fundamental.

When investigating the cosmological evolution of the string network, we obtain the characteristic length scale ξ of the strings [see Eq. (18) in [23]], which reads

$$\xi = \sqrt{\frac{\lambda^2 \Lambda_G^7 t}{a_G^2 T_\nu^8}} = 10^{14} \ \mathrm{m} \left(\frac{\lambda}{1}\right) \left(\frac{\Lambda_G}{1 \ \mathrm{meV}}\right)^{\frac{7}{2}} \left(\frac{1}{a_G}\right). \qquad (19)$$

Here, λ is the self-coupling of the scalar string field Φ, Λ_G is the infrared gravitational scale, t is the Hubble time, a_G is the scale factor of the phase transition, and $T_\nu \sim 0.2$ meV is the neutrino temperature. This scale sets the distance over which the strings are straight, which is also assumed to be the interstring separation.

When traveling around a string, the vacuum expectation value $\langle\Phi\rangle$ winds continuously by the $\mathrm{SU}(3)$ group transformation

$$\langle\Phi(\theta)\rangle = \omega^T(\theta)\langle\Phi\rangle\omega(\theta), \qquad (20)$$

where $\langle\Phi\rangle$ is defined in Eq. (9). While the angle θ runs from 0 to 2π, the path $\omega(\theta)$ interpolates between different elements $g_i \in Z_2 \times Z_2$ [Eq. (11)], which label the different strings [38]. For example, the string labeled by the g_3 element corresponds to a path $\omega(\theta)$ that has the form

$$\omega_3(\theta) = \begin{pmatrix} \cos(\theta/2) & \sin(\theta/2) & 0 \\ -\sin(\theta/2) & \cos(\theta/2) & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad (21)$$

which corresponds to a rotation around a third axis in flavor space. Thus, when a neutrino passes by a string, the mixing angles of the standard Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix [39,40] change nontrivially. The PMNS matrix $U_{\mathrm{PMNS}} = U_e^\dagger U_\nu$ is a product of two unitary matrices U_e and U_ν arising from the diagonalization of the charged lepton and neutrino mass matrices. Keeping U_e fixed, the matrix U_ν changes by $\omega(\theta)U_\nu$ when passing by a string, resulting in $U_{\mathrm{PMNS}} = U_e^\dagger \omega(\theta) U_\nu$. Thus, the parameters of the PMNS matrix and the resulting strength of *CP* violation (given by a product of these parameters) are time dependent and space dependent. As discussed in Refs. [23,35–37], the strings can only induce such observable changes of the neutrino mass matrix if the residual $Z_2 \times Z_2$ symmetry in Eq. (18) is further broken. This breaking gives rise to DWs, as we will demonstrate in the following section.

Topological domain walls.—In the original flavor string models in Refs. [35–37], the residual $Z_2 \times Z_2$ symmetry in Eq. (18) needed to be further broken at high-energy scales, $\Lambda_{Z_2} \sim (10\text{–}100)$ MeV, in order to ensure sizable lepton mass differences, $m_e \neq m_\mu$. In our case, the $Z_2 \times Z_2$ symmetry in Eq. (10) is explicitly broken at the low-energy scales given by Eq. (4), such that the resulting topological DWs have a width of

$$\delta_{\mathrm{DW}} = \frac{1}{m_{\phi_k}} = 8 \times 10^{14} \ \mathrm{m} \left(\frac{m_\tau}{m_l}\right) \left(\frac{1 \ \mathrm{meV}}{\Lambda_G}\right)^2, \qquad (22)$$

where m_l is either m_τ or m_μ, depending on the ϕ_k boson under consideration. Thus, there are two independent string-wall networks, in which the DWs of width $\delta_{\mathrm{DW}}(m_l = m_\mu)$ and $\delta_{\mathrm{DW}}(m_l = m_\tau)$ are connected to the two different types of Z_2 strings in Eq. (18), respectively. The energy density in these DWs is $\rho_{\mathrm{DW}} = \delta_{\mathrm{DW}}^3 \xi$.

As discussed above, the explicit $Z_2 \times Z_2$ symmetry breaking that gives rise to the DWs is the reason why passing by a string results in observable neutrino flavor transitions. As an example, we can consider a DW bounded by a g_3 string, described by $\omega_3(\theta)$ in Eq. (21). As we go around the string, $\langle\Phi(\theta)\rangle$ is rotated by $\omega_3^T(\theta)\langle\Phi\rangle\omega_3(\theta)$ as in Eq. (20) and returns to itself for $\theta = 2\pi$. Meanwhile, the neutrino wave function rotates by $\omega_3(\theta) \xrightarrow[\theta=2\pi]{} \mathrm{diag}(g_3, 1)$, which leads to an overall minus sign of ν_1 and ν_2. Such a

DW can arise through the vacuum expectation value (VEV) $\langle\Phi\rangle \propto (1, 0, 0)$, for which $g_3\langle\Phi\rangle \neq \langle\Phi\rangle$ with g_3 given by Eq. (11). Thus, the neutrinos change their flavor when passing by a string or through a DW.

Experimental predictions.—In principle, both the DWs and the strings that form in the Majorana neutrino case can induce time- and space-dependent effects on the neutrino mass matrix. However, the DW effects are negligibly small, as they only change the neutrino mass splittings by $\mathcal{O}(\delta_{\mathrm{DW}}^{-1}) \lesssim 2 \times 10^{-21}$ eV; see Eq. (4). While the string effects are larger, they do not change the neutrino mass splittings but only the mixing angles, as explained below Eq. (21). While a neutrino travels from one string to another, the mixing angles continuously and randomly change by $\mathcal{O}(1)$, which is the leading observational effect of the topological defects. As a secondary effect, the strings induce different mixing angles at Sun and Earth, but this effect is negligibly small because the Earth-Sun distance is smaller than the interstring separation.

The time-dependent change of the neutrino mixing angles can be observable in future neutrino experiments and already in existing neutrino data. Similar experimental signatures have been previously discussed in the context of dark-matter-neutrino interactions, which can give rise to time-dependent oscillatory contributions to the neutrino masses and mixing angles [41–45]. However, the predictions of our scenario fundamentally differ from the previously considered ones, because only the mixing angles are time dependent. Interestingly, current data allow for an $\mathcal{O}(10\%)$ variation in the neutrino parameters over the last ten years [21,46].

To determine the timescales over which the neutrino mixing angles change, we need to compare the interstring separation scale ξ in Eq. (19) to the DW width δ_{DW} in Eq. (22). Here, we can distinguish two scenarios: For $\xi > \delta_{\mathrm{DW}}$, the DWs would dominate the evolution of the string-wall network, and the network would dilute rapidly, similar to the axionic and nontopological DWs treated in Ref. [23], with $\mathcal{O}(10)$ DWs remaining per Hubble volume. In this case, an observation of variations in the mixing angles would be very unlikely. We thus focus only on the case $\xi < \delta_{\mathrm{DW}}$ in the following.

Next, we need to compare the distance scale ξ and the timescale $t = \xi/v$ to the corresponding scales of the neutrino experiments under consideration. For example, experiments like Daya Bay [47] measured $\sin^2(2\theta_{13}) = 0.0856 \pm 0.0029$ taking data for $t \sim 6$ years. During this time, the solar system moved a distance $d = vt = 4 \times 10^{13}$ m through the frozen string and DW background, where $v \sim 230$ km/s. Interestingly, this distance is similar to the interstring separation ξ in Eq. (19) and therefore similar to the distance over which the neutrino mixing angles change by $\mathcal{O}(1)$. This implies that current data already exclude the existence of the smallest possible interstring separation $\xi < \delta_{\mathrm{DW}} = 8 \times 10^8$ m for $m_l = m_\tau$ and $\Lambda_G = 1$ eV. In this case, the Solar System would only need $t = d/v \sim 1$ h to pass by a string, which would result in rapid variations of the neutrino mixing angles. While this specific case of $\xi < \delta_{\mathrm{DW}}$ and $\Lambda_G = 1$ eV is ruled out, both the cases of $\xi > \delta_{\mathrm{DW}}$ and $\xi < \delta_{\mathrm{DW}}$ with $\Lambda_G \lesssim 0.2$ meV are still viable. In the latter case, the most constraining parameter is the smallest mixing angle $\sin^2(2\theta_{13}) = 0.0856 \pm 0.0029$ [47], which could exhibit a time-dependent change Δ_{13} within the experimental uncertainty, $\Delta_{13} = \sin^2(2 \times 2\pi vt/\delta_{\mathrm{DW}}) \lesssim 2 \times 0.0029$, where $t \sim 6$ y and δ_{DW} is given by Eq. (22). This estimate strongly motivates the search for time-dependent changes in existing and future neutrino oscillation data, with a particular focus on the smallest mixing angle θ_{13}.

Conclusions.—We studied the formation and evolution of soft topological defects from a late cosmological phase transition predicted by the neutrino mass model in Ref. [9]. In the model, neutrino flavor symmetry gets spontaneously broken by a composite Higgs-like field, which is neutrino bilinear and thus does not require any new particles. We demonstrated that the Dirac case gives rise to neutrino skyrmions, global monopoles, and a hybrid string-wall network, which all quickly annihilate into dark radiation. More importantly, the Majorana case predicts the formation of strings and topological DWs that are similar to the strings and DWs of broken flavor symmetry first discussed in Refs. [35–37]. This string-wall network induces a time- and space-dependent variation of the neutrino mixing angles, which could be observable in future neutrino experiments and already in existing neutrino data. The detection of this smoking gun feature would indicate that neutrinos are Majorana particles and thus could solve one of the fundamental open questions of neutrino physics.

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