

## Protection of Noise Squeezing in a Quantum Interferometer with Optimal Resource Allocation

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Interferometers are crucial for precision measurements, including gravitational waves, laser ranging, radar, and imaging. The phase sensitivity, the core parameter, can be quantum-enhanced to break the standard quantum limit (SQL) using quantum states. However, quantum states are highly fragile and quickly degrade with losses. We design and demonstrate a quantum interferometer utilizing a beam splitter with a variable splitting ratio to protect the quantum resource against environmental impacts. The optimal phase sensitivity can reach the quantum Cramér-Rao bound of the system. This quantum interferometer can greatly reduce the quantum source requirements in quantum measurements. In theory, with a 66.6% loss rate, the sensitivity can break the SQL using only a 6.0 dB squeezed quantum resource with the current interferometer rather than a 24 dB squeezed quantum resource with a conventional squeezing-vacuum-injected Mach-Zehnder interferometer. In experiments, when using a 2.0 dB squeezed vacuum state, the sensitivity enhancement remains at  $\sim 1.6$  dB via optimizing the first splitting ratio when the loss rate changes from 0% to 90%, indicating that the quantum resource is excellently protected with the existence of losses in practical applications. This strategy could open a way to retain quantum advantages for quantum information processing and quantum precision measurement in lossy environments.

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Interferometers play a key role in precision measurements for gravitational waves [1–4], gravity fields [5–8], imaging [9–11], and so on. Quantum squeezing states, including two-mode squeezing [12,13] and vacuum squeezing states [14,15] are important, which differentiate these measurements [16–20] from classical measurements. There have been numerous proof-of-principle experiments on noise-squeezing quantum measurements in imaging [21,22], magnetometry [23–25], quantum information [26,27], etc. The typical example is the utilization of squeezing vacuum as the input of Mach-Zehnder interferometer (MZI) to improve the measurement sensitivity of gravitational waves [28–30]. However, in realistic situations, the measurement system is rarely isolated and is inevitably affected by the environment [31–33]. The losses are one of the main obstacles to the realization of quantum measurement [34,35]. The losses of quantum states bring excess noise, resulting in quick degradation of the sensitivity.

A typical quantum MZ interferometer is constructed of two input fields (a coherent field  $\varepsilon_0$  and a squeezed vacuum  $S_0$ ) and two 50:50 beam splitters for wave splitting and wave recombination as shown in Fig. 1(a), called

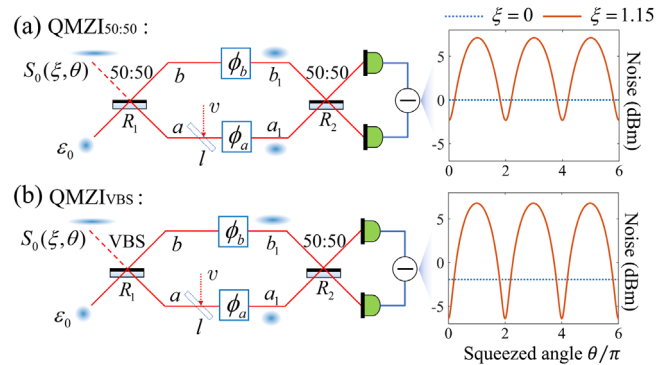


FIG. 1. Schematic of lossy quantum Mach-Zehnder interferometer.  $\varepsilon_0$ : coherent field;  $S_0(\xi, \theta)$ : squeezed vacuum field with squeezing degree  $\xi$  and squeezing angle  $\theta$ ;  $v$ : vacuum field.  $l$ : internal loss rate of arm  $a$ ;  $\phi_a, \phi_b$ : phase shifts;  $R_1, R_2$ : reflectivities of beam splitters. (a) QMZI<sub>50:50</sub> with two 50:50 beam splitters. (b) QMZI<sub>VBS</sub> with a variable beam splitter (VBS) and a 50:50 beam splitter. Blue shades: noise at the respective position of the interferometer. Two right insets show the noise squeezing of QMZI<sub>50:50</sub> and QMZI<sub>VBS</sub> with parameters  $\phi_a - \phi_b = \pi/2$ ,  $l = 0.7$  and  $\xi = 1.15$ .

QMZI<sub>50:50</sub>. A 20% loss in one arm of this interferometer results in a 3.5 dB decrease in the sensitivity when using a 10 dB-squeezed vacuum, but results in only a 0.5 dB decrease when using a vacuum state instead  $S_0$ , that is a conventional MZ interferometer (MZI<sub>50:50</sub>). The quantum enhancement of QMZI<sub>50:50</sub> is very fragile with the existence of losses in the interference arm because the advantage of noise squeezing is eliminated with considerable losses [36–38]. Therefore, studying how to protect quantum resources in practical environments is fundamentally important.

In this Letter, to resist the loss effect, we propose and experimentally demonstrate a loss-tolerant quantum interferometer, called QMZI<sub>VBS</sub>, as shown in Fig. 1(b). The difference compared with QMZI<sub>50:50</sub> is a variable beam splitter (VBS) to realize wave splitting. The central purpose is to protect the quantum resource as much as possible to minimize the noise and retain more of the coherent resource to maximize the signal and significantly improve the phase sensitivity in a lossy environment. A way to realize this goal is to optimally allocate the quantum and coherent resources by adjusting the ratio  $R_1$  of VBS.

The interference output is sensitive to the variation of the phase difference between two arms when the phase difference is fixed at the optimum point  $\pi/2$ , and a small phase shift  $\Delta\phi$  ( $\phi_a - \phi_b = \pi/2 + \Delta\phi$ ) is applied. Then, the phase sensitivity of QMZI<sub>VBS</sub> is given as (Supplemental Material, Sec. II [39])

$$\delta\phi = \sqrt{\frac{(1-R_1)l + (1-l)e^{-2\xi}}{4(1-R_1)R_1(1-l)N}}, \quad (1)$$

which depends on the loss rate  $l$ , reflectivity  $R_1$ , squeezing degree  $\xi$  and photon number  $N$  in the input laser  $\epsilon_0$ . Obviously, a large squeezing degree and a small loss are both beneficial for sensitivity. Therefore, reducing the loss by improving the reflectivity of the optical mirror and the vacuum in the optical path is an effective method [46], as is utilizing better quantum resources, which is an area that many groups have made great efforts in [47–49]. However, isolation of the surrounding environment and improvement of quantum resources are difficult to achieve in experiments. Furthermore, the quantum enhancement of the sensitivity for QMZI<sub>50:50</sub> guaranteed by the quantum resource quickly degrades with the loss. For a given  $\xi$ , there is a special loss rate that reduces the sensitivity of QMZI<sub>50:50</sub> to the standard quantum limit (SQL, defined as the sensitivity of lossless MZI<sub>50:50</sub>), called the loss rate limit  $l_{\text{SQL}} = 2(e^{2\xi} - 1)/(3e^{2\xi} - 2)$  [39]. With  $\xi = 0.69$  (6 dB squeezing) or 2.76 (24 dB squeezing), the corresponding  $l_{\text{SQL}}$  is 0.6 or 0.666, respectively, indicating that improvement of the quantum resource contributes little to the enhancement of sensitivity in a lossy environment. How to protect the quantum advantage to achieve the best

sensitivity with the use of limited resources is the focus of attention in quantum information processing and quantum measurement.

In addition to the loss rate and squeezing degree, there is a parameter  $R_1$  in Eq. (1), which can be adjusted to protect the quantum resource when  $\xi \neq 0$  and reallocate the coherent resource in two arms when  $\xi = 0$ . As shown in Fig. 2(a), when  $l = 0.7$ , the sensitivity of QMZI<sub>50:50</sub> (star on solid line) is above the SQL by 0.9 dB and the quantum enhancement is 2.3 dB compared with MZI<sub>50:50</sub> (star on dashed line). By optimizing  $R_1$ , the sensitivity of MZI<sub>VBS</sub> is slightly improved by 0.4 dB compared to MZI<sub>50:50</sub>. However, the sensitivity of QMZI<sub>VBS</sub> is significantly improved by 1.6 dB compared with QMZI<sub>50:50</sub>. Furthermore, QMZI<sub>VBS</sub> is improved to below SQL by 0.7 dB, which is better than the best MZI<sub>VBS</sub> result by 3.5 dB. Figure 2(a) clearly indicates that  $R_1$  optimization retains the quantum advantage of noise squeezing well.

Such significant quantum enhancement simply by adjusting  $R_1$  is due to the optimal balance between signal and noise in a lossy environment. The ratio  $R_1$  determines the proportion of the quantum resource and coherent field in the lossless path and lossy path. A larger  $R_1$  tends to pass more of the quantum resource into the lossless path to maintain the noise squeezing and simultaneously more of the coherent field into the lossy path, resulting in a decrease in the effective phase-sensitive photon number. At a certain loss  $l$ , the signal and the noise are

$$\text{signal} = 2\sqrt{(1-R_1)R_1(1-l)N}\Delta\phi, \quad (2)$$

$$\text{noise} = \sqrt{[(1-R_1)l + (1-l)e^{-2\xi}]N}, \quad (3)$$

which exhibit different dependence on  $R_1$  [39]. As shown in Fig. 2(b), the maximum signal always appears at  $R_1 = 0.5$ , but the noises of both QMZI<sub>VBS</sub> and MZI<sub>VBS</sub> monotonically decrease with  $R_1$ . Furthermore, the noise initially decreases faster, and then, the signal is reduced more quickly. In general, the noise of QMZI<sub>VBS</sub> decreases faster than that of MZI<sub>VBS</sub> due to the protection of quantum resource. The optimal  $R_1$  for the minimum  $\delta\phi$  can be given as

$$R_1^{\text{opt}} = \begin{cases} \frac{B - \sqrt{B^2 - Bl}}{l} & (l \neq 0, B = (1-l)e^{-2\xi} + l) \\ 0.5 & (l = 0) \end{cases}. \quad (4)$$

In Fig. 2(c),  $R_1^{\text{opt}}$  grows with  $l$ , whose growth rate depends on the squeezing degree  $\xi$ . MZI<sub>VBS</sub><sup>opt</sup> curve corresponds  $\xi = 0$ . Better squeezing degree  $\xi$  requires a larger  $R_1^{\text{opt}}$  because quantum resources with higher performance are more fragile in a lossy environment. This is the reason why quantum measurement is difficult to apply in practice.

In general, the loss has a negative effect on the sensitivity due to the decrease of phase-sensitive photon number and

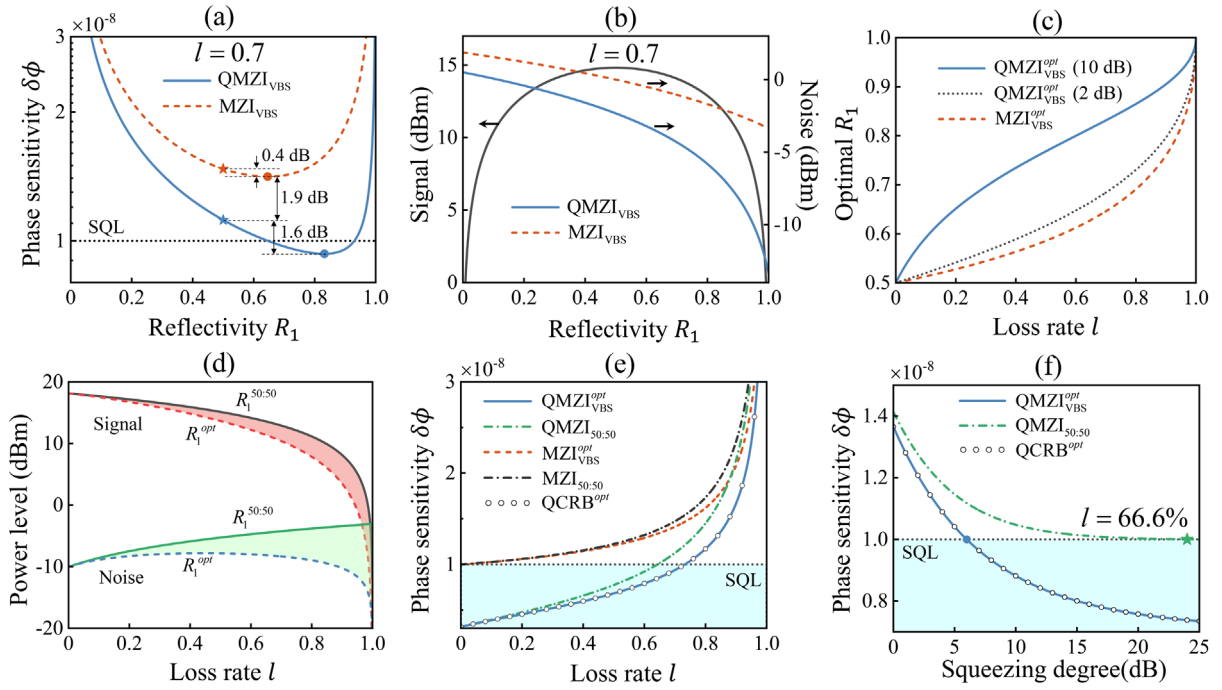


FIG. 2. Theoretical analysis. (a) Sensitivity as a function of the reflectivity  $R_1$ . Solid and dashed lines represent QMZI<sub>VBS</sub> and MZI<sub>VBS</sub>. SQL: dotted line, lossless MZI<sub>50:50</sub>. (b) Signal (left axis, solid line) and noise (right axis, blue solid line) of QMZI<sub>VBS</sub>. Noise of MZI<sub>VBS</sub> (right axis, brown dashed line) is set to 0 dBm as a noise reference when  $R_1 = 0.5$ . (c) Optimal reflectivity  $R_1$  for minimum sensitivity versus loss  $l$  in MZI and QMZI (10 dB or 2 dB squeezing input). (d) Signal and noise as a function of loss rate  $l$ . QMZI<sub>50:50</sub>: solid line; QMZI<sub>VBS</sub>: dashed line. Sensitivity as a function of (e) the loss rate  $l$  and (f) the squeezing degree at  $l = 66.6\%$ . QCRB<sup>opt</sup>: quantum Cramér-Rao bound with optimal  $R_1$ . QMZI<sub>VBS</sub><sup>opt</sup>: QMZI<sub>VBS</sub> with optimal  $R_1$ . The SQL is obtained by  $1/\sqrt{N}$ , with the total number of photons  $N = 10^{16}$ . In (a),(f), stars represent the sensitivity with  $R_1 = 0.5$ , and dots represent the sensitivity with optimal  $R_1$ . The squeezing degree of QMZI in (a),(b),(d),(e) is 10 dB.

the reduction of quantum performance. However, with the optimal  $R_1$ , both the sensitivity and quantum enhancement can be significantly improved. In Fig. 2(d), when the loss increases, the signal reduces while the noise increases for QMZI<sub>50:50</sub>, but both the signal and noise decrease for the optimal QMZI<sub>VBS</sub>, which differentiates QMZI<sub>VBS</sub> from QMZI<sub>50:50</sub>. Therefore, the sensitivity of QMZI<sub>VBS</sub><sup>opt</sup> is always better than QMZI<sub>50:50</sub> and MZI<sub>VBS</sub><sup>opt</sup> [Fig. 2(e)], clearly revealing the advantage of an adjustable  $R_1$  in a lossy environment. Here, we show the results of sensitivity improvement via the  $R_1$  optimization when the loss of arm  $b$ ,  $l_b = 0$ . In fact, in practical applications, both arms may suffer losses simultaneously. However, sensitivity improvement can still be achieved via  $R_1$  optimization with the same trend as the results of  $l_b = 0$  [39]. Furthermore, in many applications, arm  $b$  can be retained in a local environment that can be controlled to be nearly lossless, so we theoretically study and experimentally demonstrate the optimization effect in a simple and clear way.

Another advantage of the current interferometer is that the performance requirements for quantum sources can be greatly reduced. In the QMZI<sub>50:50</sub> case, the improvement of the quantum resource helps little in enhancing the phase sensitivity  $\delta\phi$  in a lossy environment. According to the loss rate limit  $l_{SQL}$ , when the loss rate  $l \geq 2/3$ ,  $\delta\phi$  is worse than

the SQL regardless of the squeezing degree of the squeezing vacuum. However, in the QMZI<sub>VBS</sub> case,  $\delta\phi$  can be better than the SQL by optimizing  $R_1$  at  $l \geq 2/3$  using a squeezing vacuum with a relatively poor squeezing degree. In Fig. 2(f), when  $l$  is 66.6%, the achievement of SQL-broken sensitivity requires 24 dB squeezing with 50:50 beamsplitters, while only 6.0 dB squeezing is required with the optimal VBS of  $R_1 = 0.72$ .

To note that, in Figs. 2(e) and 2(f), the quantum Cramér-Rao bound (QCRB) curve is also given to evaluate the performance of interferometers [39]. QCRB is the detection-independent ultimate limit of the sensitivity [43–45]. The optimal sensitivity of QMZI<sub>VBS</sub> can saturate the optimal QCRB at all loss rates and squeezing degrees, showing that the sensitivity of QMZI<sub>VBS</sub> reaches the ultimate limitation.

In the experiment, the VBS of QMZI<sub>VBS</sub> consists of a polarization beam splitter (PBS2) and a half-wave plate (HWP1), as shown in Fig. 3.  $R_1$  can be changed by adjusting HWP1.  $S_0$  and  $\epsilon_0$ , spatially overlapping with orthogonal polarizations, are divided into two interference arms after passing through VBS- $R_1$  and recombined to realize a quantum interferometer. HWP2 and PBS4 are utilized as the traditional 50:50 beam splitter. The visibility of lossless MZI<sub>50:50</sub> is 99%. A variable attenuation plate is

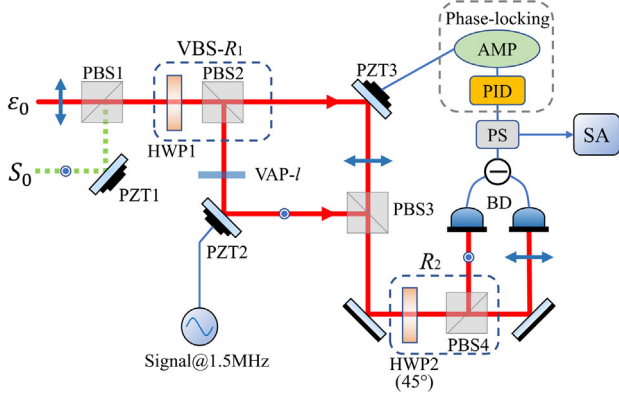


FIG. 3. Experimental setup. Coherent field  $\epsilon_0$  with horizontal polarization and squeezed vacuum field  $S_0$  with vertical polarization. The power of  $\epsilon_0$  is 2.5 mW. The squeezing degree of  $S_0$  is 2 dB. The interference signal is measured by balanced intensity-difference detection (BD) and analyzed by a spectrum analyzer (SA). Polarization beam splitter (PBS); variable attenuation plate (VAP); half-wave plate (HWP); variable beam splitter (VBS); amplifier (AMP); power splitter (PS); piezoelectric transducer (PZT).

placed in one interference arm to change the loss rate  $l$ . The phase difference between two arms is locked at  $\pi/2$ . Squeezing degree of  $S_0$  is 2.0 dB, generated via polarization self-rotation effect in atomic vapor [50,51].

The dependences of the signal, noise and sensitivity on  $R_1$  were measured to further experimentally explore the physical

mechanism for  $R_1$  optimization. In Fig. 4(a), with the increase of  $R_1$ , the noise level of  $\text{QMZI}_{\text{VBS}}$  and that of  $\text{MZI}_{\text{VBS}}$  monotonically decrease and the best signal appears at  $R_1 = 0.5$ , matching the theoretical results well. In particular, as  $R_1$  increases, the noise of  $\text{QMZI}_{\text{VBS}}$  drops faster than that of  $\text{MZI}_{\text{VBS}}$  due to the protection of quantum noise. The four figures in Fig. 4(b) show the sensitivity as a function of  $R_1$  at different losses. The optimal  $R_1$  is 0.5, 0.6, 0.66, and 0.8 for loss rates  $l = 0, 0.427, 0.7$ , and  $0.9$ , respectively. A larger loss requires a larger optimal  $R_1$ . In general, the absolute sensitivities of both  $\text{MZI}_{50:50}$  and  $\text{QMZI}_{\text{VBS}}$  increase with the loss, mainly due to the decrease in the number of phase-sensitive photons. The best sensitivity of  $\text{QMZI}_{\text{VBS}}$  is always better than that of  $\text{QMZI}_{50:50}$  and reaches the optimal QCRB because of the protection of the quantum resource via  $R_1$  optimization. In particular, at  $l = 0.427$ , the phase sensitivity of  $\text{QMZI}_{50:50}$  is worse than SQL. By adjusting  $R_1$  to the optimal value, the sensitivity of  $\text{QMZI}_{\text{VBS}}^{\text{opt}}$  can still go beyond SQL.

The optimization ratios (ORs) of  $\text{QMZI}_{\text{VBS}}^{\text{opt}}/\text{MZI}_{50:50}$  and  $\text{MZI}_{\text{VBS}}^{\text{opt}}/\text{MZI}_{50:50}$  at different losses are given in Fig. 4(c) to show the optimization degree. The results of  $\text{QMZI}_{50:50}$  are also compared under the same initial conditions. The sensitivities of the two quantum interferometers ( $\text{QMZI}_{50:50}$  and  $\text{QMZI}_{\text{VBS}}^{\text{opt}}$ ) are both beyond the SQL by 2.0 dB at  $l = 0$ . With the growth of the loss from 0% to 90%, the OR monotonically decreases to 0.5 dB in the  $\text{QMZI}_{50:50}$  case, while it is maintained at

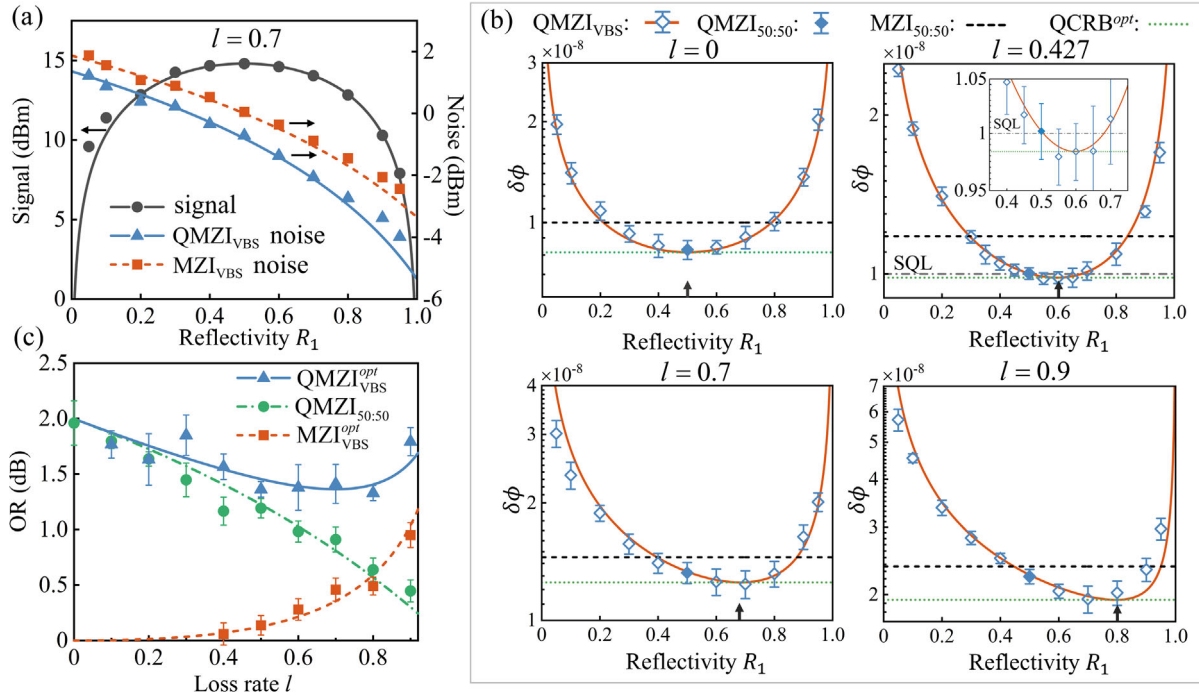


FIG. 4. Experimental results. Squeezing degree of  $S_0$  is 2.0 dB. (a) Signal and noise versus reflectivity  $R_1$  at  $l = 0.7$ . Blue triangles and black dots are the noise and signal of  $\text{QMZI}_{\text{VBS}}$ , respectively. Brown squares: noise of  $\text{MZI}_{\text{VBS}}$ . Lines represent theoretical fittings. (b) Sensitivity versus reflectivity  $R_1$  with different losses. The arrows indicate the location of optimal  $R_1$ . (c) Optimization ratio (OR) is defined as  $-20\log_{10}(\delta\phi_i/\delta\phi_{\text{MZI}_{50:50}})$ , where  $i$  is  $\text{QMZI}_{\text{VBS}}^{\text{opt}}$ ,  $\text{QMZI}_{50:50}^{\text{opt}}$  or  $\text{MZI}_{\text{VBS}}^{\text{opt}}$ .

1.6 dB in the  $\text{QMZI}_{\text{VBS}}^{\text{opt}}$  case. It experimentally demonstrates that the conventional quantum interferometer is very sensitive to losses, but the utilization of an adjustable beam splitter to adjust  $R_1$  is an effective way to optimize the sensitivity of quantum measurements in a lossy environment. As a result,  $\text{QMZI}_{\text{VBS}}^{\text{opt}}$  is loss tolerant. Furthermore, we emphasize that in physics varying the beam splitter ratio optimizes the performance of the interferometer in the presence of losses for both vacuum and squeezing-vacuum injections, as shown in Fig. 4(c). While in the case of squeezing-vacuum injection, squeezing protection through the allocation in two arms of the  $\text{QMZI}_{\text{VBS}}^{\text{opt}}$  shows better performance compared with vacuum injection case [39].

So far, we demonstrate the quantum protection of  $\text{QMZI}_{\text{VBS}}$  with the loss in one arm. In fact, such a proposal can also be applied to improve the performance with the existence of internal losses in two arms. As long as the losses are unbalanced, whether with or without detection loss [41,42], both the sensitivity and quantum protection can be improved by the  $R_1$  optimization. Current  $\text{QMZI}_{\text{VBS}}^{\text{opt}}$  is better than  $\text{QMZI}_{50:50}$  [39]. We have proven the effectiveness of the  $\text{QMZI}_{\text{VBS}}$  model.

The purpose of introducing the VBS into a quantum interferometer is to protect the quantum resource as much as possible to significantly improve the phase sensitivity in a lossy environment. The phase sensitivity at the optimal  $R_1$  reaches the ultimate limit,  $\text{QCRB}^{\text{opt}}$ , and is much better than that with  $\text{QMZI}_{50:50}$ . The performance requirements for quantum sources are greatly reduced, alleviating the vulnerability of the quantum resource to some extent. In experiments, when using a 2.0 dB-squeezed vacuum, as high as 1.6 dB sensitivity enhancement is retained at the loss rate from 0 to 90%. In practical applications, quantum resources are difficult to improve, but coherent resources are much easier to achieve. The current technique promises full use of quantum resources in a practical environment, which has important application prospects in quantum precision measurement, quantum information, and biotechnology.

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- [1] M. e. Tse, H. Yu, N. Kijbunchoo, A. Fernandez-Galiana, P. Dupej, L. Barsotti, C. Blair, D. Brown, S. Dwyer, A. Effler *et al.*, *Phys. Rev. Lett.* **123**, 231107 (2019).
  - [2] F. Acernese, M. Agathos, L. Aiello, A. Allocca, A. Amato, S. Ansoldi, S. Antier, M. Arène, N. Arnaud, S. Ascenzi *et al.* (Virgo Collaboration), *Phys. Rev. Lett.* **123**, 231108 (2019).
  - [3] L. M. Krauss, S. Dodelson, and S. Meyer, *Science* **328**, 989 (2010).
  - [4] H. J. Kimble, Y. Levin, A. B. Matsko, K. S. Thorne, and S. P. Vyatchanin, *Phys. Rev. D* **65**, 022002 (2001).
  - [5] J. B. Fixler, G. T. Foster, J. M. McGuirk, and M. A. Kasevich, *Science* **315**, 74 (2007).
  - [6] G. Lamporesi, A. Bertoldi, L. Cacciapuoti, M. Prevedelli, and G. M. Tino, *Phys. Rev. Lett.* **100**, 050801 (2008).
  - [7] G. Rosi, F. Sorrentino, L. Cacciapuoti, M. Prevedelli, and G. Tino, *Nature (London)* **510**, 518 (2014).
  - [8] A. Peters, K. Y. Chung, and S. Chu, *Metrologia* **38**, 25 (2001).
  - [9] W. Gault and G. Shepherd, *Adv. Space Res.* **2**, 111 (1982).
  - [10] E. Thiébaud, in *Optical and Infrared Interferometry*, Vol. 7013, edited by M. Schöller, W. C. Danchi, and F. Delplancke, International Society for Optics and Photonics (SPIE, Marseille, 2008), pp. 479–490.
  - [11] W. F. Hemsing, A. R. Mathews, R. H. Warnes, and G. R. Whittemore, in *Ultrahigh- and High-Speed Photography, Videography, Photonics, and Velocimetry '90*, edited by P. A. Jaanimagi, B. T. Neyer, and L. L. Shaw, International Society for Optics and Photonics (SPIE, San Diego, 1991), Vol. 1346, pp. 133–140.
  - [12] P. M. Anisimov, G. M. Raterman, A. Chiruvelli, W. N. Plick, S. D. Huver, H. Lee, and J. P. Dowling, *Phys. Rev. Lett.* **104**, 103602 (2010).
  - [13] O. Steuernagel and S. Scheel, *J. Opt. B* **6**, S66 (2004).
  - [14] A. B. Matsko, I. Novikova, G. R. Welch, D. Budker, D. F. Kimball, and S. M. Rochester, *Phys. Rev. A* **66**, 043815 (2002).
  - [15] D. F. Walls, *Nature (London)* **306**, 141 (1983).
  - [16] V. Giovannetti, S. Lloyd, and L. Maccone, *Science* **306**, 1330 (2004).
  - [17] D. S. Simon, G. Jaeger, and A. V. Sergienko, *Quantum metrology*, in *Quantum Metrology, Imaging, and Communication* (Springer International Publishing, Cham, 2017), pp. 91–112.
  - [18] V. Giovannetti, S. Lloyd, and L. Maccone, *Nat. Photonics* **5**, 222 (2011).
  - [19] M. Xiao, L.-A. Wu, and H. J. Kimble, *Phys. Rev. Lett.* **59**, 278 (1987).
  - [20] M. D. Lang and C. M. Caves, *Phys. Rev. Lett.* **111**, 173601 (2013).
  - [21] H. Katsuki, H. Chiba, C. Meier, B. Girard, and K. Ohmori, *Phys. Rev. Lett.* **102**, 103602 (2009).
  - [22] D.-S. Ding, Z.-Y. Zhou, B.-S. Shi, and G.-C. Guo, *Nat. Commun.* **4**, 1 (2013).
  - [23] G. Bao, S. Wu, S. Liu, W. Huang, Z. Li, L. Chen, C.-H. Yuan, and W. Zhang, *Phys. Rev. Appl.* **11**, 054075 (2019).

- [24] J. B. Brask, R. Chaves, and J. Kołodyński, *Phys. Rev. X* **5**, 031010 (2015).
- [25] W. Wasilewski, K. Jensen, H. Krauter, J. J. Renema, M. V. Balabas, and E. S. Polzik, *Phys. Rev. Lett.* **104**, 133601 (2010).
- [26] H. Yuen and J. Shapiro, *IEEE Trans. Inf. Theory* **24**, 657 (1978).
- [27] S. L. Braunstein and P. van Loock, *Rev. Mod. Phys.* **77**, 513 (2005).
- [28] D. Gatto, P. Facchi, F. A. Narducci, and V. Tamma, *Phys. Rev. Res.* **1**, 032024(R) (2019).
- [29] C. M. Caves, *Phys. Rev. D* **23**, 1693 (1981).
- [30] R. S. Bondurant and J. H. Shapiro, *Phys. Rev. D* **30**, 2548 (1984).
- [31] R. Nichols, T. R. Bromley, L. A. Correa, and G. Adesso, *Phys. Rev. A* **94**, 042101 (2016).
- [32] C. Oh, S.-Y. Lee, H. Nha, and H. Jeong, *Phys. Rev. A* **96**, 062304 (2017).
- [33] B. Escher, R. de Matos Filho, and L. Davidovich, *Braz. J. Phys.* **41**, 229 (2011).
- [34] R. Demkowicz-Dobrzanski, U. Dorner, B. J. Smith, J. S. Lundeen, W. Wasilewski, K. Banaszek, and I. A. Walmsley, *Phys. Rev. A* **80**, 013825 (2009).
- [35] A. Parks, S. Spence, J. Troupe, and N. Rodecap, *Rev. Sci. Instrum.* **76**, 043103 (2005).
- [36] H. Vahlbruch, M. Mehmet, S. Chelkowski, B. Hage, A. Franzen, N. Lastzka, S. Goßler, K. Danzmann, and R. Schnabel, *Phys. Rev. Lett.* **100**, 033602 (2008).
- [37] J. Xin, H. Wang, and J. Jing, *Appl. Phys. Lett.* **109**, 051107 (2016).
- [38] X.-X. Zhang, Y.-X. Yang, and X.-B. Wang, *Phys. Rev. A* **88**, 013838 (2013).
- [39] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/PhysRevLett.130.073601> for the universal Quantum Mach-Zehnder interferometer, which includes Refs. [40–45].
- [40] M. O. Scully and M. S. Zubairy, *Quantum Optics* (Cambridge University Press, Cambridge, England, 1997), pp. 60–65.
- [41] G. Frascella, S. Agne, F. Y. Khalili, and M. V. Chekhova, *npj Quantum Inf.* **7**, 72 (2021).
- [42] D. Gatto, P. Facchi, and V. Tamma, *Phys. Rev. A* **105**, 012607 (2022).
- [43] B. Escher, R. de Matos Filho, and L. Davidovich, *Nat. Phys.* **7**, 406 (2011).
- [44] A. Monras, *Phys. Rev. A* **73**, 033821 (2006).
- [45] D. Triggiani, P. Facchi, and V. Tamma, *Eur. Phys. J. Plus* **137**, 125 (2022).
- [46] A. Freise and K. Strain, *Living Rev. Relativity* **13**, 1 (2010).
- [47] S. Shi, Y. Wang, W. Yang, Y. Zheng, and K. Peng, *Opt. Lett.* **43**, 5411 (2018).
- [48] M. Mehmet, S. Ast, T. Eberle, S. Steinlechner, H. Vahlbruch, and R. Schnabel, *Opt. Express* **19**, 25763 (2011).
- [49] H. Vahlbruch, M. Mehmet, K. Danzmann, and R. Schnabel, *Phys. Rev. Lett.* **117**, 110801 (2016).
- [50] S. Barreiro, P. Valente, H. Failache, and A. Lezama, *Phys. Rev. A* **84**, 033851 (2011).
- [51] E. E. Mikhailov and I. Novikova, *Opt. Lett.* **33**, 1213 (2008).