Cross-Channel Constraints on Resonant Antikaon-Nucleon Scattering

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Chiral perturbation theory and its unitarized versions have played an important role in our understanding of the low-energy strong interaction. Yet, so far, such studies typically deal exclusively with perturbative or nonperturbative channels. In this Letter, we report on the first global study of meson-baryon scattering up to one-loop order. It is shown that covariant baryon chiral perturbation theory, including its unitarization for the negative strangeness sector, can describe meson-baryon scattering data remarkably well. This provides a highly nontrivial check on the validity of this important low-energy effective field theory of QCD. We show that the $\bar{K}N$ related quantities can be better described in comparison with those of lower-order studies, and with reduced uncertainties due to the stringent constraints from the πN and KN phase shifts. In particular, we find that the two-pole structure of $\Lambda(1405)$ persists up to one-loop order reinforcing the existence of two-pole structures in dynamically generated states.

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Introduction.—Resolving the patterns of low-energy hadron-hadron interactions constitutes one of the most important goals of modern theoretical physics, fostering not only a better understanding of the nonperturbative nature of QCD but also motivating novel tests of fundamental symmetries including searches for beyond standard model physics. Furthermore, the quantitative understanding of hadron-hadron interactions in the different strangeness sectors has important implications for matter on the largest scales in the universe. For example, in the strangeness zero sector, πN scattering is related to the $\sigma_{\pi N}$ term, crucial for dark matter direct-detection efforts [1,2]. Another example with negative strangeness relates antikaon-nucleon scattering to the properties of neutron stars, where matter compressed to multiples of nuclear matter densities may allow for the appearance of kaon condensates [3,4]. Related to this is an ongoing controversy about the existence of deeply bound K^- nuclear clusters [5–11], sensitive to the $\bar{K}N$ interaction. The interest in the strangeness sector has motivated several ongoing or future experiments including SIDDHARTA-2 [12–14]; AMADEUS [15,16]; BGO-OD [17]; J-PARC E15, E31, E57 and E62 [18–21]; PANDA [22,23]; and the proposed secondary K_L beam in Hall D of Jefferson Lab [24,25].

Bridging the different strangeness sectors of the meson-baryon interaction in a systematic fashion has been a major challenge, which is faced in this Letter. Systematic theoretical approaches to such interactions are provided by lattice QCD and chiral perturbation theory (CHPT). Following the latter methodology (For reviews on recent progress in extracting resonant hadron-hadron dynamics from lattice QCD see, e.g., Refs. [26,27].) baryon chiral perturbation theory (BCHPT) is known to be able to describe both pion-nucleon [28–34] and kaon-nucleon [35,36] scattering data rather well up to one-loop order. However, in a strict perturbative application of BCHPT, such calculations do not allow one to simultaneously study the antikaon-nucleon channel due to the existence of the subthreshold $\Lambda(1405)$ resonance. In this, unitarized CHPT with kernels from both leading [37–40] and next-to-leading order (NLO) [41–46] CHPT became the predominant tool, including a prediction of a second pole [39,47]. See Refs. [47–50] for recent reviews. Such models have also been shown to be consistent with modern experimental data, such as $K^- p \to \pi^0\pi^0\Sigma^0$ and $\gamma p \to K^+(\pi\Sigma)$ [51–55]. Nevertheless, on a quantitative level, various NLO studies obtain different results depending on the details of the implementation [48,56,57], particularly for the pole positions of the $\Lambda(1405)$. One hypothesis for the origin of this ambiguity is related to the large number of unknown low-energy constants (LECs) appearing in the NLO kernel. In this Letter, we aim to improve on this by employing novel constraints from SU(3)-flavor symmetry and its breaking, simultaneously addressing the πN, KN, and $\bar{K}N$ channels.

A simultaneous study of all the meson-baryon scattering data is difficult for a number of reasons. First, it is well known that an adequate description of pion-nucleon scattering at energies below the first resonance [$\Delta(1232)$], i.e., $\sqrt{s} \approx 1.16$ GeV, requires the inclusion of the next-to-next-to-leading order (NNLO) contributions [31,35]. Second, the convergence of BCHPT in the three-flavor sector is controversial. Only in recent years has it been shown that this problem can be circumvented or alleviated in the extended-on-mass-shell (EOMS) formulation of BCHPT [58,59]. For some recent applications, see, e.g., Ref. [60] for the case of baryon magnetic moments, Ref. [61] for the case of baryon masses, and Ref. [62] for a review.

Of particular importance for the present work is the unifying NNLO BCHPT calculation of πN and KN scattering performed in Ref. [35]. Unifying this with a nonperturbative formulation of the $\bar{K}N$ amplitude is the main challenge attacked in this Letter, which allows one to reliably determine the free parameters (more than 30 LECs) taking advantage of SU(3) flavor symmetry and abundant scattering data.

Notably, including $\bar{K}N$ data allows one to disentangle all LECs, instead of determining only certain combinations. In addition, for the $\bar{K}N$ channel, there is *less* freedom in the NNLO fits with KN and πN constraints than in the NLO fits without them, which is reflected in the reduced uncertainty. Indeed, apart from providing the first NNLO study of the $\bar{K}N$ sector, this Letter also estimates, for the first time, systematic uncertainties for the meson-baryon sector from the truncation of the chiral expansion.

Formalism.—In the following, we outline the workflow connecting the three meson-baryon channels πN, KN, and $\bar{K}N$. Beginning with the low-energy regime, we use BCHPT in the EOMS formulation [58,59] to write the scattering amplitude as

$$T^{l\pm}_{\mathrm{BCHPT}}(s) = \mathcal{T}^{(1),l\pm}(s) + \mathcal{T}^{(2),l\pm}(s) + \mathcal{T}^{(3),l\pm}(s) \quad (1)$$

up to the NNLO with respect to small meson momenta and masses, where the superscript $l\pm$ denotes that the amplitudes are projected to the partial waves with orbital angular moment l and total angular momentum $J = l \pm \frac{1}{2}$. The total energy squared is denoted by s. The terms $\mathcal{T}^{(i)}$ can be found in Ref. [35]. As shown in the latter reference such expressions indeed allow one to address the perturbative regime of the meson-baryon scattering below the lowest resonances quite successfully. Specifically, this is the case for the KN and πN channels close to respective thresholds. The situation is entirely different in the $S = -1$ channel, where the $\Lambda(1405)$ resonance appears around the $\bar{K}N$ threshold. Addressing this issue is guided by restoring two-body unitarity—the fundamental principle of S-matrix theory—for which we follow the method proposed in Ref. [39]:

$$T^{l\pm}_{\bar{K}N}(s) = [1 + N^{l\pm}(s) \cdot G(s)]^{-1} \cdot N^{l\pm}(s),$$
$$N^{l\pm}(s) = T^{l\pm}_{\mathrm{BCHPT}}(s) + \mathcal{T}^{(1),l\pm}(s) \cdot G(s) \cdot \mathcal{T}^{(1),l\pm}(s). \quad (2)$$

Here, all elements are promoted to matrices in channel space ($\pi^0\Lambda$, $\pi^-\Sigma^+$, $\pi^0\Sigma^0$, $\pi^+\Sigma^-$, K^-p, $\bar{K}^0 n$, $\eta\Lambda$, $\eta\Sigma^0$, $K^0\Xi^0$, $K^+\Xi^-$), while $G(s)$ denotes a loop function, given explicitly in the Supplemental Material [63].

Characteristic to all unitarization procedures is the introduction of certain model dependence [48] being also related to the appearance of the new unknown parameters $\{a_{\pi\Lambda}, a_{\pi\Sigma}, a_{\bar{K}N}, a_{\eta\Lambda}, a_{\eta\Sigma}, a_{K\Xi}\}$, corresponding to each combination of multiplets of the aforementioned channel space. However, the crucial feature of the employed program lies in the fact that both amplitudes (1), (2) coincide exactly when expanded to the third chiral order. Ultimately, this allows one to reduce uncertainties of extracted amplitudes and poles compared with the NLO calculations of individual strangeness sectors, by combining the analysis of the perturbative ($\pi N, KN$) and nonperturbative sector ($\bar{K}N$) corresponding to Eqs. (1) and (2). The larger number of fit parameters at NNLO compared with NLO is outweighed by the larger data base of the global analysis.

Results and discussions.—The formalism proposed in this Letter is derived from the $\mathcal{O}(p^3)$ chiral Lagrangian [79–81]. The counterterms appearing in it contain 27 LECs, 14 of $\mathcal{O}(p^2)$ ($b_0, b_D, b_F, b_{1,\ldots,8}$, and $c_{1,2,3}$) and 13 of $\mathcal{O}(p^3)$ ($d_{1,\ldots,10,48,49,50}$). Taking into account the six subtraction constants $\{a_{\pi\Lambda}, a_{\pi\Sigma}, a_{\bar{K}N}, a_{\eta\Lambda}, a_{\eta\Sigma}, a_{K\Xi}\}$ yields a total of 33 free parameters. Note that SU(3) flavor symmetry is broken both at the Lagrangian level and through the use of physical hadron masses and channel specific subtraction constants.

The challenging task of determining them is approached in a multistep fitting strategy. In that, we note that πN scattering data can be described well [35] using $\mathcal{O}(p^3)$ amplitudes with eight combinations of the 27 LECs; see the Supplemental Material [63]. Additionally, for each of the two KN isospin channels, eight combinations of LECs contribute up to this order which indeed decouple among all the three reaction channels $\{\pi N, KN_{I=0}, KN_{I=1}\}$. Therefore, we leave the LECs related to the πN scattering data and baryon masses fixed at the values determined in Ref. [35], denoting them with "NNLO*" in the corresponding tables in the Supplemental Material [63]. For the remaining 23 free parameters, we first search for reasonable values for the 16 effective LECs which describe the KN elastic scattering data. Then, taking these numbers as initial

TABLE I. Best χ^2/d.o.f. obtained for the four isospin-strangeness meson-baryon channels. For the selection of the πN and KN data, see Ref. [35]. Note that the χ^2's for πN and KN defined in Ref. [35] do not have statistical rigor and only serve to show the goodness of the fits.

	$\bar{K}N$	πN	$KN_{I=0}$	$KN_{I=1}$
χ^2/d.o.f.	1.56	1.28	0.46	1.46
Data	173	78	60	60

values, we perform a global fit for all the 23 parameters to the experimental data in the $\{KN, \bar{K}N\}$ sector. (We also perform an alternative fit where the constraints from baryon masses are excluded, referred to as "NNLO*" afterward. See the Supplemental Material [63] for details.) Here, the latter consists of cross sections $\{K^-p \to X | X = K^-p, \bar{K}^0n, \pi^-\Sigma^+, \pi^0\Sigma^0, \pi^+\Sigma^-, \pi^0\Lambda, \eta\Lambda\}$ [82–87] as well as threshold ratios $\{\gamma, R_c, R_n\}$ [88,89] and K^-p scattering length [90] extracted from the SIDDHARTA data [91]. Note that we only fit to the experimental data that are directly related to the two-body scattering amplitude $T_{M_iB_i \to M_jB_j}$ but do not consider the $\pi\Sigma$ spectra in the γp [92–94], K^-p [95], pp [96,97], and e^-p [98] reactions, which involve at least three final-state particles. Theoretical efforts in this direction are being made currently, see, e.g., Refs. [53,54,57,99].

The best fit χ^2 per degree of freedom (χ^2/d.o.f.) are listed in Table I. The corresponding LECs are shown in the

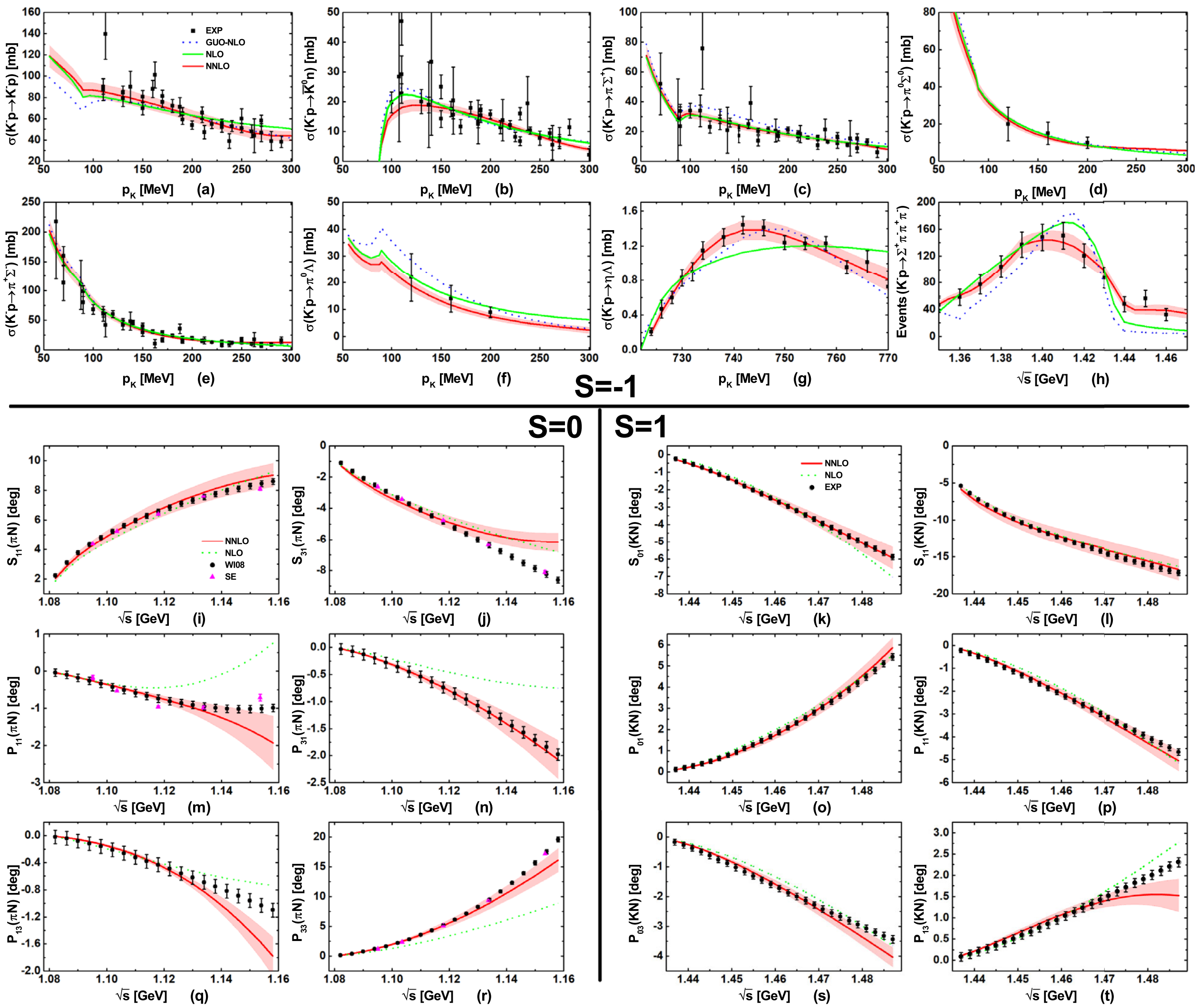

FIG. 1. Compilation of fit results. Upper panel: cross sections for $\{K^-p \to X | X = K^-p, \bar{K}^0n, \pi^-\Sigma^+, \pi^0\Sigma^0, \pi^+\Sigma^-, \pi^0\Lambda, \eta\Lambda\}$ with p_K denoting the K^- laboratory momentum. The black dots with error bars denote the experimental data for total cross sections [82–87] and $\pi^-\Sigma^+$ mass spectrum [104], while the red bands (green dotted lines) refer to our NNLO (NLO) results, and the blue dashed lines denote the results of the NLO study [43]. Lower panel: phase shifts of πN and KN elastic scattering compared with (black dots) SAID WI08 [105] and SP92 solutions [106], respectively. The green dashed lines and red solid lines denote our NLO and NNLO results, respectively, with uncertainty bands estimated by the Bayesian model. We also show the single energy (SE) πN phase shifts (magenta triangles) obtained with a correlated analysis of different partial waves in Ref. [107].

TABLE II. Threshold parameters, pole positions, and couplings of the two $I = 0$ states obtained in the present work in comparison with experimental data and the results of Ref. [43].

	a_{K^-p} (fm)	γ	R_c	R_n
NNLO	$(-0.71 \pm 0.07) + i(0.84 \pm 0.07)$	2.35 ± 0.19	0.684 ± 0.033	0.198 ± 0.019
NLO [43]	$-0.61^{+0.07}_{-0.08} + i(0.89^{+0.09}_{-0.08})$	$2.36^{+0.17}_{-0.22}$	$0.661^{+0.12}_{-0.11}$	$0.188^{+0.028}_{-0.029}$
EXP	$(-0.64 \pm 0.10) + i(0.81 \pm 0.15)$	2.36 ± 0.12	0.664 ± 0.033	0.189 ± 0.015

	Pole positions (MeV)	$\lvert g_{\pi\Sigma}\rvert$ (GeV)	$\lvert g_{\eta\Lambda}\rvert$ (GeV)	$\lvert g_{\bar{K}N}\rvert$ (GeV)	$\lvert g_{K\Xi}\rvert$ (GeV)
$\Lambda(1380)$	$1392 \pm 8 - i(102 \pm 15)$	6.40 ± 0.10	3.01 ± 0.15	2.31 ± 0.10	0.45 ± 0.01
$\Lambda(1405)$	$1425 \pm 1 - i(13 \pm 4)$	2.15 ± 0.07	5.45 ± 0.24	4.99 ± 0.08	0.58 ± 0.02

Supplemental Material [63]. They show that BCHPT and its unitarized version can provide a good description of the meson-baryon scattering data for all the three strangeness sectors simultaneously. For the $\bar{K}N$ channel, with all the constraints from the KN and πN channels, we obtain a $\chi^2/\text{d.o.f} = 1.56$ weighting different observables by the respective number of data points [41,43,44,100], which should be compared with the equivalent value of about 2 from the NLO study [43]. The $\chi^2/\text{d.o.f.}$ for the KN channels decrease considerably [from 3.93(2.24) to 0.46(1.46) for $KN_{I=0}(KN_{I=1})$] compared with those obtained in Ref. [35] since we take into account the $\mathrm{O}(p^3)$ tree level contributions which were omitted there.

In Fig. 1, we show the cross sections from the global NLO (The NLO study is presented only for the sake of comparison. The description of the $\bar{K}N$ channel is acceptable but that of the πN channel is much worse. See the Supplemental Material [63] for details.) and NNLO fits for the $\bar{K}N$ coupled channels as well as πN and KN phase shifts. The error bands are produced by the Bayesian model for a degree of belief of 68% [101–103] (see the Supplemental Material [63] for details). The comparison with the best NLO fits of Guo [43] reveals that the $\bar{K}N$ cross sections can be described rather well already at NLO, but quantitatively better results are obtained at NNLO, in particular, those of $\{\pi^-\Sigma^+, \pi^0\Lambda, \eta\Lambda\}$ final states. It is important to note that compared with the NLO fits, only NNLO fits allow also for a simultaneous description of the πN and KN phase shifts [35].

In Fig. 1(h), we also show the $\pi^-\Sigma^+$ mass spectrum in the vicinity of $\Lambda(1405)$. As explained above, these data are not fitted. They are calculated following the approach of Refs. [39,43] but including the contributions from $\pi\Lambda$ and $\eta\Lambda$. The $\eta\Sigma$ and $K\Xi$ channels are neglected because they are too far away from the energy region of our interest. While we are faced with the well-known problem that the left-hand cuts overlap with the unitary cuts below the $\bar{K}N$ threshold (see Supplemental Material [63] for details), the data are indeed described well.

In Table II we compare the scattering length and three ratios with the experimental data. Clearly the agreement is very good. We show as well the results of Fit II of the NLO study of Ref. [43], which agree with ours within uncertainties.

The double pole structure of $\Lambda(1405)$ is the most interesting nonperturbative phenomenon in this coupled-channel problem. Studies on this special resonance date back to the 1960s [108] where it was suggested as a $\bar{K}N$ bound state (see also review in Ref. [48]). It was then found that $\Lambda(1405)$ is actually a superposition of two poles [39,109–111]. Recent discussions on this issue can be found in Refs. [42,43,53,112–114]. Note that a recent lattice QCD study also supports the $\bar{K}N$ bound state interpretation of $\Lambda(1405)$ [115]; see also Refs. [116,117]. In order to obtain the pole position, one needs to extend the amplitudes to the second Riemann sheet. This can be achieved by analytically extrapolating the loop function $G(s)$ to the second Riemann sheet following the standard prescription, see, e.g., Refs. [27,43,56]. The poles

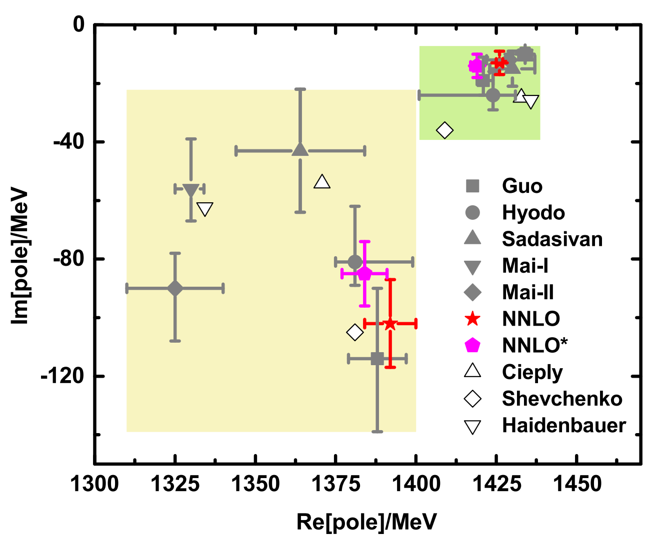

FIG. 2. Positions of the two $\Lambda(1405)$ poles obtained in the present study ("NNLO" and "NNLO*" corresponding to results with or without baryon mass constraints) in comparison with those of the NLO studies, i.e., Guo [43], Hyodo [42], Mai-I [53], Mai-II [53], Sadasivan [113], Cieply [118], Shevchenko [119], and Haidenbauer [120].

discussed in the following are all situated on the respective sheet that is closest to the physical axis. The couplings of the poles to various channels i, j are obtained from the residues of the poles on the complex plane as $T^{ij}(s) = \lim_{s\to s_R} g_i g_j/(s - s_R)$. With the LECs determined above, we can predict the positions of the two poles and the corresponding couplings to various channels, which are shown in Table II. In the $I = 0$ sector, the lower pole is located at $(1392, -102)$ MeV while the higher one is at $(1425, -13)$ MeV. We also find a state located at $(1676, -25)$ MeV corresponding to the $\Lambda(1670)$ resonance. A selected compilation of the two-pole positions is shown in Fig. 2 including the two-pole position from the NNLO and NNLO* fits corresponding to results with or without baryon mass constraints. It is clear that though the positions of the lower pole from different studies are quite scattered, those of the higher pole are determined much more precisely. We note that compared with the NLO results, the uncertainties in the NNLO results are smaller, due to the stringent constraints from the πN and KN scattering data. It is interesting to point out that the $\Lambda(1405)$ pole positions are similar to those of Fit II of Ref. [43].

Conclusion and outlook.—We have performed for the first time a global study of meson-baryon scattering in all three strangeness sectors $S = 0, +1, -1$. The crucial step for this was the derivation of the formalism based on covariant baryon chiral perturbation theory including next-to-next-to-leading order contributions while employing a consistent unitarization procedure for the nonperturbative $S = -1$ sector. Besides theoretical relevance, this formalism allows one to put tighter constraints on extracted amplitudes and resonances, by connecting data from the different reactions $\{\pi N, KN, \bar{K}N\}$, ensured by the $\mathrm{SU}(3)_{\mathrm{f}}$ symmetry and its breaking. Indeed, this is only possible at NNLO due to the known poor convergence of the chiral expansion in the $S = 0$ sector.

Focusing on the $\bar{K}N$ sector, we confirmed the two-pole structure of $\Lambda(1405)$ in this novel approach, simultaneously ensuring for the first time an agreement with the perturbative channels. For the corresponding pole positions, we found results consistent with most NLO studies but with reduced uncertainties due to the stringent constraints from the πN and KN scattering data. It should be stressed that for dynamically generated states, the existence of two-pole structures seems to be a common phenomenon [47]. Some recent examples that have attracted considerable attention include the $K_1(1270)$ [121] and $D_0^*(2300)$ [122]. The two-pole structure of $\Lambda(1405)$ can be understood by following trajectories on which symmetries of the hadron-hadron interactions are restored [47,111,123,124]. As a result, the emergence of a two-pole structure can be viewed as strong evidence supporting the molecular nature of the state under investigation.

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