Comment on "Turbulence Statistics of Arbitrary Moments of Wall-Bounded Shear Flows: A Symmetry Approach"

In a recent Letter [1], no progress has been made as claimed *"to overcome the problem related to the closure problem of turbulence"* (p. 5). Figure 1 briefly proves this in three parts. All further details may be taken from [2].

First, a trivial aspect of [1] is shown by Fig. 1(a), which is to be compared with Fig. 3 in [1]: The structure of parallel lines in channel center is a trivial feature, which, as shown, can also be found in laminar flow, or for any other function (see Figs. 1,3 in [3]). Hence, the claim in [1] that this type of scaling is the result of anomalous scaling due to *"intermittent behavior"* (p. 5) cannot be confirmed.

Second, a misleading aspect of [1] is shown by Fig. 1(b), to be compared with Fig. 1(a) in [1]. The scaling behavior shown is clearly dominated by the mean flow and says nothing about the fluctuating part. Because of the overwhelming dominance of the mean flow $\bar{U}_1$ in this region, the near-perfect fitting result only reflects the trivial aspect that $\overline{U_1^n} \approx \bar{U}_1^n$. And any *unbiased* indicator function reaffirms it ([2], Sec. B.1). Therefore, it is not surprising that only a few parameters are needed to fit the data up to high order. In fact, with the symmetry method used in [1], it is easy to prove [2] that even fewer parameters are already sufficient to achieve this invariant scaling, as shown in Fig. 1(b). Here, only the mean velocity ($n = 1$, bottom curve) was fitted according to Eq. (15), i.e., instead of seven parameters $(\kappa, B, \omega, \alpha, \beta, \tilde{\alpha}, \tilde{\beta})$, as in [1], only two (κ, B) were used.

Third, an erroneous aspect of [1] is shown by Figs. 1(c) and 1(d), to be compared with Figs. 3 and 1(a) in [1], respectively. The scaling laws in [1] are not solutions to the statistical Navier-Stokes equations as claimed, otherwise they would also match to the data of the fluctuation correlations. The reason is that the underlying "statistical symmetries" Eqs. (8)–(9) violate the classical principle of cause and effect between the fluctuations and the mean fields [2,4–6]. This violation is suppressed when analyzing the symmetry-based scaling of the full-field correlations, but becomes measurable and clearly visible when analyzing the corresponding *fluctuation* correlations, as shown by the fitting failure in Figs. 1(c)–1(d) already for the lowest moments where the DNS data of [1] can be trusted. To note is that this failure is structural and not due to a numerical stability issue ([2], Sec. 3), or any DNS uncertainty when transforming to the fluctuation moments ([2], Sec. C). If one attempts to use the method of Lie groups in turbulence correctly, then a consistent approach is presented in Sec. 4 in [2].

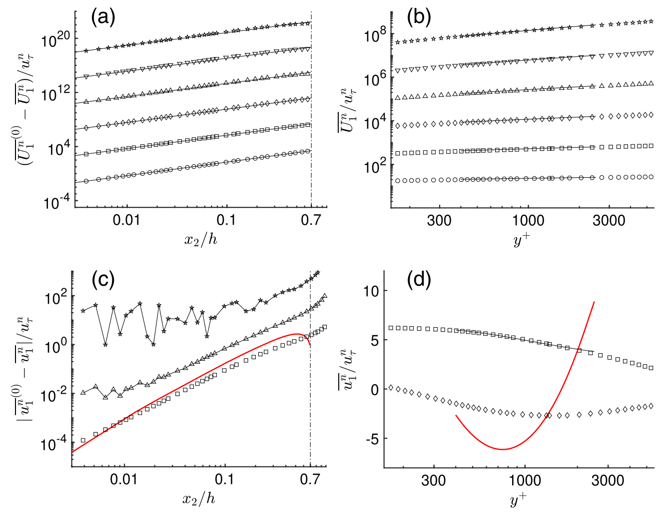

FIG. 1. (a) Laminar case. The markers display the different powers of the laminar profile up to order $n = 6$ (from bottom to top). The solid lines show the best-fit of the turbulent scaling law Eqs. (19–20) from [1]. (b) Turbulent case. The markers display the turbulent full-field correlations $\overline{U_1^n}$ up to order $n = 6$, exactly as in [1]. The solid lines show $\bar{U}_1^n$, the powers of the fitted mean velocity profile $\bar{U}_1$. (c) Best-fit (red line) to the fluctuation moment $n = 2$ (squares). The thin line with triangle- and star-markers show the moments $n = 4, 6$, respectively. Obviously, the higher moments in the DNS of [1] are not yet fully converged. (d) Best-fit (red line) to the fluctuation moment $n = 3$ (diamonds), based on the best-fit (black line) of the second moment $n = 2$ (squares). Note that although the second moment can be fitted here, it fits the data very unnaturally (see fig. 6(a) in [2]).

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