Top-Down Holography in an Asymptotically Flat Spacetime

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We propose a holographic duality for a four dimensional Wess-Zumino-Witten model with target manifold SO(8), coupled to scalar-flat Kähler gravity on an asymptotically flat, four dimensional background known as the Burns metric. The holographic dual is a two dimensional chiral algebra built out of gauged β-γ systems with SO(8) flavor. We test the duality by matching two-point correlators of soft gluon currents with two-point gluon amplitudes, and their leading operator product expansion coefficients with collinear limits of three-point gluon amplitudes.

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Introduction.—Holography on asymptotically flat spacetimes has been a long-standing open problem, with existing gauge-gravity dualities relating matrix quantum mechanics to flat space string and M theory (e.g., [1–5]). In this Letter, we describe a new kind of holographic duality wherein the bulk is a four-dimensional asymptotically flat, self-dual spacetime. The duality relates certain models of self-dual gauge and gravitational theories in the bulk to a two-dimensional chiral algebra living on a Riemann sphere.

This example arises from a generalization of twisted holography [6,7], whereby the B-model topological string [8] on a "backreacted," i.e., complex structure-deformed geometry is dual to a B-brane world-volume theory. The latter is typically a chiral algebra, supported on a complex submanifold of the undeformed flat space geometry. In these twisted examples, open-closed duality has recently been formalized mathematically as Koszul duality [7,11,12], which permits efficient bulk computations that match the boundary chiral algebra operator product expansions (OPEs).

The present example is rooted in recent developments connecting the celestial holography program to twisted holography and twistor theory [13,14]. In flat space, it was observed that the physics of soft gluons and gravitons is governed by certain 2D chiral algebras [15–17]. These chiral algebras often fail to be associative when one incorporates quantum effects [18] or higher derivative couplings [19]. If, however, the 4D theory can be uplifted to a local holomorphic theory on twistor space [20], an associative chiral algebra, including quantum effects corresponding to higher-loop collinear singularities, is guaranteed. The $G = \mathrm{SO}(8)$ 4D Wess-Zumino-Witten (WZW_4) theory is one of these distinguished theories that can therefore serve as a simple toy model, and its quantum consistency when coupled to the gravitational sector is guaranteed by a Green-Schwarz anomaly cancellation mechanism in the type I topological string [21] on twistor space.

The resulting twisted open-closed string theory is our undeformed bulk theory, and we systematically incorporate backreaction, per the standard holographic setup, by wrapping N D1-branes on the zero section of the twistor fibration $\simeq \mathbb{CP}^1$, the Riemann sphere. This deforms the spacetime geometry from flat space to the asymptotically flat *Burns space* [22]. The D1-branes, as usual, support the dual, boundary chiral algebra. The usual identification of the twistor $\mathbb{CP}^1$ with the celestial sphere at null infinity then makes this a concrete, top-down toy model of celestial holography as envisioned by, e.g., [23,24] and references therein. We will elaborate on the string theory uplift of this example, and present additional details and checks of our proposed duality, in a forthcoming companion paper [25].

In the remainder of this Letter we introduce our bulk gauge + gravitational theory and the dual chiral algebra, describe the holographic dictionary between bulk states and chiral algebra operators, and provide explicit checks of the duality in 2- and 3-point computations. Supplemental

Material [26] contains computational details of chiral algebra OPEs and sample computations of bulk 2- and 3-gluon amplitudes to leading nontrivial order.

Bulk theory.—Our bulk spacetime is self-dual (i.e., it has a self-dual Weyl tensor), so it is most conveniently formulated in Euclidean signature. Let $x^\mu \in \mathbb{R}^4$ and introduce double-null coordinates on $\mathbb{R}^4$ [27]

$$x^{\alpha\dot{\alpha}} \coloneqq \frac{1}{\sqrt{2}}\begin{pmatrix} x^0 + ix^3 & x^2 + ix^1 \\ -x^2 + ix^1 & x^0 - ix^3 \end{pmatrix}, \tag{1}$$

where $\alpha = 1, 2$, $\dot{\alpha} = \dot{1}, \dot{2}$ are Weyl spinor indices. In terms of these, we can set up complex coordinates on $\mathbb{R}^4 \simeq \mathbb{C}^2$,

$$u^{\dot{\alpha}} \coloneqq x^{1\dot{\alpha}}, \qquad \hat{u}^{\dot{\alpha}} \coloneqq x^{2\dot{\alpha}} = (-\overline{u^{\dot{2}}}, \overline{u^{\dot{1}}}). \tag{2}$$

Also let $\|u\|^2 \coloneqq |u^{\dot{1}}|^2 + |u^{\dot{2}}|^2 = \frac{1}{2}\delta_{\mu\nu}x^\mu x^\nu$, and denote by $\partial = du^{\dot{\alpha}}\partial/\partial u^{\dot{\alpha}}$, $\bar{\partial} = d\hat{u}^{\dot{\alpha}}\partial/\partial\hat{u}^{\dot{\alpha}}$ the holomorphic and antiholomorphic exterior derivatives on $\mathbb{C}^2$.

We equip $\mathbb{C}^2 - \{0\}$ with the Riemannian metric

$$ds^2 = \|du\|^2 + \frac{|u^{\dot{1}}du^{\dot{2}} - u^{\dot{2}}du^{\dot{1}}|^2}{\|u\|^4}. \tag{3}$$

This is a self-dual metric known as the *Burns metric* [22,28,29]. It is asymptotically flat, in the sense that $g_{\mu\nu} = \delta_{\mu\nu} + O(\|u\|^{-2})$ at large $\|u\|$ [30]. It has zero scalar curvature but is *not* Ricci flat. Moreover, it is Kähler with Kähler potential

$$K = \|u\|^2 + \log\|u\|^2. \tag{4}$$

Let $\tilde{\mathbb{C}}^2 = \{(u^{\dot{\alpha}}, [\zeta^{\dot{\alpha}}]) \in \mathbb{C}^2 \times \mathbb{CP}^1 | u^{\dot{1}}\zeta^{\dot{2}} = u^{\dot{2}}\zeta^{\dot{1}}\}$ denote the blowup of $\mathbb{C}^2$ at the origin. Away from $u^{\dot{\alpha}} = 0$, it is diffeomorphic to $\mathbb{C}^2 - \{0\}$, whereas the origin $u^{\dot{\alpha}} = 0$ is replaced by a copy of $\mathbb{CP}^1$ called the exceptional divisor. The metric (3) extends smoothly to $\tilde{\mathbb{C}}^2$ [31].

We will refer to $\tilde{\mathbb{C}}^2$ equipped with the Burns metric as *Burns space*. In the past, it has featured in the spacetime foam models of [32], being conformally diffeomorphic to (an affine patch of) $\mathbb{CP}^2$ equipped with its Fubini-Study metric via the inversion map $u^{\dot{\alpha}} \mapsto u^{\dot{\alpha}}/\|u\|^2$. It has also been conjectured to result from the backreaction of D1-branes in topological string theory on twistor space [33], a fact that we will prove in the companion paper [34].

The field theory we study on Burns space is the WZW_4 model [20,35–37] for SO(8). This is a σ model governing maps $g(u, \hat{u})$ from Burns space to the group manifold of SO(8), with action

$$\frac{N}{8\pi^2}\int_{\tilde{\mathbb{C}}^2} \partial\bar{\partial}K \wedge \mathrm{tr}(g^{-1}\partial g \wedge g^{-1}\bar{\partial}g) - \frac{N}{24\pi^2}\int_{\tilde{\mathbb{C}}^2\times[0,1]} \partial\bar{\partial}K \wedge \mathrm{tr}(\tilde{g}^{-1}d\tilde{g})^3 \tag{5}$$

where $N > 0$ is a 4D analog of the Kac-Moody level, $\tilde{g}$ is an extension of g to $\tilde{\mathbb{C}}^2 \times [0, 1]$ satisfying the boundary conditions $\tilde{g}|_{\tilde{\mathbb{C}}^2\times\{0\}} = \mathbb{1}$, $\tilde{g}|_{\tilde{\mathbb{C}}^2\times\{1\}} = g$, and $d\tilde{g}$ represents its exterior derivative on $\tilde{\mathbb{C}}^2 \times [0, 1]$. The first term in this action is the standard kinetic term for a σ model, and the second is a 5D Wess-Zumino term.

The Wess-Zumino term is independent of the 5D extension of g iff $(iN/2\pi)\partial\bar{\partial}K \in H^2(\tilde{\mathbb{C}}^2, \mathbb{Z})$ [37]. In particular, integrating it over the exceptional divisor of $\tilde{\mathbb{C}}^2$ yields

$$\int_{\mathbb{CP}^1} \frac{iN}{2\pi}\partial\bar{\partial}K = N. \tag{6}$$

So, we demand the quantization condition $N \in \mathbb{Z}_+$ [38]. In this case, WZW_4 provides a gauge-fixed formulation of self-dual Yang-Mills [27,39] at tree level. This match with gauge theory does not persist at loop level; instead the model is conjecturally related to 4D $\mathcal{N} = 2$ heterotic strings [40].

We couple this model to a gravitational field ρ describing Kähler metrics with Kähler potential $K + \rho$ that classically obey $R = 0$ [20]. Such scalar-flat Kähler metrics are automatically self-dual [41,42]. They can have a nonvanishing Ricci tensor, so are *a priori* unrelated to Einstein gravity. Nevertheless, the coupled system admits an anomaly free uplift to the type I open + closed topological B model on twistor space [20,43,44], to which one can apply twisted holography [6,7]. We refer to our gravitational theory as scalar-flat Kähler gravity.

Our main proposal is a holographic duality matching the collinear singularities in scattering amplitudes of WZW_4 + scalar-flat Kähler gravity on Burns space with OPE coefficients of the large N limit of a family of 2D chiral algebras (with defects). In this Letter, we present explicit checks of the duality in the planar limit for open string operators in the chiral algebra, which correspond to tree-level Yang-Mills collinear singularities; additional planar computations matching the gravitational degrees of freedom will appear in [25].

Compared to previous tree and 1-loop results in celestial holography, our proposal describes the dual chiral algebra *nonperturbatively* in the bulk coupling $1/\sqrt{N}$. We conjecture that this strong form of the duality, analogous to the strong form of the AdS/CFT correspondence, holds. Moreover, WZW_4 is a nonrenormalizable quantum field theory, placed on a curved manifold, but our duality gives an exact (conjectural) description of collinear limits in this theory at finite coupling. This should be compared to known nonperturbative results about nonrenormalizable 2D integrable theories [45].

The holographic dual.—The dual theory is a 2D chiral algebra living on $\mathbb{CP}^1$. It is given by the BRST reduction of a collection of symplectic bosons by the gauge group $\mathrm{Sp}(N)$ (with the conventions that the fundamental representation of $\mathrm{Sp}(N)$ is the symplectic vector space of dimension $2N$). Additionally, it has matter content consisting of spin $\frac{1}{2}$ fields valued in symplectic representations of $\mathrm{Sp}(N)\times \mathrm{SO}(8)$, where SO(8) is a flavor symmetry.

There are two kinds of fields (arising from 1-5 and 1-1 strings [46], respectively):

$$I = I_{im} \in \mathbb{C}^8 \otimes \mathbb{C}^{2N}, \tag{7}$$

$$X = X_{\dot{\alpha}mn} \in \mathbb{C}^2 \otimes {\textstyle\bigwedge}^2_{\substack{\text{trace}\\\text{free}}} \mathbb{C}^{2N}. \tag{8}$$

Here, m is an index in $\mathbb{C}^{2N}$, i in the fundamental of SO(8), and $\dot{\alpha}$ in the fundamental of SU(2), which acts by isometries of Burns space. The tensor X is trace-free: $X_{\dot{\alpha}mn}\omega^{mn} = 0$, with ω^{mn} the symplectic form on $\mathrm{Sp}(N)$.

The OPEs are

$$I_{im}(z_1) I_{jn}(z_2) \sim \frac{\delta_{ij}\omega_{mn}}{z_{12}}, \tag{9}$$

$$X_{\dot{\alpha}mn}(z_1) X_{\dot{\beta}rs}(z_2) \sim \frac{\epsilon_{\dot{\alpha}\dot{\beta}}}{z_{12}}\left(\omega_{m[r|}\omega_{n|s]} - \frac{\omega_{mn}\omega_{rs}}{2N}\right), \tag{10}$$

where $z_{ij} \equiv z_i - z_j$, $\epsilon_{\dot{\alpha}\dot{\beta}}$ is the two-dimensional Levi-Civita symbol, and ω_{mn} is the inverse of ω^{mn}. To this algebra, we adjoin b, c ghosts living in the adjoint of $\mathrm{Sp}(N)$, with the usual BRST operator. The BRST cohomology in the large N limit is our conjecture for the holographic dual chiral algebra.

These fields can be written down for any $\mathrm{SO}(k)$ flavor symmetry group. Only when $k = 8$ does the BRST operator square to zero. This is the conformal field theory (CFT) counterpart of the fact [20,21] that the type I topological string is anomaly free only for SO(8). It is worth remarking that this algebra is the chiral algebra associated by the construction of [47] to a well-known family of 4D $\mathcal{N} = 2$ superconformal field theorie, with SO(8) flavor symmetry, $\mathrm{Sp}(N)$ gauge symmetry and matter as above. For $N = 1$, this is SU(2) gauge theory with $N_f = 4$, as studied by Seiberg and Witten [48].

At large N, the BRST cohomology can be readily computed using the method explained in [6], based on classic results of homological algebra (this computation is done in detail in [49]). To write down the operators, we use ω^{mn} to raise an index so that $X^n_{\dot{1}m}$, $X^n_{\dot{2}m}$ are $2N\times 2N$ matrices. By raising and lowering $\mathrm{Sp}(N)$ indices we can treat I_{im} as being a collection of eight vectors or covectors.

To describe the open string operators, let us introduce an SU(2) doublet $\tilde{\lambda}_{\dot{\alpha}}$. A basis for open string operators are the coefficients of $\tilde{\lambda}^l_{\dot{1}}\tilde{\lambda}^l_{\dot{2}}$ in the generating function

$$\begin{aligned} J_{ij}[\tilde{\lambda}](z) &= I_i \exp[(2N)^{-1/2}\epsilon^{\dot{\alpha}\dot{\beta}}\tilde{\lambda}_{\dot{\alpha}}X_{\dot{\beta}}]I_j \\ &= \sum_{k,l=0}^{\infty}\frac{1}{k!l!}\tilde{\lambda}^k_{\dot{1}}\tilde{\lambda}^l_{\dot{2}}J_{ij}[k,l](z). \end{aligned} \tag{11}$$

This is in the adjoint of SO(8), so we will freely replace composite indices like ij with adjoint indices $a, b, c, \ldots$ in what follows. Similarly, the terms with $k + l = n$ are in the spin $n/2$ representation of SU(2).

There are also two towers of closed-string states, which will not play an important role in this Letter:

$$\begin{aligned} E[\tilde{\lambda}](z) &= \mathrm{Tr}\{\exp[(2N)^{-1/2}\epsilon^{\dot{\alpha}\dot{\beta}}\tilde{\lambda}_{\dot{\alpha}}X_{\dot{\beta}}]\}, \\ F[\tilde{\lambda}](z) &= \mathrm{Tr}\{\epsilon^{\dot{\alpha}\dot{\beta}}X_{\dot{\alpha}}\partial X_{\dot{\beta}}\exp[(2N)^{-1/2}\epsilon^{\dot{\gamma}\dot{\delta}}\tilde{\lambda}_{\dot{\gamma}}X_{\dot{\delta}}]\} + \cdots, \end{aligned}$$

where the ellipses in $F[\tilde{\lambda}]$ indicate terms involving ghosts.

In the planar limit the OPEs can be computed by elementary Wick contractions:

$$J_a[\tilde{\lambda}_1](z_1)J_b[\tilde{\lambda}_2](z_2) \sim \frac{f^c_{ab}}{z_{12}}J_c[\tilde{\lambda}_1+\tilde{\lambda}_2](z_2) - \frac{[12]f^c_{ab}}{z_{12}^2}\int_0^1 d\omega_1\int_0^1 d\omega_2 J_c[\omega_1\tilde{\lambda}_1+\omega_2\tilde{\lambda}_2](z_2) + \cdots - 2N\sum_{n=0}^{\infty}\frac{(-[12])^n}{n!n!}\frac{\kappa_{ab}\mathbf{1}}{z_{12}^{n+2}}, \tag{12}$$

where $\kappa_{ab} = \delta_{il}\delta_{jk} - \delta_{ik}\delta_{jl}$ in terms of composite indices $a = ij$, $b = kl$, and we used the standard conventions $[1\,2] = \epsilon^{\dot{\alpha}\dot{\beta}}\tilde{\lambda}_{1\dot{\beta}}\tilde{\lambda}_{2\dot{\alpha}}$ and $\epsilon^{\dot{\gamma}\dot{\alpha}}\epsilon_{\dot{\gamma}\dot{\beta}} = \delta^{\dot{\alpha}}_{\dot{\beta}}$ for Weyl spinor contractions. The ellipsis denotes terms involving three or more contractions, whereas the contribution of the identity arises from maximal contractions. The negative level of $-2N$ indicates that our celestial holographic dual is nonunitary. Some minor subtleties of this computation are also discussed in Supplemental Material [26].

We will relate OPEs of this chiral CFT to amplitudes on Burns space. The isometry group of Burns space is U(2). Its complexification $\mathrm{GL}(2,\mathbb{C})$ is a symmetry of the analytically continued scattering amplitudes. This is in addition to the SO(8) symmetry. As written, however, the CFT has an $\mathrm{SL}(2,\mathbb{C})\times\mathrm{SL}(2,\mathbb{C})\times\mathrm{SO}(8)$ symmetry, where one $\mathrm{SL}(2,\mathbb{C})$ is a flavor symmetry rotating the dotted index on $X_{\dot{\alpha}}$ and the other is 2D conformal symmetry.

The more precise statement of our dictionary involves a CFT with defects at 0 and ∞, which break the $SL(2,\mathbb{C})$ conformal symmetry to $\mathbb{C}^\times$. The defects are modules generated by vacuum vectors $|v_0\rangle$ and $|v_\infty\rangle$ annihilated by certain modes of $J_a[k,l]$:

$$\oint dz\, z^n J_a[k,l]|v_0\rangle = 0 \quad \text{for } n \geq k+l+1,$$
$$\oint dz\, z^n J_a[k,l]|v_\infty\rangle = 0 \quad \text{for } n \leq -1. \tag{13}$$

We will expand on the geometric and stringy origin of these defects in the companion paper [25]. They have also been studied from the perspective of half-BPS surface defects in 4D $\mathcal{N}=2$ theories in [50,51].

The holographic dictionary.—To study gluon perturbation theory, we parametrize the WZW_4 field as $g=e^\phi$, where ϕ is an adjoint valued scalar field. As its extension, we take $\tilde{g}=e^{t\phi}$ for $t\in[0,1]$. The action (5) then expands to

$$\int_{\tilde{\mathbb{C}}^2} \frac{N\partial\bar{\partial}K}{8\pi^2} \wedge \mathrm{tr}\left(\partial\phi\wedge\bar{\partial}\phi - \frac{1}{3}\phi[\partial\phi,\bar{\partial}\phi] + O(\phi^4)\right). \tag{14}$$

The linearized field equation of ϕ obtained from this action is simply the Laplace equation on Burns space,

$$\left(\epsilon^{\dot{\alpha}\dot{\beta}} + \frac{u^{\dot{\alpha}}\hat{u}^{\dot{\beta}}}{\|u\|^4}\right)\frac{\partial^2\phi}{\partial u^{\dot{\alpha}}\partial\hat{u}^{\dot{\beta}}} = 0. \tag{15}$$

As in flat space, we find a family of analytic solutions of (15) labeled by a complex null momentum $p_{\alpha\dot{\alpha}} = \lambda_\alpha\tilde{\lambda}_{\dot{\alpha}}$, where $\tilde{\lambda}_{\dot{\alpha}}\in\mathbb{C}^2$ whereas $\lambda_\alpha = (z^{-1},1)$ for some $z\in\mathbb{C}^\times$:

$$\phi_a(z,\tilde{\lambda}) = \frac{\mathsf{T}_a e^{ip\cdot x/2}}{z}\left(\cos\frac{\psi p\cdot x}{2} + \frac{i}{\psi}\sin\frac{\psi p\cdot x}{2}\right) \tag{16}$$

(suppressing $x^{\alpha\dot{\alpha}}$ dependence on the left). In this expression, T_a is a generator of the gauge algebra $\mathfrak{so}(8)$ and

$$p\cdot x = \frac{[u\tilde{\lambda}]}{z} + [\hat{u}\,\tilde{\lambda}], \qquad \psi = \sqrt{1+\frac{4[u\tilde{\lambda}][\hat{u}\,\tilde{\lambda}]}{z\|u\|^2(p\cdot x)^2}}. \tag{17}$$

Here, $[u\tilde{\lambda}] = u^{\dot{\alpha}}\tilde{\lambda}_{\dot{\alpha}}$ and $[\hat{u}\,\tilde{\lambda}] = \hat{u}^{\dot{\alpha}}\tilde{\lambda}_{\dot{\alpha}}$ are spinor contractions. The state (16) extends to the blowup $\tilde{\mathbb{C}}^2$. It also admits a very useful "large $\|u\|$" series expansion

$$\frac{\mathsf{T}_a}{z}\sum_{j=0}^{\infty}\left(-\frac{1}{z}\frac{[u\tilde{\lambda}][\hat{u}\,\tilde{\lambda}]}{\|u\|^2}\right)^j \frac{{}_1F_1(j+1,2j+1|ip\cdot x)}{(2j)!}. \tag{18}$$

Its leading term is the flat space momentum eigenstate $e^{ip\cdot x}$, up to the important normalization factor of $1/z$.

We can also Taylor expand this state in $\tilde{\lambda}_{\dot{\alpha}}$ to write

$$\phi_a(z,\tilde{\lambda}) = \sum_{k,l=0}^{\infty}\frac{1}{k!l!}\tilde{\lambda}_1^k\tilde{\lambda}_2^l\phi_a[k,l](z). \tag{19}$$

We will refer to $\phi_a[k,l](z)$ as soft modes, as they are analogous to the coefficients in the flat space soft expansion of $e^{ip\cdot x}$. We propose the holographic dictionary

$$\phi_a[k,l](z) \leftrightarrow J_a[k,l](z), \tag{20}$$

$$\phi_a(z,\tilde{\lambda}) \leftrightarrow J_a[\tilde{\lambda}](z). \tag{21}$$

In the presence of the defects, correlators of such dual operators will be given by polynomials in $(z_i-z_j)^{-1}$ and in z_i^{-1}. For the purposes of this Letter, we may state our holographic dictionary as a match between OPEs in the 2D defect CFT and collinear limits of scattering amplitudes in the bulk. Equivalently, we conjecture a match between CFT correlators and WZW_4 amplitudes on Burns space, up to terms depending on the z_i^{-1}.

If we wanted to recover scattering amplitudes, and not just their collinear limits, we would need to determine the one-point functions of $J_a[k,l]$ in our defect CFT. We hope to return to this in the future.

Tests of the duality.—Two-point functions: Amplitudes in Euclidean signature can be defined and computed via the on-shell effective action [52,53], and then analytically continued to more physical signatures if necessary. To test our duality, we begin by computing the two point amplitude of WZW_4.

Let $\phi_i \equiv \phi_{a_i}(z_i,\tilde{\lambda}_{i\dot{\alpha}})$, where i is a particle label. The two-point tree amplitude of WZW_4 is given by the symmetrized on-shell kinetic term

$$A(1,2) = \frac{N}{8\pi^2}\int_{\tilde{\mathbb{C}}^2}\partial\bar{\partial}K\wedge\mathrm{tr}(\partial\phi_1\wedge\bar{\partial}\phi_2) + (1\leftrightarrow 2). \tag{22}$$

Calculating this on Burns space by plugging in the expansion (18) for ϕ_1, ϕ_2 reveals a delightfully simple result:

$$A(1,2) = -\frac{N}{z_{12}^2}J_0\left(2\sqrt{\frac{[12]}{z_{12}}}\right)\mathrm{tr}(\mathsf{T}_{a_1}\mathsf{T}_{a_2}), \tag{23}$$

with J_0 denoting a Bessel function of the first kind. Up to the color factor, this matches the two-point amplitude of a conformally coupled scalar on $\mathbb{CP}^2$ first found in [32].

Under our proposed identification (21), if we normalize the generators so that $\mathrm{tr}(\mathsf{T}_a\mathsf{T}_b) = 2\kappa_{ab}$, then this exactly matches with the coefficient of the identity operator in the OPE of $J_a[\tilde{\lambda}](z)$. Indeed, we can resum the coefficient of the identity operator $\mathbf{1}$ in (12) to get

$$J_a[\tilde{\lambda}_1](z_1)J_b[\tilde{\lambda}_2](z_2) \sim -\frac{2N\kappa_{ab}\mathbf{1}}{z_{12}^2} J_0\left(2\sqrt{\frac{[12]}{z_{12}}}\right). \quad (24)$$

We derive (23) at $O(z_{12}^{-2})$ in Supplemental Material [26], with more details to appear in [25].

OPE coefficients: Define the "holographic OPE" $\phi_1 \cdot \phi_2$ between two states ϕ_1, ϕ_2 by demanding that the linear combination

$$\varepsilon_1\phi_1 + \varepsilon_2\phi_2 + \varepsilon_1\varepsilon_2\phi_1 \cdot \phi_2 \quad (25)$$

satisfy the nonlinear equations of motion of (14) modulo ε_i^2. The two-point amplitude of the (off-shell) state $\phi_1 \cdot \phi_2$ with a third state ϕ_3 gives rise to the three-point amplitude of ϕ_1, ϕ_2, ϕ_3 (on symmetrization over 1,2,3) [54,55].

From the equation of motion of (14), the OPE satisfies

$$\partial\bar{\partial}K \wedge \left(\partial\bar{\partial}(\phi_1 \cdot \phi_2) + \frac{[\partial\phi_1, \bar{\partial}\phi_2] + [\partial\phi_2, \bar{\partial}\phi_1]}{2}\right) = 0, \quad (26)$$

having used $\partial\bar{\partial}K \wedge \partial\bar{\partial}\phi_i = 0$ to drop $\partial\bar{\partial}K \wedge [\partial\bar{\partial}\phi_1, \phi_2]$, etc. The solution of (26) for $\phi_1 \cdot \phi_2$ is found to be

$$-\int_{v\in\tilde{\mathbb{C}}^2} G(u,v)\, \partial\bar{\partial}K \wedge ([\partial\phi_1, \bar{\partial}\phi_2] + [\partial\phi_2, \bar{\partial}\phi_1])|_v, \quad (27)$$

where $G(u,v)$ for $u^{\dot{\alpha}}, v^{\dot{\alpha}} \in \mathbb{C}^2 - \{0\}$ is the massless scalar propagator on Burns space

$$G(u,v) = -\frac{1}{8\pi^2}\frac{1}{\|u-v\|^2 + \|u\|^{-2}\|v\|^{-2}|[uv]|^2}, \quad (28)$$

and $[uv] = \epsilon_{\dot{\alpha}\dot{\beta}}u^{\dot{\beta}}v^{\dot{\alpha}}$ as usual. Equation (28) can be derived by a conformal rescaling of the propagator of a conformally coupled scalar on $\mathbb{CP}^2$ given in [56].

Plugging in (28), we have evaluated (27) on Burns space to first order in $[1\,2]$ (and singular orders in z_{12})

$$\phi_1 \cdot \phi_2 \sim \frac{f^c_{a_1a_2}}{z_{12}}\phi_c(z_2, \tilde{\lambda}_1 + \tilde{\lambda}_2) - \frac{[12] f^c_{a_1a_2}}{z_{12}^2}\int_0^1 d\omega_1 \int_0^1 d\omega_2\, \phi_c(z_2, \omega_1\tilde{\lambda}_1 + \omega_2\tilde{\lambda}_2) + O([12]^2). \quad (29)$$

The result (29) matches the chiral algebra OPE displayed in (12) under the dictionary (21).

Last, one can also directly compute the three-point amplitude of WZW_4 by plugging the ϕ_i into the cubic interaction vertex of (14). At zeroth order in the square brackets $[1\,2]$, $[2\,3]$, $[3\,1]$, we find a current algebra three-point function

$$\frac{-2Nf^c_{a_1a_2}\kappa_{ca_3}}{z_{12}z_{13}z_{23}} \quad (30)$$

which enables us to directly probe the $j_a(z_1)j_b(z_2)$ OPE. We derive (30) in Supplemental Material [26].

Discussion.—We have presented a top-down toy model of holography in an asymptotically flat spacetime, motivated by constructions from celestial holography, twisted holography, and twistor theory. Following the standard AdS/CFT dictionary, we have presented explicit two- and three-point checks relating chiral algebra OPEs with asymptotic bulk observables in Burns space.

As is familiar from the AdS case, these computations did not rely on the presence of dynamical gravity in the bulk. At tree level, the closed string sector of our duality describes the Kähler subsector of self-dual conformal gravity on Burns space. The presence of defects at the antipodes of $\mathbb{CP}^1$ in our dual CFT is closely related to fixing the diffeomorphism invariance of bulk gravity to a gauge whose coordinates are adapted to the Kähler structure. We will return to this as well as present checks of our duality involving the closed string sector in [25].

Clearly, it would be desirable to build models in which one could better access features of dynamical (quantum) gravity like bulk reconstruction, black hole transitions, spacetime entanglement, and quantum information, etc. Likewise, since our twistorial theory is highly nongeneric, it will be important to better understand which features may be relaxed (and how), and which break down, in more realistic examples.

Nevertheless, we believe our toy model, with its twisted string origins, is a concrete and illustrative starting point for asymptotically flat holography in string theory. Twisted holography is also sensitive to certain nonperturbative effects in N [57], and it will be enlightening to formalize the map between $\mathrm{O}(N)$ effects in the bulk and boundary (gauge theory instantons/Skyrmions, bulk giant gravitons,…), as well as $\mathrm{O}(N^2)$ effects, perhaps by studying the chiral algebra on other boundary geometries (see also [58]) and/or finding connections to Kleinian black holes analogous to those in [59].

It is also worth exploring twisted holography for self-dual Einstein gravity, starting from its twistorial uplifts like the ones studied in [60]. Candidate theories for such a duality could be built along the lines of the twistor sigma models of [61–63]. The backreacted geometry could again be Burns space, which curiously can also be viewed as an Einstein-Maxwell instanton [64,65]. Many other curved backgrounds relevant to celestial holography have also arisen in [66–69], and it will be worthwhile trying to find their uplifts to possible string theory backgrounds.

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