β Symmetry of Supergravity

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Continuous $O(d,d)$ global symmetries emerge in Kaluza-Klein reductions of D-dimensional string supergravities to $D-d$ dimensions. We show that the nongeometric elements of this group effectively act in the D-dimensional parent theory as a hidden bosonic symmetry that fixes its couplings: the β symmetry. We give the explicit β transformations to first order in α' and verify the invariance of the action as well as the closure of the transformation rules.

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Introduction.—String theory provides a quantum completion of general relativity, predicting precise higher-derivative corrections to the Einstein equations. The interactions, determined from scattering amplitudes of string states or conformal invariance of the world sheet theory, contain hidden footprints of the string dualities. In particular, closed strings can wrap around noncontractible cycles in spacetime, giving winding states that have no analog for particle theories and allow T duality. This is a discrete $O(d,d)$ symmetry that establishes the physical equivalence of string theories on dual backgrounds with very different geometries [1]. Its footprints appear as a continuous $O(d,d)$ rigid symmetry in compactifications of the string effective field theories on d-dimensional tori [2], to all orders in α' [3].

The couplings in the higher-derivative expansion of the string (super)gravities can then be predicted by demanding the emergence of $O(d,d)$ symmetries after compactification. Although this procedure is in general tedious, as it requires nontrivial field redefinitions to make the symmetry manifest, it has been successfully pursued up to order α'^3 [4]. An alternative procedure explores symmetry principles that determine double field theory interactions, either through higher-derivative deformations of generalized diffeomorphisms [5] or double Lorentz symmetries [6,7]. The invariant action can then be downgraded to supergravity with all the couplings fixed.

The former method involves heavy brute force computations that become nonviable after a few orders, while the latter is currently confronted with an obstruction [8] starting at the quartic Riemann interactions common to all string theories [9]. We are then at a stage that requires simplifications in the first approach and clarifications in the second one.

The key observation introduced in this Letter is that the appearance of $O(d,d)$ symmetries in the $D-d$-dimensional theory can be assessed already in the D-dimensional parent action. The idea is extremely simple and goes as follows. Starting with a string effective field theory in D dimensions, the Kaluza-Klein reduction to $D-d$ dimensions, keeping only the massless modes, consists of three steps: (i) Split the D spacetime coordinates into $D-d$ external and d internal directions and impose that the fields are independent of the internal ones. (ii) Propose a Kaluza-Klein parametrization of the higher-dimensional fields in terms of those in lower dimensions. The purpose of this step is to obtain fields with standard transformation properties with respect to the local symmetries. (iii) Enforce higher-derivative field redefinitions that allow assembling the degrees of freedom into $O(d,d)$ multiplets, so as to make the $O(d,d)$ symmetry manifest and not corrected by higher derivatives. Some cases require including extra gauge degrees of freedom [13].

The last two items are just field redefinitions. What they do is to take the $D-d$ effective action obtained directly from the D-dimensional one, in which derivatives are nonvanishing only in the external directions, to a scheme in which the symmetries are manifest. These redefinitions are purely aesthetical, since the symmetries, though hidden, are still there. Hence, there must be a way to identify the $O(d,d)$ symmetry directly in the D-dimensional action. This is what we will show in this Letter.

While the *geometric* subgroup of $O(d,d)$, consisting of rigid d-dimensional diffeomorphisms and 2-form shifts, acts trivially with no higher-derivative corrections, the *nongeometric* sector parametrized by a bivector β [14] fixes entirely the effective action in the scheme in which it looks exactly like the higher-dimensional theory. In other words, the nongeometric sector fixes the higher-dimensional action, and it does so by acting effectively as if it were a symmetry in D dimensions.

The Letter is organized as follows. In the next section, we expose the β invariance of the two-derivative universal string supergravity. Then, we derive the first order α' corrections to the β transformations in the generalized Bergshoeff–de Roo scheme and verify closure together with the local symmetries. This is followed by some final remarks in the last section.

The β symmetry to lowest order.—Each term in the universal two-derivative Neveu Schwarz-Neveu Schwarz action

$$S=\int d^D x\sqrt{-g}e^{-2\phi}\left(R-4(\nabla\phi)^2+4\Box\phi-\frac{1}{12}H^2\right) \tag{1}$$

is manifestly invariant under local D-dimensional diffeomorphisms and gauge transformations of the two-form. These symmetries in turn contain $GL(D)\times R^{[D(D-1)/2]}$ as a rigid continuous subgroup, infinitesimally parametrized by a^μ_ν and $B_{\mu\nu}$ acting on $E_{\mu\nu}=g_{\mu\nu}+b_{\mu\nu}$ and ϕ as follows:

$$\delta\partial_\mu=-a^\rho_\mu\partial_\rho, \tag{2a}$$

$$\delta E_{\mu\nu}=B_{\mu\nu}-a^\rho_\mu E_{\rho\nu}-a^\rho_\nu E_{\mu\rho}, \tag{2b}$$

$$\delta\phi=-\frac{1}{2}a^\mu_\mu. \tag{2c}$$

This is the geometric subgroup of $O(D,D)$, which additionally contains nongeometric elements parametrized by a constant bivector $\beta^{\mu\nu}$,

$$\delta E_{\mu\nu}=-E_{\mu\rho}\beta^{\rho\sigma}E_{\sigma\nu}, \tag{3a}$$

$$\delta\phi=\frac{1}{2}\beta^{\mu\nu}E_{\mu\nu}. \tag{3b}$$

These nongeometric transformations are *not* symmetries of supergravity (1). Demanding invariance under the full $O(D,D)$ group requires doubling the spacetime coordinates and adding extra terms in the action, as is known from double field theory [16]. This is not the route that we follow in this Letter: here we deal with pure supergravity.

Even if D-dimensional supergravity is not invariant under $O(D,D)$, we know that its compactification on T^d must be $O(d,d)\in O(D,D)$ symmetric. Operationally, the compactification amounts to the assumption that the fields do not depend on the internal directions, which implies truncating the *derivatives* to be purely *external*. In such case, the action gains the full $O(d,d)$ symmetry, given by the trivial embedding into $O(D,D)$ such that the *parameters* contain only *internal* components. Then, (3) effectively becomes a symmetry of (1) under the constraint

$$\beta^{\mu\nu}\partial_\nu\ldots=0. \tag{4}$$

As a consequence, the $O(d,d)$ symmetry of (1) compactified on T^d can be determined, for all practical purposes, directly in (1) through the action of (3) constrained as in (4).

Checking the β invariance of the action turns out to be easier in the frame formulation, where flattening the indices of the fields with the frame and defining flattened variations

$$\delta e_{ab}=e^\mu_a\delta e_{\mu b},\qquad \delta b_{ab}=e^\mu_a e^\nu_b\delta b_{\mu\nu}, \tag{5}$$

the transformations take the form

$$\delta e_{ab}=-b_{ac}\beta^c_b,\quad \delta b_{ab}=-\beta_{ab}-b_{ac}\beta^{cd}b_{db},\quad \delta\phi=\frac{1}{2}\delta e^a_a. \tag{6}$$

These in turn dictate the variations of the tensors and connections that appear in the action (see the Supplemental Material [17] for details on the notation)

$$[\delta,D_a]=0, \tag{7a}$$

$$\delta w_{cab}=\beta^d_{[a}H_{b]cd}-\frac{1}{2}\beta^d_c H_{abd}, \tag{7b}$$

$$\delta H_{abc}=6w^d_{[ac}\beta_{b]d}, \tag{7c}$$

$$\delta(\nabla_a\phi)=\frac{1}{2}\beta^{cd}H_{acd}, \tag{7d}$$

$$\delta(\nabla_a\nabla_b\phi)=\frac{1}{2}\beta^{cd}\nabla_{(a}H_{b)cd}-\beta^{ce}w^d_{e(a}H_{b)cd} -\beta^c_{(a}H_{b)cd}\nabla^d\phi. \tag{7e}$$

To derive these expressions, we have used (4) and the fact that $\beta^{\mu\nu}$ is constant and antisymmetric, which in turn implies

$$D_a\beta^{bc}=4\beta^{d[b}\omega^{c]}_{[da]},\qquad \beta^{ab}\omega_{abc}=0. \tag{8}$$

To prove the invariance of the action (1) is now trivial, taking into account that the above transformations yield

$$\delta(\sqrt{-g}e^{-2\phi})=0, \tag{9a}$$

$$\delta R=-2\beta^{cd}\nabla^b H_{bcd}+5\beta^{cd}\omega_{cab}H^{ab}_d, \tag{9b}$$

$$\delta(\nabla\phi)^2=\beta^{cd}H_{bcd}\nabla^b\phi, \tag{9c}$$

$$\delta\Box\phi = \frac{1}{2}\beta^{cd}\nabla^b H_{bcd} - \beta^{cd}\omega_{cab}H^{ab}_{d} + \beta^{cd}H_{bcd}\nabla^b\phi, \tag{9d}$$

$$\delta H^2 = 12\beta^{cd}\omega_{cab}H^{ab}_{d}. \tag{9e}$$

In fact, the β invariance of a generic combination of terms preserved by the local symmetries

$$\begin{aligned} 0 &= \delta[R + m(\nabla\phi)^2 + n\Box\phi + pH^2] \\ &= \beta^{cd}\nabla^b H_{bcd}\left(-2 + \frac{n}{2}\right) + \beta^{cd}\omega_{cab}H^{ab}_{d}(5 - n + 12p) \\ &\quad + \beta^{cd}H_{bcd}\nabla^b\phi(m + n) \end{aligned} \tag{10}$$

fixes the value of the coefficients to

$$m = -4, \qquad n = 4, \qquad p = -\frac{1}{12}, \tag{11}$$

selecting (1) as the unique β-symmetric theory.

The β transformations realize the nongeometric sector of $O(d,d)$ as a hidden symmetry in the standard supergravity scheme. Instead, they become both geometric and manifest in the so-called β-supergravity scheme [18], where the nongeometric sector is realized by the B shifts, which should then fix the corresponding couplings.

Together with Lorentz transformations, (6) close into the bracket

$$[\delta_1, \delta_2] = -\delta_{12}, \quad \Lambda_{12ab} = 2\beta^c_{1[a}\beta_{2b]c} + 2\Lambda^c_{1[a}\Lambda_{2b]c}. \tag{12}$$

The β symmetry to first order.—In the biparametric (a, b) generalized Bergshoeff–de Roo scheme, all string effective actions up to first order in α' are included in [19]

$$S = \int d^Dx\sqrt{-g}e^{-2\phi}(L^{(0)} + aL^{(1)}_a + bL^{(1)}_b), \tag{13}$$

where the lowest order Lagrangian $L^{(0)}$ is defined in (1), and the first order one can be written in a flattened fashion with

$$L^{(1)}_a = \frac{1}{4}H^{abc}\Omega^{(-)}_{abc} - \frac{1}{8}R^{(-)}_{abcd}R^{(-)abcd}, \tag{14a}$$

$$L^{(1)}_b = -\frac{1}{4}H^{abc}\Omega^{(+)}_{abc} - \frac{1}{8}R^{(+)}_{abcd}R^{(+)abcd}. \tag{14b}$$

Defining $\omega^{(\pm)}_{abc} = \omega_{abc} \pm \frac{1}{2}H_{abc}$, these expressions contain

$$\Omega^{(\pm)}_{abc} = \omega^{(\pm)e}_{[\underline{a}d}D_{\underline{b}}\omega^{(\pm)d}_{\underline{c}]e} + \omega^{(\pm)e}_{[\underline{a}d}\omega^{(\pm)d}_{fe}\omega^{f}_{\underline{bc}]} + \frac{2}{3}\omega^{(\pm)e}_{[\underline{a}d}\omega^{(\pm)f}_{\underline{b}e}\omega^{(\pm)d}_{\underline{c}]f}, \tag{15}$$

$$R^{(\pm)}_{abcd} = 2D_{[a}\omega^{(\pm)}_{b]cd} + 2\omega^{e}_{[ab]}\omega^{(\pm)}_{ecd} + 2\omega^{(\pm)e}_{[\underline{a}c}\omega^{(\pm)}_{\underline{b}]ed}. \tag{16}$$

We look for measure preserving β transformations that tie the variation of the dilaton to that of the frame field to all orders,

$$\delta(\sqrt{-g}e^{-2\phi}) = 0 \qquad \Rightarrow \qquad \delta\phi = \frac{1}{2}\delta e^a_a. \tag{17}$$

The β invariance up to first order is then guaranteed by

$$\int d^Dx\sqrt{-g}e^{-2\phi}(\delta^{(1)}L^{(0)} + a\delta^{(0)}L^{(1)}_a + b\delta^{(0)}L^{(1)}_b) = 0, \tag{18}$$

where $\delta^{(0)}$ denotes the lowest order variations of the previous section.

To find $\delta^{(1)}$, the first order α' corrections to the β transformations, we will consider an expansion in powers of the fluxes ω_{abc}, H_{abc}, and $D_a\phi$. This is a useful strategy that serves as an organizing principle, mimicking a background field expansion. The difference is that fluxes are composite fields, and hence obey Bianchi identities (BIs) that relate different orders, namely, (A.5), (A.7), and (A.8). To remove ambiguities, one uses the leading terms in the BIs to take the leading order to a minimal form at the expense of introducing subleading terms. Once the leading order is fixed, one moves to the next order and again takes it to a minimal form using BIs at the expense of inducing further higher-order terms, and so on. As an example, the lowest order equations of motion admit a flux expansion of the form

$$\Delta b_{ab} = \frac{1}{2}\nabla_c H^c_{ab} - \nabla_c\phi H^c_{ab} = \frac{1}{2}D_c H^c_{ab} + \cdots, \tag{19a}$$

$$\begin{aligned} \Delta e_{ab} &= -2\left(R_{ab} + 2\nabla_{(a}\nabla_{b)}\phi - \frac{1}{4}H_{acd}H^{cd}_{b}\right) \\ &= -4D_aD_b\phi - 2D_a\omega^c_{cb} + 2D_c\omega^c_{ab} + \cdots, \end{aligned} \tag{19b}$$

where the dots represent quadratic terms, which are subleading with respect to those that we have written explicitly. The way the lowest order in (19b)) looks like can be changed using the BI (A.8), but once it is fixed, the subleading terms are also fixed. Note that flat derivatives commute at leading order.

Integrating by parts, the first term in (18) can be taken to the form

$$\int d^Dx\sqrt{-g}e^{-2\phi}\delta^{(1)}L^{(0)} = \int d^Dx\sqrt{-g}e^{-2\phi}(\delta^{(1)}b^{ab}\Delta b_{ab} + \delta^{(1)}e^{ab}\Delta e_{ab}). \tag{20}$$

On the other hand, since the lowest order transformation rules (6) are known, we can readily compute $\delta^{(0)}L^{(1)}$ and determine the first order deformations by requiring invariance of the action (18). To this end, it is convenient to consider the particular case $b=0$ and then infer the general transformations from the fact that $L_b^{(1)} = L_a^{(1)}[H\to -H]$. The leading terms in the flux expansion of $\delta^{(0)}L_a^{(1)}$ turn out to be cubic, i.e.,

$$\delta^{(0)}L_a^{(1)} = \sum_{h=0}^{3}[\delta^{(0)}L_a^{(1)}]_{(h,3-h)} + \cdots, \tag{21}$$

where $(h, 3-h)$ denotes terms with h fluxes H and $3-h$ fluxes ω, and the dots represent subleading expressions. Each term in this expansion can be taken to the form

$$[\delta^{(0)}L_a^{(1)}]_{(0,3)} = \Delta e^{ab}\left[\frac{1}{4}\beta^c_a\omega_{bde}\omega_c^{de}\right] - [\mathcal{D}_aT^a]_{(0,3)}, \tag{22a}$$

$$[\delta^{(0)}L_a^{(1)}]_{(1,2)} = \Delta b^{ab}\left[-\beta^{ec}\omega^d_{ea}\omega_{bcd} + \beta^{ec}\omega^d_{ae}\omega_{bcd} + \frac{1}{2}\beta^c_a\omega_{bde}\omega_c^{de}\right] + \Delta e^{ab}\left[-\frac{1}{8}\beta^e_a\omega_{bcd}H_e^{cd} - \frac{1}{8}\beta^e_aH_{bcd}\omega_e^{cd}\right] - [\mathcal{D}_aT^a]_{(1,2)}, \tag{22b}$$

$$[\delta^{(0)}L_a^{(1)}]_{(2,1)} = \Delta b^{ab}\left[\frac{1}{2}\beta^{ec}\omega^d_{ea}H_{bcd} - \frac{1}{2}\beta^{ec}\omega^d_{ae}H_{bcd} - \frac{1}{2}\beta^c_a\omega_{bde}H_c^{de} - \frac{1}{2}\beta^c_aH_{bde}\omega_c^{de}\right] + \Delta e^{ab}\left[\frac{1}{16}\beta^e_aH_{bcd}H_e^{cd}\right] - [\mathcal{D}_aT^a]_{(2,1)}, \tag{22c}$$

$$[\delta^{(0)}L_a^{(1)}]_{(3,0)} = \Delta b^{ab}\left[\frac{1}{8}\beta^c_aH_{bde}H_c^{de}\right] - [\mathcal{D}_aT^a]_{(3,0)}, \tag{22d}$$

where Δb_{ab} and Δe_{ab} contain the leading order of the equations of motion (19). The derivative $\mathcal{D}_aT^a$ gives rise to a total derivative when introduced in the action

$$\mathcal{D}_aT^a = D_aT^a - 2D_a\phi T^a - \omega^b_{ba}T^a, \qquad \sqrt{-g}e^{-2\phi}\mathcal{D}_aT^a = \partial_\mu(\sqrt{-g}e^{-2\phi}e^\mu_aT^a), \tag{23}$$

and hence it is not relevant for our purposes. Nevertheless, for completeness we give the explicit expression of the vector T^a to cubic order in [20].

Written like this, it is now trivial to extract the first order corrections to the β transformations proportional to the parameter a, introducing (20) and (22) into (18). Note that there is no room for deformations with higher powers of fluxes, as those would be of higher order in α'. Reinserting the parameter b, we obtain the full first order corrections to the β transformations in the generalized Bergshoeff–de Roo scheme,

$$\delta^{(1)}e_{ab} = \frac{a+b}{8}\beta^e_{(a}(\omega_{b)cd}H_e^{cd} + H_{b)cd}\omega_e^{cd}) + \frac{b-a}{4}\beta^e_{(a}\left(\omega_{b)cd}\omega_e^{cd} + \frac{1}{4}H_{b)cd}H_e^{cd}\right), \tag{24a}$$

$$\begin{aligned}\delta^{(1)}b_{ab} = {}&(a+b)\left[\beta^{ec}\omega^d_{e[a}\omega_{b]cd} - \beta^{ec}\omega^d_{[\underline{a}e}\omega_{\underline{b}]cd} - \frac{1}{2}\beta^c_{[a}\omega_{b]de}\omega_c^{de} - \frac{1}{8}\beta^c_{[a}H_{b]de}H_c^{de}\right]\\ &+\frac{b-a}{2}\left[\beta^{ec}\omega^d_{e[a}H_{b]cd} - \beta^{ec}\omega^d_{[\underline{a}e}H_{\underline{b}]cd} - \frac{1}{2}\beta^c_{[a}\omega_{b]de}H_c^{de} - \frac{1}{2}\beta^c_{[a}H_{b]de}\omega_c^{de}\right].\end{aligned} \tag{24b}$$

We have verified that these transformations preserve the action to all orders in the flux expansion. Interestingly, one can check that, in fact, the Lagrangian itself is invariant. As a final test we have also verified that these transformations close in combination with the local symmetries of the theory, with respect to the following α'-corrected brackets:

$$\begin{aligned}\Lambda_{12ab} = {}&2\beta^c_{1[a}\beta_{2b]c} + 2\Lambda^c_{1[a}\Lambda_{2b]c} + 2\xi^\mu_{[1}\partial_\mu\Lambda_{2]ab} - 4F_{c[a}\beta_{[1b]d}\beta_{2]}^{cd} - 4F^{cd}\beta_{1c[a}\beta_{2b]d} - \left[\frac{a+b}{4}H_{ecd} + \frac{b-a}{2}\omega_{ecd}\right]\beta^e_{[1[a}D_{b]}\Lambda^{cd}_{2]}\\ &-\left[(a+b)\omega_d^{ef} + \frac{b-a}{2}H_d^{ef}\right](\omega^{cd}_{[a} + \omega^{cd}_{[a})\beta_{[1b]e}\beta_{2]cf},\\ \lambda_{12\mu} = {}&4\xi^\nu_{[1}\partial_{[\nu}\lambda_{2]\mu]} - \frac{a+b}{2}(\Lambda^{ab}_{[1}\partial_\mu\beta_{2]ab} - 2\beta^{ab}_{[1}\partial_\mu\Lambda_{2]ab}) - (b-a)(\Lambda^{ab}_{[1}\partial_\mu\Lambda_{2]ab} + \beta^{ab}_{[1}\partial_\mu\beta_{2]ab}),\\ \xi^\mu_{12} = {}&2\xi^\nu_{[1}\partial_\nu\xi^\mu_{2]} + \beta^{\mu\nu}_{[1}\lambda_{2]\nu},\end{aligned} \tag{25}$$

where ξ is the vector that generates diffeomorphisms and λ is the one-form that generates the gauge transformations of the two-form, and we have also defined

$$F_{ab} = \frac{a+b}{8}\left(\omega_{acd}\omega_b^{cd} + \frac{1}{4}H_{acd}H_b^{cd}\right) + \frac{b-a}{8}\omega_{(a}^{cd}H_{b)cd}. \tag{26}$$

Conclusions.—We showed that the nongeometric sector of $O(d,d)$ effectively acts in D dimensions as a hidden symmetry that fixes the couplings predicted by string theory to correct the Einstein-Hilbert action. In this sense it plays the same role as (i) supersymmetry, but applies, in general, to all string effective actions even if not supersymmetric, as it is purely bosonic; (ii) Kaluza-Klein reductions, but does not require the redefinitions intended to make the duality symmetry manifest; (iii) double field theory, but is free from the obstructions signaled in that context due to the constraint (4).

The explicit transformation rules that preserve the action (13) up to first order in α' are given in (6) and (24), and the verification that they close together with the local symmetries is presented in (25).

We would like to highlight two potential important applications of our Letter: (i) The invariance of the action implies that the equations of motion are covariant under β transformations. They can then be used as a solution generating technique, becoming a powerful tool to compute higher-derivative corrections to nongeometric backgrounds like T folds, from standard solutions in supergravity. (ii) Most of the current efforts in the field are focused in understanding the duality structure of the quartic Riemann interactions common to all string theories. Although the full set of couplings were recently computed using duality arguments [4], finding the β transformations that preserve the action will help in identifying the field redefinitions that connect with duality covariant variables, as in the Bergshoeff–de Roo action described here. This would not only clarify the origin of the obstructions that prevent uploading these couplings to a double field theory [8], but could also lead to finding a systematic iterative method to obtain higher-derivative terms, such as the biparametric generalized Bergshoeff–de Roo identification of [7].

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[19] In this action [6], the parameters a, b interpolate between the bosonic $(a = b = -\alpha')$ and heterotic string $(a = -\alpha', b = 0)$ and include, for instance, the Hohm-Siegel-Zwiebach gravity theory $(a = -b = -\alpha')$ [5], which is $O(d, d)$ symmetric but is not a string theory.

[20] The explicit expression of the tensor T^a that appears in (22) is

$$
\begin{aligned}
T^a = & -\frac{1}{16} D_b H^{bcd} H_{cde} \beta^{ae} - \frac{1}{16} D_b H_{cde} H^{bcd} \beta^{ae} + \frac{1}{8} D_b H^{bcd} \beta^{ae} \omega_{ecd} + \frac{1}{8} D_b H_{cde} \beta^{ac} \omega^{bde} \\
& + \frac{1}{8} D_b \omega^{bcd} H_{cde} \beta^{ae} + \frac{1}{8} D_b \omega_{cde} H^{bde} \beta^{ac} - \frac{1}{4} D_b \omega^{bcd} \beta^{ae} \omega_{ecd} - \frac{1}{4} D_b \omega_{cde} \beta^{ac} \omega^{bde} \\
& + \frac{1}{8} D_b \phi H^{bcd} H_{cde} \beta^{ae} - \frac{1}{4} D_b \phi H^{bcd} \beta^{ae} \omega_{ecd} - \frac{1}{4} D_b \phi H_{cde} \beta^{ac} \omega^{bde} + \frac{1}{2} D_b \phi \beta^{ac} \omega^{bde} \omega_{cde} \\
& - \frac{1}{16} H^a_{\ bc} H^{bde} H_{def} \beta^{cf} + \frac{1}{4} H^{abc} H^e_{\ bd} \beta^{df} \omega_{cef} + \frac{1}{4} H^{abc} H_{bde} \beta^{df} \omega^e_{\ fc} - \frac{1}{8} H^a_{\ bc} H_{def} \beta^{bd} \omega^{cef} \\
& + \frac{1}{16} H^{bcd} H_{bce} \beta^{ae} \omega^f_{\ df} + \frac{1}{8} H_{bcd} H^{bc}_{\ \ e} \beta^{ef} \omega^{ad}_{\ \ f} + \frac{1}{4} H^a_{\ bc} \beta^{bd} \omega^{cef} \omega_{def} + \frac{1}{2} H^{abc} \beta^{de} \omega_{bdf} \omega^f_{\ ce} \\
& - \frac{1}{8} H_{bcd} \beta^{ab} \omega^{cd}_{\ \ e} \omega^{ef}_{\ \ f} + \frac{1}{8} H^{bcd} \beta^{ae} \omega_{bef} \omega^f_{\ cd} - \frac{1}{8} H^{bcd} \beta^{ae} \omega_{ebc} \omega^f_{\ fd} - \frac{1}{8} H^{bcd} \beta^{ae} \omega^f_{\ bc} \omega_{fde} \\
& - \frac{1}{4} H_{bcd} \beta^{be} \omega^{af}_{\ \ e} \omega^{cd}_{\ \ f} + \frac{1}{4} H_{bcd} \beta^{ef} \omega^{ab}_{\ \ e} \omega^{cd}_{\ \ f} - \frac{1}{2} H^{abc} \beta^{de} \omega_{bdf} \omega^f_{\ ec} + \frac{1}{4} \beta^{ab} \omega_{bcd} \omega^{cd}_{\ \ e} \omega^{ef}_{\ \ f} \\
& - \frac{1}{4} \beta^{ab} \omega^c_{\ de} \omega^{de}_{\ \ f} \omega^f_{\ bc} - \frac{1}{2} \beta_{bc} \omega^{\mathrm{bad}} \omega^{cef} \omega_{def} + \frac{1}{8} H_{abc} H^{bde} \beta^{cf} \omega_{fde} + \frac{1}{16} H_{bcd} H^{bce} \beta^{af} \omega^d_{\ ef} + \cdots,
\end{aligned}
$$

where the dots represent terms of quartic order in fluxes.