Experimental Limit on Nonlinear State-Dependent Terms in Quantum Theory

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Linear time evolution is one of the fundamental postulates of quantum theory. Past theoretical attempts to introduce nonlinearity into quantum evolution have violated causality. However, a recent theory has introduced nonlinear state-dependent terms in quantum field theory, preserving causality [D. E. Kaplan and S. Rajendran, Phys. Rev. D **105**, 055002 (2022)]. We report the results of an experiment that searches for such terms. Our approach, inspired by the Everett many-worlds interpretation of quantum theory, correlates a binary macroscopic classical voltage with the outcome of a projective measurement of a quantum bit, prepared in a coherent superposition state. Measurement results are recorded in a bit string, which is used to control a voltage switch. Presence of a nonzero voltage reading in cases of no applied voltage is the experimental signature of a nonlinear state-dependent shift of the electromagnetic field operator. We implement blinded measurement and data analysis with three control bit strings. Control of systematic effects is realized by producing one of the control bit strings with a classical random-bit generator. The other two bit strings are generated by measurements performed on a superconducting qubit in an IBM Quantum processor and on a ^{15}N nuclear spin in a nitrogen-vacancy center in diamond. Our measurements find no evidence for electromagnetic quantum state-dependent nonlinearity. We set a bound on the parameter that quantifies this nonlinearity $|\epsilon_\gamma| < 4.7 \times 10^{-11}$, at 90% confidence level.

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Quantum mechanics has proven to be a very successful theory of physics at microscopic scales. It describes the behavior of systems at the nuclear and atomic scale, as well as properties of materials and radiation. There are two fundamental principles at the foundation of quantum theory: the concept of probabilistic, rather than deterministic, measurement and the requirement of linear time evolution. The often counterintuitive predictions of quantum theory have generated numerous scientific debates and interpretations, as well as a number of theoretical modifications [1–3]. Nevertheless, it has withstood all experimental tests [4–10]. Quantum field theory forms the foundation of modern physics.

Despite this clear success, a number of extensions of quantum theory have been proposed, such as the wave function collapse models, designed to explain the absence of certain quantum effects in the macroscopic world [2,11,12]. Past theoretical attempts of introducing nonlinear time evolution into quantum theory, such as Weinberg's nonlinear framework, have generically suffered from causality problems [13–16]. In addition, experiments with ^{9}Be$^+$ ions rapidly placed stringent bounds on Weinberg's framework [17]. However, in a recent theory advance, a different approach was taken to formulate a causal quantum field theory with nonlinear time evolution [18]. In Ref. [18], nonlinearity is introduced within the field theory framework by shifting bosonic field operators by a small amount, proportional to their expectation value in the full quantum state $|\psi\rangle$. For example, in linear quantum electrodynamics, the interaction between the electromagnetic field, given by the four-vector potential A_μ, and a current four-vector J^μ is given by the Lagrangian $A_\mu J^\mu$. In the proposed nonlinear theory, this electromagnetic interaction is modified to $(A_\mu + \epsilon_\gamma \langle\psi|A_\mu|\psi\rangle)J^\mu$, where ϵ_γ is the parameter that quantifies the degree of nonlinearity for electromagnetic fields, and we keep only lowest-order terms. The current J^μ thus effectively interacts with the vector potential,

$$A'_\mu = A_\mu + \epsilon_\gamma \langle\psi|A_\mu|\psi\rangle. \quad (1)$$

This modification preserves causality, energy conservation, and gauge invariance of the theory, as well ensuring that quantum states have a conserved norm [18]. The same nonlinear construction could be extended to

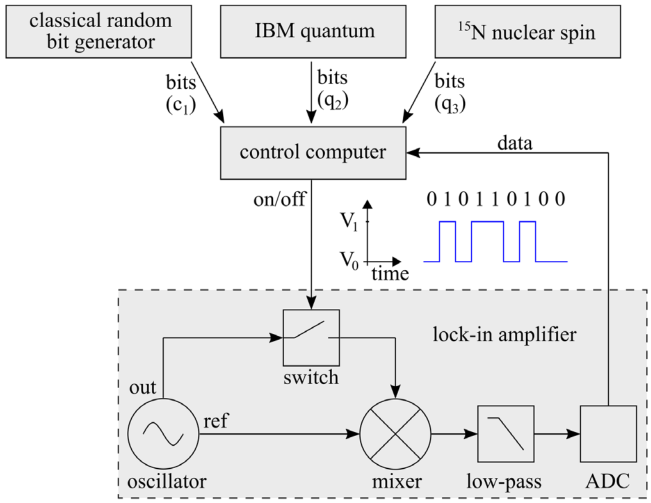

FIG. 1. Experimental setup. Three bit strings c_1, q_2, q_3 were randomly permuted and used to manipulate the switch that controls the output voltage of the Zurich Instruments MFLI lock-in amplifier. The lock-in output was set to 3V amplitude at 1 MHz carrier frequency. The low-pass filter time constant was set to 1 ms. The analog-to-digital converter (ADC) digitized the voltage readings, that were then recorded by the control computer. The figure shows an example of a 9-bit control string and the corresponding voltage waveform.

gravitational fields by modifying the $g_{\mu\nu}$ metric tensor as $g'_{\mu\nu} = g_{\mu\nu} + \epsilon_g \langle\psi|g_{\mu\nu}|\psi\rangle$. This theoretical advance opens up a number of intriguing prospects, including the possibility to solve the black hole information problem [18,19]. Nevertheless, experimental constraints that can currently be placed on such causal nonlinear modifications are weak. The strongest bounds $\epsilon_\gamma \lesssim 10^{-5}$ can be deduced from ion trapping experiments, and the experimental bounds on ϵ_g are even weaker [18].

We report experimental limits on the electromagnetic nonlinearity parameter ϵ_γ. Our approach is similar to the "Everett phone" proposed in Ref. [15]. We correlate a macroscopic (classical) voltage with the outcome of the measurement of a quantum two-level system (qubit). We denote the stationary states of the qubit as $|0\rangle$ and $|1\rangle$. The classical apparatus that creates and detects the voltage is the Zurich Instruments MFLI lock-in amplifier, whose oscillator output is connected directly to its voltage input, Fig. 1. Our measurement procedure is as follows: (1) Initialize the qubit in state $|\chi\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$. (2) Perform a qubit measurement and record the resulting qubit state. (3) Set the lock-in output voltage amplitude according to the following rule: output voltage $V_0 = 0\text{V}$, if the qubit measurement detects it in state $|0\rangle$; output voltage $V_1 = 3\text{V}$, if the qubit measurement detects it in state $|1\rangle$. (4) Record the resulting voltage reading.

Our system is described by the density matrix $\rho = (|0\rangle\langle 0|\rho_0 + |1\rangle\langle 1|\rho_1)/2$, where ρ_0 and ρ_1 are the density matrices that describe the classical environment, corresponding to the two possible voltage readings $\text{Tr}(\rho_0 \hat{V}) = V_0 = 0$ and $\text{Tr}(\rho_1 \hat{V}) = V_1 = 3\text{V}$, where $\hat{V}$ is the voltage operator. We emphasize that the classical apparatus is not being observed to be in a coherent superposition state [20,21]. The nonlinear modification, introduced in Eq. (1), modifies the voltage operator to

$$\hat{V}' = \hat{V} + \epsilon_\gamma \text{Tr}(\rho \hat{V}) = \hat{V} + \epsilon_\gamma V_1/2. \tag{2}$$

Now, to lowest order in ϵ_γ, the two possible voltage measurement outcomes are $\text{Tr}(\rho_0 \hat{V}') = \epsilon_\gamma V_1/2$ when the observer records $|0\rangle$, and $\text{Tr}(\rho_1 \hat{V}') = V_1 = 3\text{V}$ when the observer records $|1\rangle$. Thus we search for a nonzero voltage reading, proportional to ϵ_γ, in cases when the qubit is measured to be in state $|0\rangle$. Note that, in the Everett many-worlds interpretation, the trace in Eq. (2) is over the entire density matrix [22]. In the Copenhagen interpretation, the quantum state collapses after the measurement in step (2) of our procedure, and this effect vanishes. Specifically, in the Copenhagen framework, if the qubit is measured to be in state $|0\rangle$, the density matrix of the system collapses to ρ_0, and the voltage measurement outcome is $\text{Tr}\{\rho_0[\hat{V} + \epsilon_\gamma \text{Tr}(\rho_0 \hat{V})]\} = 0$. If the qubit is measured to be in state $|1\rangle$, the density matrix of the system collapses to ρ_1, and the voltage measurement outcome is $\text{Tr}\{\rho_1[\hat{V} + \epsilon_\gamma \text{Tr}(\rho_1 \hat{V})]\} = (1 + \epsilon_\gamma) \times 3\text{V}$, which amounts to a small renormalization of voltage. Therefore, the nonlinear modification of bosonic operators, proposed in Ref. [18], creates an opportunity to experimentally distinguish between the Copenhagen and the many-worlds interpretations of quantum theory and search for the existence of other worlds created by quantum measurements.

The lock-in amplifier is controlled by a computer program, which takes as inputs three bit strings. The first bit string c_1 consists of 60000 classical bits, generated by a random-bit generator (implemented in MATLAB R2021a software). The second bit string q_2 consists of 30000 bits, generated by the cloud-based IBM Quantum processor. To obtain each bit in q_2, a single transmon superconducting qubit was initialized into a superposition state $|\chi\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$ and measured, assigning the bit value 0 to measurement result $|0\rangle$ and bit 1 to measurement result $|1\rangle$. The third bit string q_3 consists of 10717 bits, generated by a ^{15}N nuclear spin $I = 1/2$ qubit that is part of a single nitrogen-vacancy (NV) center in a diamond crystal in our laboratory. We label the quantum states of this qubit as $|\downarrow\rangle, |\uparrow\rangle$, indicating the sign of the projection of the nuclear spin along the NV center axis. To obtain each bit in q_3, the ^{15}N nuclear spin was initialized into the $|\chi\rangle = (|\downarrow\rangle + |\uparrow\rangle)/\sqrt{2}$ state and measured, assigning the bit value 0 to measurement result $|\downarrow\rangle$ and bit 1 to measurement result $|\uparrow\rangle$.

We use the three bit strings to control systematic errors in our experiment. The most important systematic is leakage of the lock-in oscillator output through the open switch, which leads to a nonzero open-switch detected voltage V_s, mimicking the signal due to quantum nonlinearity. We control this systematic by checking the voltage detected when classical bits are used to control the switch. A statistically significant difference in the detected voltage between classical and quantum bit control is a signature of quantum nonlinearity ϵ_γ. The two quantum measurement-generated bit strings q_2 and q_3 are a mechanism for incorporating the effects of classical noise and qubit readout fidelity. In this context, we define readout fidelity f as the probability of measuring the qubit to be in the same quantum state to which it had just been initialized (see Supplemental Material [23]). Classical readout noise degrades this fidelity, and random bits generated by a classical computer correspond to $f = 1/2$. For readout fidelity $1/2 \leq f \leq 1$, and in the presence of an open-switch leakage voltage $V_s \ll V_1$, our experiment's possible voltage outcomes are $V_s + \epsilon_\gamma V_1 (f - 1/2)$ when the observer records $|0\rangle$ and $V_1 = 3\text{V}$ when the observer records $|1\rangle$.

The fidelity of the IBM Quantum measurements that generated the q_2 bit string was 99% [23]. The imperfect readout of the NV center severely degrades the fidelity of direct ^{15}N nuclear spin measurements, which are limited by photon shot noise. To circumvent this limitation, we implemented the repetitive readout scheme, which improved this fidelity by repeating the measurement cycle that consisted of controlled-NOT (CNOT) gates, that correlated the NV center ^{15}N nuclear spin with the electron spin and subsequent projective optical measurements of the electron spin state [23,30–32]. A narrow band selective microwave pulse realized the CNOT gate between the NV center nuclear and the electron spin qubits. Each ^{15}N nuclear spin measurement consisted of 50 repetitive readout cycles, achieving 55% readout fidelity (see Supplemental Material [23]).

The experimental run took place in November 2021. Our approach implemented a blinded measurement and data analysis procedure. The control computer software combined and randomly permuted the three bit strings, saving the permutation key for final unblinding. The resulting bit string controlled the output switch of the lock-in amplifier. The length of each switch cycle was 2 s. In order to allow switching transients to decay, the computer recorded voltage data during the last second of each switch cycle. After linear drift subtraction and averaging, a single voltage reading was obtained for each switching cycle. Blinded data analysis divided these voltage readings into "high" and "low," based on whether they were greater or less than 1 V. The histogram of the 49699 low voltage readings is shown in Fig. 2(a). The distribution is consistent with the Gaussian shape with mean $V_L = (-0.309 \pm 0.015)$ nV. Here and below, we quote the Gaussian standard error of the mean as the uncertainty. The average of the 51018 high voltage readings is $V_H = (2.98 \pm 8 \times 10^{-7})$ V. This matches the amplitude of the lock-in amplifier output waveform.

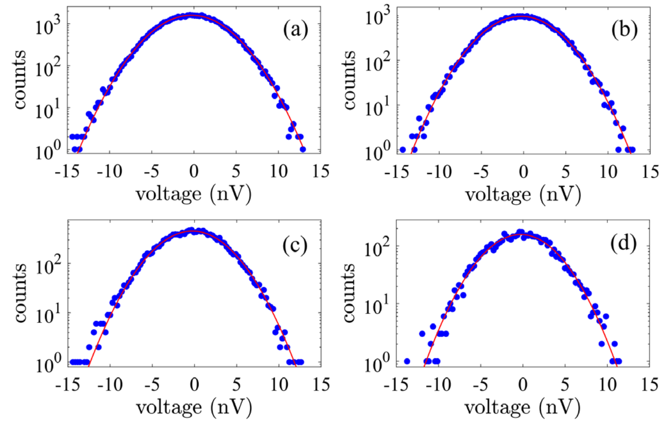

FIG. 2. Histograms of the low voltage readings recorded during the experimental run. (a) Histogram of all the recorded low voltage readings (blinded analysis). (b) Histogram of the low voltage readings for control bits in the classically generated bit string c_1. (c) Histogram of the low voltage readings for control bits in the bit string q_2, generated by IBM Quantum. (d) Histogram of the low voltage readings for control bits in the bit string q_3, generated by the NV center. All histograms are consistent with Gaussian distributions, shown as red lines.

The larger uncertainty for the mean V_H value is due to the less-sensitive range used by the lock-in to make volt-level measurements, when the switch is closed.

After the analysis procedure was finalized, the bit permutation key was used to unblind the results. The voltage measurements for each of the three control bit strings were collected and analyzed separately. The results are shown in Figs. 2(b)–2(d). All three histograms are consistent with Gaussian-distributed data. The mean of the low voltage readings for bits from classically generated bit string c_1 is $V_L^{(1)} = (-0.307 \pm 0.020)$ nV. This is a measurement of the open-switch leakage voltage V_s. The mean of the low voltage

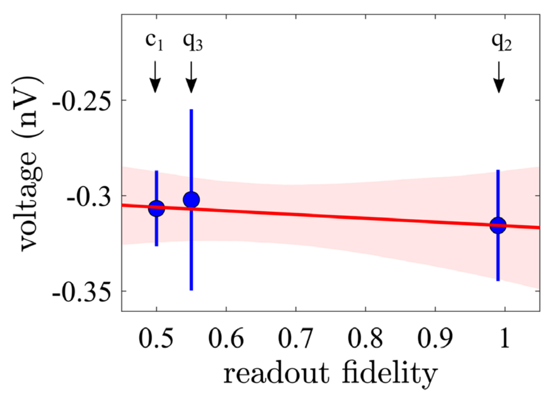

FIG. 3. Detected open-switch voltage as a function of readout fidelity. Mean voltages obtained for each of the bit strings c_1, q_2, q_3 are marked with arrows. Each error bar corresponds to Gaussian standard error of the mean. The red line shows the best linear fit, and the shaded region indicates the 1-standard-deviation uncertainties in the linear fit parameters, obtained using a Monte Carlo simulation with 10^4 realizations.

readings for bits in string q_2, generated by the IBM Quantum processor, is $V_L^{(2)} = (-0.316 \pm 0.029)$ nV. The mean of the low voltage readings for bits in string q_3, generated by the NV center in diamond, is $V_L^{(3)} = (-0.302 \pm 0.047)$ nV. These values are plotted in Fig. 3, with the corresponding readout fidelity on the horizontal axis. Linear regression gives the best-fit value of the open-switch leakage voltage: $V_s = (-0.306 \pm 0.019)$ nV. The quantum nonlinearity parameter is extracted from the slope of the linear fit: $\epsilon_\gamma = (0.7 \pm 2.3) \times 10^{-11}$. Our measurements find no evidence for electromagnetic quantum nonlinearity, and we set a 90% confidence bound at the level $|\epsilon_\gamma| < 4.7 \times 10^{-11}$.

As discussed in Ref. [18], the observable effects of nonlinear quantum mechanics may be extremely sensitive to the full unknown cosmic history of the quantum state $|\psi\rangle$. Our Letter makes the plausible assumption that the Universe has dominantly evolved classically, with negligible quantum spread [18]. Within the Everett many-worlds interpretation of quantum theory, our measurements place limits on the electromagnetic interaction between different branches of the Universe, created by initializing the qubit into a superposition state [22]. In the usual linear quantum mechanics, the Copenhagen and Everett interpretations produce identical measurement outcomes. A significant detection of nonlinear state-dependent operator shift in our experiment would have favored the Everett many-worlds theory, but our null result does not favor one interpretation over the other. We note that the open-switch voltage V_s could have a contribution due to some unknown physical effects that modify the system Hamiltonian; however, our approach is not sensitive to such effects, since we search for the difference between measurement results for the qubit in the superposition state and for a mixture of classical bits.

It may be possible to improve experimental sensitivity to the electromagnetic quantum nonlinearity. One possible avenue is to extend our approach to a larger dynamic range of macroscopic electromagnetic fields, controlled by a qubit measurement outcome. For example, switching a Tesla-level magnetic field with detection by a superconducting quantum interference device may lead to a significant sensitivity gain. Another possibility is to make use of an ion interferometer, where nonlinear effects cause the Coulomb field of one arm of the interferometer to affect the phase on the other arm. The possibility of a solution to the black hole information problem is a strong motivation for experimental searches for gravitational nonlinearity, which could make use of an accelerometer, such as an atomic interferometer, to measure the gravity gradient created by a test mass, whose position is modulated based on a qubit measurement. Given the importance of testing the foundations of quantum mechanics, as well as potential applications, such as schemes that use nonlinear time evolution of quantum states to solve NP-complete problems in polynomial time [18,33–36], there is a strong case to explore all available experimental avenues.

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