Editors' Suggestion

Synchronous Observation of Bell Nonlocality and State-Dependent Contextuality

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Bell nonlocality and Kochen-Specker contextuality are two remarkable nonclassical features of quantum theory, related to strong correlations between outcomes of measurements performed on quantum systems. Both phenomena can be witnessed by the violation of certain inequalities, the simplest and most important of which are the Clauser-Horne-Shimony-Holt (CHSH) and the Klyachko-Can-Binicioğlu-Shumovski (KCBS), for Bell nonlocality and Kochen-Specker contextuality, respectively. It has been shown that, using the most common interpretation of Bell scenarios, quantum systems cannot violate both inequalities concomitantly, thus suggesting a monogamous relation between the two phenomena. In this Letter, we show that the joint consideration of the CHSH and KCBS inequalities naturally calls for the so-called generalized Bell scenarios, which, contrary to the previous results, allows for joint violation of them. In fact, this result is not a special feature of such inequalities: We provide very strong evidence that there is no monogamy between nonlocality and contextuality in any scenario where both phenomena can be observed. We also implement a photonic experiment to test the synchronous violation of both CHSH and KCBS inequalities. Our results agree with the theoretical predictions, thereby providing experimental proof of the coexistence of Bell nonlocality and contextuality in the simplest scenario, and lead to novel possibilities where both concepts could be jointly employed for quantum information processing protocols.

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Introduction.—Quantum theory is notable for being intriguing and counterintuitive, a fact due, mostly, to predictions and concepts that diverge from those of classical theories. Among such nonclassical concepts are Bell nonlocality [1,2] and Kochen-Specker contextuality [3,4].

Classical reasoning assumes the possibility of well-defined values for every physical quantity. Probabilities are used as a consequence of partial knowledge one has regarding *the real state of affairs*. Prior to quantum theory, there was no reason to distrust such a view.

Bell nonlocality refers to stronger-than-classical correlations on outcomes of measurements performed by distant parties on composite systems. Classical reasoning allows for the possibility of the so-called *local hidden variables* (LHVs) as a mathematical description of every correlation in spacelike separated measurements. In a seminal paper [1], Bell showed that quantum theory admits correlations that cannot be explained by any LHV model, a result later known as *Bell's theorem*. The *nonlocal* correlations violate inequalities that are satisfied in any LHV theory, the so-called *Bell inequalities*, the simplest and best known of which is the *Clauser-Horne-Shimony-Holt* (CHSH) inequality [5]. It is worth mentioning that Bell nonlocality in quantum systems has been extensively tested and verified in several seminal experiments [6–11].

Contextuality is a concept similar to nonlocality; in fact, it can be understood as a generalization of nonlocality that manifests also for single systems. While, in classical theories, every pair of measurements can be jointly performed, at least in principle [12], in quantum theory, pairs of measurements are usually *incompatible*, preventing their joint measurability. Sets of compatible measurements are called *contexts*, and theories in which it is possible to assign values to the outcomes of measurements irrespective of the context in which they are measured are called *noncontextual hidden variable* (NCHV) theories. Kochen and Specker [4] and Bell [3] were the first to note that quantum theory admits correlations that cannot be explained by any NCHV model, a result later known as *Kochen-Specker theorem*. In 2008, Klyachko *et al.* [13] noticed that, as for nonlocality, there are inequalities that hold for all noncontextual correlations which can be violated for single quantum systems. The *Klyachko-Can-Binicioğlu-Shumovski* (KCBS) inequality became the simplest example of contextuality inequalities. Interestingly, there are other contextuality inequalities that, in contrast with KCBS, can be violated by *every* quantum state [14,15], thus revealing a property of the measurements known as *state-independent contextuality* [16–18].

Despite their common roots related to the search for hidden variables in quantum theory, nonlocality and contextuality were developed through very distinct research programs, and it was only a few years ago that mathematical approaches to unify both concepts were proposed [19–21]. The first proposal to consider nonlocality and contextuality in the same system is due to Kurzyński, Cabello, and Kaszlikowski [22]: Two parties could make a CHSH inequality test while one of the parties would also evaluate the KCBS inequality in a subsystem, using, among others, the same incompatible measurements applied in the nonlocality test. The authors proved that in quantum theory—and in more general nondisturbing theories—there exists a trade-off relation between nonlocality and contextuality indicators, allowing for the violation of only one of these tested inequalities, and conjectured a fundamental *monogamy* relation between nonlocality and (state-dependent) contextuality. This monogamy relation was experimentally verified by Zhan *et al.* [23]; more general monogamy relations were also identified in other scenarios [24–30]. It is worth mentioning, though, that such monogamy relations do not hold for state-independent contextuality, since the contextuality test is trivial. Recently, simultaneous observation of Bell nonlocality and state-independent Kochen-Specker contextuality was reported [31].

In the work by Kurzyński, Cabello, and Kaszlikowski [22], there was the implicit assumption that nonlocality tests could consider only one measurement from each party. Since, for instance, a test of the KCBS inequality demands five different measurements, each of which is performed in a context with a second compatible one, each measurement used in CHSH violation could also be supplemented by a compatible one, leading to new generalized Bell inequalities, in the sense of Ref. [32].

In this Letter, we revisit the scenario considered in Ref. [22], and, by considering compatible measurements in the nonlocality test, we experimentally demonstrate that quantum systems can, actually, lead to the synchronous violation of both CHSH and KCBS inequalities. We prove, thus, that there is no fundamental monogamy relation between nonlocality and state-dependent contextuality, even in this simplest scenario where such monogamy was believed to hold. More generally, we show that quantum systems can lead to the concomitant observation of both phenomena in thousands of important scenarios, thus providing strong evidence that, in fact, monogamy between Bell nonlocality and contextuality may never hold. Given that both Bell nonlocality and contextuality are important resources for quantum information processing protocols, we believe that this work may be a first step in the direction of devising novel information processing tasks where both nonclassical resources can be used concomitantly.

The scenario.—Consider the following measurement scenario (its main ideas and concepts can be extended in a straightforward manner to more general scenarios): Two parties, Alice and Bob, run several rounds of experiments on spatially separated laboratories, each on its respective subsystem of a composite physical system, identically prepared in every round. Let Alice be able to perform $m_A = |\mathcal{X}|$ possible measurements, labeled by $x \in \mathcal{X}$, each with $o_A = |\mathcal{A}|$ possible outcomes, labeled by $a \in \mathcal{A}$. Let Bob be able to perform $m_B = |\mathcal{Y}|$ possible measurements, labeled by $y \in \mathcal{Y}$, each with $o_B = |\mathcal{B}|$ possible outcomes, labeled by $b \in \mathcal{B}$. Assume, additionally, that some measurements of Bob are *compatible*, meaning that, in each round, Bob is able to perform subsets of measurements concomitantly. Let $\mathcal{C} = \{\mathbf{y}\}$ be the set *contexts* of Bob, each element $\mathbf{y}$ of which represents a tuple of compatible measurements. Let $\mathbf{b} \in \mathcal{B}^{|\mathbf{y}|}$ be the (ordered) tuple of outcomes of the tuple of measurements $\mathbf{y}$. After sufficiently many rounds, the parties are able to estimate the following set of probabilities, the so-called *behavior* of the experiment:

$$\mathbf{p} = \{p(a, \mathbf{b}|x, \mathbf{y}) | a \in \mathcal{A}, \mathbf{b} \in \mathcal{B}^{|\mathbf{y}|}, x \in \mathcal{X}, \mathbf{y} \in \mathcal{C}\}. \tag{1}$$

Let the measurements be performed in an informationally separated way, so that the following *no-signalling conditions* hold:

$$\sum_a p(a, \mathbf{b}|x, \mathbf{y}) = p(\mathbf{b}|x, \mathbf{y}) = p(\mathbf{b}|\mathbf{y}), \quad \forall \ \mathbf{b}, \mathbf{y}, \tag{2a}$$

$$\sum_{\mathbf{b}} p(a, \mathbf{b}|x, \mathbf{y}) = p(a|x, \mathbf{y}) = p(a|x), \quad \forall \ a, x. \tag{2b}$$

Following the formalism presented in Ref. [32], we define the behavior to be *local* in this scenario or, in other words, to admit an LHV model, if there are a variable λ and probability distributions $p(\lambda)$, $p(a|x,\lambda)$, and $p(\mathbf{b}|\mathbf{y},\lambda)$ such that, for all outcomes and measurements,

$$p(a, \mathbf{b}|x, \mathbf{y}) = \int p(a|x,\lambda) p(\mathbf{b}|\mathbf{y},\lambda) p(\lambda) d\lambda. \tag{3}$$

Consider the marginal behavior of Bob's experiment:

$$\mathbf{p}_B = \{p(\mathbf{b}|\mathbf{y}) | \mathbf{b} \in \mathcal{B}^{|\mathbf{y}|}, \mathbf{y} \in \mathcal{C}\}. \tag{4}$$

Assume it obeys the *no-disturbance* conditions:

$$\sum_{\mathbf{b}/b} p(\mathbf{b}|\mathbf{y}) = p(b|\mathbf{y}) = p(b|y), \quad \forall \ b, y, \tag{5}$$

where $\mathbf{b}/b$ means that the sum is over all labels in $\mathbf{b}$ except b. We define the marginal behavior of Bob to be *noncontextual*, or to admit an NCHV model, if there are a variable σ and probability distributions $p(b|y,\sigma)$ and $p(\sigma)$ such that, for all outcomes of all contexts,

$$p(\mathbf{b}|\mathbf{y}) = \int \left[\prod_{y\in\mathbf{y}} p(b|y,\sigma)\right] p(\sigma) d\sigma. \tag{6}$$

With all these definitions in place, let us focus on the particular scenario we are interested in. Let Alice choose between two dichotomic measurements $\mathcal{X} = \{0,1\}$, $\mathcal{A} = \{-1,1\}$, and let Bob have five dichotomic measurements $\mathcal{Y} = \{0,1,2,3,4\}$, $\mathcal{B} = \{-1,1\}$, available with measurement contexts $\mathcal{C} = \{\{0,1\},\{1,2\},\{2,3\},\{3,4\},\{4,0\}\}$. The compatibility relations between all measurements are represented in Fig. 1. The following version of the CHSH inequality holds for all behaviors that are local [according to the definition in Eq. (3)]:

$$\alpha_{\mathrm{CHSH}} = \langle A_0B_0\rangle + \langle A_0B_2B_3\rangle + \langle A_1B_0\rangle - \langle A_1B_2B_3\rangle \overset{\mathrm{LHV}}{\leqslant} 2, \tag{7}$$

where

$$\langle A_xB_y\rangle = p(a=b|x,y) - p(a\neq b|x,y), \tag{8a}$$

$$\langle A_xB_yB_{y'}\rangle = p(a=b\cdot b'|x,y,y') - p(a\neq b\cdot b'|x,y,y'). \tag{8b}$$

Note that, according to the definitions above, the joint measurement of B_2B_3 can be regarded as a single dichotomic measurement whose outcome is given by the product $b\cdot b'$, where b and b' are the outcomes of B_2 and B_3, respectively. The left-hand side of inequality (7) is, in essence, equivalent to the left-hand side of the standard CHSH inequality, hence the same local bound.

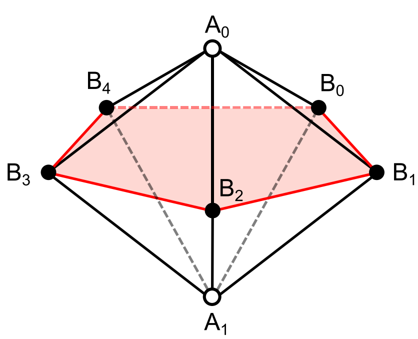

FIG. 1. Representation of the compatibility relations between all measurements in the experiment. Each measurement is represented by a vertex, and vertices connected by an edge represent compatible measurements. Both measurements of Alice (white vertices) are compatible with all measurements of Bob (black vertices); compatibility of measurements of Bob is represented by a pentagon. Sets of measurements that are two-by-two compatible are jointly compatible (e.g., $\{A_0, B_0, B_1\}$).

The marginal scenario of Bob is exactly the one considered by Klyachko and co-authors [13]. The marginal behavior $\mathbf{p}_B$ is contextual if and only if it violates the KCBS inequality (or one of the inequalities obtained from it by relabelings of measurements and/or outcomes):

$$\begin{aligned}\beta_{\mathrm{KCBS}} &= \langle B_0B_1\rangle + \langle B_1B_2\rangle + B_2B_3\rangle \\ &\quad + \langle B_3B_4\rangle - \langle B_4B_0\rangle \overset{\mathrm{NCHV}}{\leqslant} 3,\end{aligned} \tag{9}$$

where

$$\langle B_yB_{y'}\rangle = p(b=b'|y,y') - p(b\neq b'|y,y'). \tag{10}$$

Theoretical results.—The first theoretical result of this Letter is the following theorem.

Theorem 1.—The CHSH inequality (7) and the KCBS inequality (9) can be concomitantly violated in quantum theory.

Proof.—We give a direct proof by showing state spaces, measurements, and a parametrized family of states obeying all the conditions of the scenario and leading to concomitant violations of both inequalities, for some values of the parameters. The systems of Alice and Bob are a qubit and a qutrit, respectively, whose corresponding basis states are $\{|0\rangle,|1\rangle\}$ and $\{|0\rangle,|1\rangle,|2\rangle\}$. (i) For Alice, choose measurement x to be given by the Pauli observables A_x:

$$A_0 = \sigma_z, \qquad A_1 = \sigma_x. \tag{11}$$

(ii) Measurement y of Bob can be represented by an observable B_y with eigenvalues in $\{\pm 1\}$ given as

$$B_j = (-1)^j(\mathbb{1} - 2|v_j\rangle\langle v_j|), \tag{12a}$$

for $j\in\{0,1,2,3,4\}$, where $\mathbb{1}$ is the identity matrix and

$$|v_j\rangle \propto \left[\cos\left(\frac{4\pi j}{5}\right)|0\rangle + \sin\left(\frac{4\pi j}{5}\right)|1\rangle + \sqrt{\cos\left(\frac{\pi}{5}\right)}|2\rangle\right]. \tag{12b}$$

Notice that $\langle v_j|v_{(j+1)\bmod 5}\rangle = 0$, which implies that B_j and $B_{(1+j)\bmod 5}$ commute and, hence, are compatible. (iii) Consider the one-parameter family of states:

$$|\Psi(\phi)\rangle = \cos(\phi)|u\rangle + \sin(\phi)|v\rangle, \tag{13a}$$

where, for $\theta_u \sim 2.868$ and $\theta_v \sim 1.449$,

$$|u\rangle = [\cos(\theta_u)|0\rangle + \sin(\theta_u)|1\rangle]\otimes|2\rangle, \tag{13b}$$

$$|v\rangle = [\cos(\theta_v)|0\rangle + \sin(\theta_v)|1\rangle]\otimes|0\rangle. \tag{13c}$$

TABLE I. Experimental data of α_{CHSH} and β_{KCBS} for 11 input states. Error bars are due to the statistical uncertainty in photon-number counting. States 1–4 violate the KCBS inequality but not the CHSH inequality; states 5–7 violate both inequalities; and states 8–11 violate the CHSH inequality only.

State	ϕ(rad)	$\alpha_{\mathrm{CHSH}}^{\mathrm{th}}$	$\alpha_{\mathrm{CHSH}}^{\mathrm{exp}}$	$\beta_{\mathrm{KCBS}}^{\mathrm{th}}$	$\beta_{\mathrm{KCBS}}^{\mathrm{exp}}$
$\vert\Psi_1\rangle$	0	1.1188	1.1043(438)	3.9443	3.9069(518)
$\vert\Psi_2\rangle$	0.096	1.4293	1.4141(448)	3.9129	3.8728(514)
$\vert\Psi_3\rangle$	0.192	1.7269	1.7083(438)	3.8199	3.7826(510)
$\vert\Psi_4\rangle$	0.288	2.0005	1.9813(450)	3.6688	3.6339(536)
$\vert\Psi_5\rangle$	0.351	2.1622	2.1382(442)	3.5405	3.5034(529)
$\vert\Psi_6\rangle$	0.421	2.3215	2.2972(446)	3.3739	3.3397(446)
$\vert\Psi_7\rangle$	0.487	2.4495	2.4246(423)	3.1964	3.1580(506)
$\vert\Psi_8\rangle$	0.553	2.5536	2.5291(435)	3.0021	2.9684(517)
$\vert\Psi_9\rangle$	0.631	2.6433	2.6164(451)	2.7556	2.7277(537)
$\vert\Psi_{10}\rangle$	0.708	2.6955	2.6739(468)	2.4998	2.4726(555)
$\vert\Psi_{11}\rangle$	0.785	2.7075	2.6871(465)	2.2379	2.2065(553)

According to Born's rule, we have

$$\langle A_x B_y B_{y'}\rangle = \langle\Psi(\phi)|(A_x \otimes B_y B_{y'})|\Psi(\phi)\rangle, \tag{14a}$$

$$\langle A_x B_y\rangle = \langle\Psi(\phi)|(A_x \otimes B_y)|\Psi(\phi)\rangle. \tag{14b}$$

Any choice of $\phi \in [0.288, 0.553]$ completes the proof. ■

In the proof above, the measurements of Alice are usual for the maximal CHSH violation, and the measurements of Bob are usual for the maximal KCBS violation. The parameters θ_u and θ_v were obtained by numerical optimization in the corresponding family, for fixed values of ϕ. Table I may help in understanding the role played by ϕ.

We now extend the result to more general scenarios, by stating the following theorem, the proof of which we show in Supplemental Material [33].

Theorem 2.—Bell nonlocality and contextuality can be concomitantly observed in all scenarios where Alice performs two dichotomic measurements and Bob performs n dichotomic measurements, pairwise compatible according to an n-cycle graph, for $4 \le n \le 10^4$.

It is worth mentioning that the scenario considered in Theorem 1 is a particular case of the scenarios mentioned in Theorem 2, for $n = 5$. Even though we provide numerical proof for only a finite number of scenarios, we state the following.

Conjecture.—Bell nonlocality and contextuality can be concomitantly observed in all scenarios where Alice performs two dichotomic measurements and Bob performs n dichotomic measurements, pairwise compatible according to an n-cycle graph, for all n.

The n-cycle scenarios [40] are building blocks for the study of contextuality: It was proven in Ref. [41] that quantum contextuality is possible only in measurement scenarios where the compatibility graph of measurements has an induced n-cycle graph, with $n \ge 4$. Hence, in all scenarios where nonlocality and contextuality are possible, there will be a subset composed of two measurements of Alice and n of Bob, each of which have a subset of two outcomes, such that a scenario as the one in Theorem 2 can be effectively considered [33]. Hence, we state the following.

Remark.—If the conjecture holds, there is no monogamy relation between Bell nonlocality and state-dependent contextuality in any scenario where both phenomena can be observed in quantum systems.

Experimental realization.—To experimentally test the concomitant violations of both KCBS and CHSH inequalities, we set up an experiment where pairs of photons were employed to encode pairs of qubit-qutrit systems. Schematics of the setup are represented in Fig. 2.

In each round of the experiment, the photons are prepared in one of the states $|\Psi(\phi)\rangle$ of the one-parameter family defined in Eq. (13). The qubit system is encoded in the polarization degree of freedom of one photon of the entangled pair, and the qutrit system is hybridly encoded in both the polarizations and the spatial modes of the other photon of the pair. Measurement of one of the observables given in Eq. (11) is, then, performed in the qubit photon, while sequential measurements of a pair of compatible observables given in Eq. (12) are performed in the qutrit photon. Details of the implementation are provided in Supplemental Material [33].

We produce 11 points $\alpha_{\mathrm{CHSH}} - \beta_{\mathrm{KCBS}}$, corresponding to 11 different input states $|\Psi_i(\phi)\rangle$ $(i = 1, \ldots, 11)$. The experimental results on the average values of the CHSH and KCBS operators are shown in Fig. 3 and Table I. Synchronous violation of both KCBS and CHSH inequalities is observed for the states $|\Psi_5(\phi)\rangle$, $|\Psi_6(\phi)\rangle$,

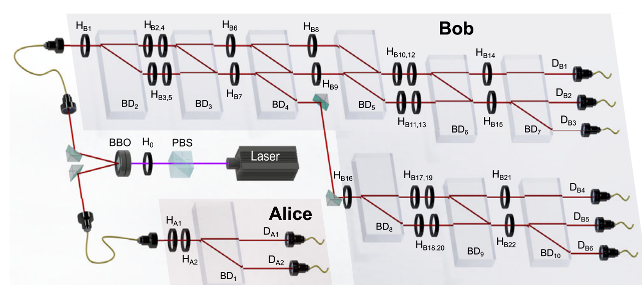

FIG. 2. Illustration of the experimental setup. Polarization-entangled photon pairs are generated via type-I spontaneous parametric down-conversion where two joint β-BBO crystals are pumped by a continuous wave diode laser. Qubit is encoded in the horizontal and vertical polarizations of one photon of each pair, while qutrit is encoded in both polarizations and spatial modes of the other photons of the entangled pairs, which are split in different paths dependent on their polarizations via a beam displacer (BD). For Alice, observables A_i are measured via standard polarization measurements using a half-wave place (HWP) and a BD. For Bob, cascade Mach-Zehnder interferometers for sequentially measuring observables B_j and $B_{(j+1)\ \mathrm{mod}\ 5}$ are used to test the KCBS inequality.

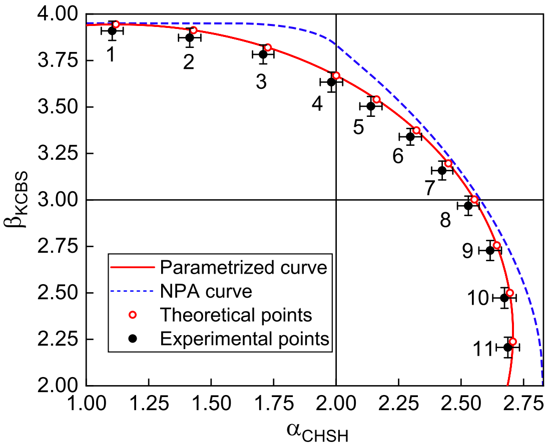

FIG. 3. Experimental results. The measurements in Eqs. (11) and (12) and the one-parameter family of states in Eq. (13) lead to the solid (red) line. Experimental data of α_{CHSH} and β_{KCBS} for specific values of the parameter ϕ are represented by the black dots and compared to their theoretical predictions (red circles). The points can be separated in three sets: Points 1–4 exhibit only contextuality; points 5–7 exhibit both nonlocality and contextuality; and points 8–11 exhibit only nonlocality. Error bars are due to the statistical uncertainty in photon-number counting. The traced (blue) curve is an outer bound to the set of quantum behaviors calculated by means of the Navascués-Pironio-Acín (NPA) hierarchy [42].

and $|\Psi_7(\phi)\rangle$ with $\phi = 0.351,\ 0.421,\ 0.487$, respectively. For $|\Psi_5(\phi)\rangle$, $\alpha_{\mathrm{CHSH}} = 2.1382 \pm 0.0442$ violates the local bound of the inequality by 3 standard deviations and is in great agreement with the quantum prediction 2.1622. Also, $\beta_{\mathrm{KCBS}} = 3.5034 \pm 0.0529$ violates the noncontextual bound of the KCBS inequality by 9 standard deviations and is in great agreement with quantum prediction 3.5405. For $|\Psi_6(\phi)\rangle$, the CHSH and KCBS inequalities are violated by 6 and 7 standard deviations, respectively. For $|\Psi_7(\phi)\rangle$, the violations are by 10 and 3 standard deviations, respectively.

To validate nondisturbance in the data and the compatibility between pairs of observables of Bob, we computed, for each state, the distance $\sum_{j=1}^{5}(p_j - p'_j)^2$, where p_j is the estimated probability of outcome $b = 1$ of the observable B_j measured in one context and p'_j is the corresponding probability of the same observable measured in the other context. As shown in Supplemental Material [33], the distances for all the states being tested are small enough (<0.0005), which indicates that a very good level of nondisturbance and compatibility between observables holds in our experiment.

Conclusion and discussion.—In Ref. [22], the authors showed that the CHSH inequality and the KCBS inequality could not be violated simultaneously by quantum systems. Their proof uses a usual hypothesis for Bell scenarios, that each part uses a single measurement per round in the nonlocality test. However, since one part is necessarily measuring other compatible observables in order to show contextuality, it is natural to use, instead, the locality concept of Ref. [32], that makes use of all data available.

In this Letter, we show that, using a more natural notion of locality, the data available in a joint test of CHSH and KCBS can produce joint violation of both inequalities, defying the concept of monogamy between contextuality and nonlocality. In fact, by constructing examples of states and measurements that lead to joint violations of Bell and noncontextuality inequalities, we show that both nonlocality and contextuality can be observed in thousands of relevant scenarios. Since such scenarios necessarily appear from subsets of measurements in all scenarios where quantum nonlocality and contextuality are possible, we provide clear evidence that monogamy of both phenomena may never hold in its stronger sense. We also employ a photonic implementation to experimentally demonstrate, for the first time, the synchronous observation of both Bell nonlocality and state-dependent contextuality in a simple scenario. Our experimental results agree with theoretical predictions, providing a proof that these phenomena not only are nonmonogamous, but may be observed and manipulated in simple quantum systems.

There are several results on monogamy between Bell inequalities and noncontextuality inequalities, such as the results in Refs. [22,29,30]. They, however, rely on the assumption that nonlocality tests are carried out with a subset of the available data, namely, multipartite correlations that take into account only one measurement of each part. In essence, the inequalities we investigate in this work are the same as some considered in previous works; the main difference is that we take into account local compatible measurements in their evaluation. Given that we manage to observe nonmonogamy in pairs of inequalities that were proven to be monogamous (under different hypotheses), our findings show that such monogamy results cannot be directly extended to the formalism that we adopt here, introduced in Ref. [32].

From the fundamental point of view, the results presented in this Letter shine new light on the relationship between two of the most important phenomena of the foundations of quantum physics. From the practical point of view, on the other hand, considering the individual importance of Bell nonlocality and contextuality to quantum information science, quantum cryptography, and quantum computing, we believe that this work may lead to novel possibilities where both concepts could be jointly employed for quantum information processing protocols.

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