Dynamical Instability of Self-Gravitating Membranes

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We show that a generic relativistic membrane with in-plane pressure and surface density having the same sign is unstable with respect to a series of warping mode instabilities with high wave numbers. We also examine the criteria of instability for commonly studied exotic compact objects with membranes, such as gravastars, anti–de Sitter bubbles, and thin-shell wormholes. For example, a gravastar which satisfies the weak energy condition turns out to be dynamically unstable. A thin-layer black hole mimicker is stable only if it has positive pressure and negative surface density (such as a wormhole), or vice versa.

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Introduction.—The detection of binary black hole (BH) mergers with ground-based gravitational-wave (GW) detectors [1–4], images of the supermassive BHs M87 and Sgr A* ith radio interferometry [5,6], and the observation of *S* stars orbiting a small dark region in the Galactic Center [7], all point to the existence of BHs, of which a description was first obtained by Schwarzschild more than one hundred years ago using general relativity. The study of BHs is not limited to astrophysics and general relativity, but also plays a role in other major areas of physics, such as quantum fields and strings, condensed matter physics, and quantum information. Because of its unparalleled conceptual and observational importance, it is paramount to test the more refined features of BHs against all viable alternatives allowed by the laws of nature. Any signal, e.g., ringdown quasinormal modes [8,9], that favors a BH mimicker over BHs themselves would represent a fundamental breakthrough or revolution in physics. In the coming decades the third-generation ground-based GW detectors [10,11], the space-borne GW detectors [12,13], and the next-generation Event Horizon Telescope, will likely improve the precision of such tests by many orders.

Horizonless compact objects are important candidates for BH mimickers [14]. One class of them, such as boson stars, has smooth distributions of matter and fields that are convenient for stability analysis and numerical simulations. However, it appears difficult to construct stable configurations of these compact stars that approach the compactness of BHs. For example, a fluid star with causal equation of state can achieve maximum compactness at around $M/R \leq 0.355$ [15,16], with M its mass and R its radius as measured from the surface area. The bound for boson stars is around 0.44 [17]. There are proposals for constructing compact stars with anisotropic stress [18–24] to increase the maximum compactness, but they often feature problems such as superluminal sound speed, violation of energy conditions, and lack of stability analysis. A recent study showed that the bound can be improved to ∼0.376 by including various prescriptions of elastic stress [25].

Another class of compact objects often includes a (or multiple) membrane(s) that separates spacetime regions, such as gravastars [26] and thin-shell wormholes [27,28]. These are interesting because this type of construction allows the transition to the exterior spacetime, which is the same as the BH spacetime, to be arbitrarily close to the horizon of a corresponding BH. Therefore, these models can have compactness arbitrarily close to that of BHs. In addition, membranes are often invoked if there is interesting physics happening near a certain surface, such as proposals considering hard structures near BH horizons motivated by firewalls or $2-2$ holes [29,30]. Moreover, compact objects with membranes are expected to have distinct strong-gravity dynamic behavior from more uniform compact objects. For this latter reason, two-dimensional domain walls have been extensively studied in cosmology.

Here, we present a perturbation study of self-gravitating membranes with nontrivial energy and stress. We find that if the signs of the in-plane pressure and surface density in the membrane are the same, there is a generic warping instability for modes with sufficiently high wave numbers. We apply these results to commonly studied compact objects, and find that a significant portion of the parameter space of gravastars—which are usually modeled by a de Sitter interior and Schwarzschild exterior with a spherical shell of matter at the boundary—and AdS bubbles (with anti–de Sitter interiors) are dynamically unstable. Static thin-shell wormholes always have positive pressure and negative surface density, so that they are free from these instabilities. Therefore, requiring membranes to have

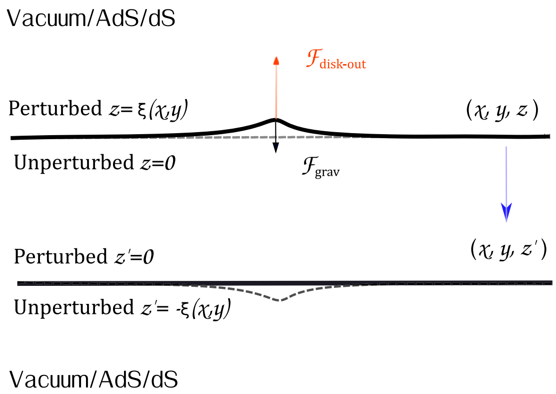

FIG. 1. A self-gravitating membrane separating two spacetime regions, which are vacuum solutions to Einstein's equations with possibly a nonzero cosmological constant. The positive pressure generally produces an antispring force out of the plane and the gravitational pull may act as a spring force trying to bring back the displacement to its equilibrium position. For the analysis presented in both the Newtonian and relativistic regime, we use a coordinate transformation to map the membrane to the equatorial plane of the new coordinates to facilitate the derivation of the equation of motion.

negative pressure (for positive surface density) and positive pressure (for negative density) becomes a powerful qualifier for the stability of compact objects. Throughout this Letter, we adopt geometric units ($c = G = 1$)

Membrane instability.—Let us consider a self-gravitating membrane with intrasurface pressure surrounded by vacuum. If the pressure is positive, any local vertical displacement results in an "antirestoring" force that pushes the mass element away from equilibrium, see Fig. 1. On the other hand, the gravitational attraction from surrounding mass elements tends to bring it back to equilibrium. We shall show that the antispring force always wins in the eikonal limit, leading to a series of instabilities with high wave number. To illustrate the basic picture, we present the analysis in the Newtonian regime first before proceeding to the relativistic case.

Consider a membrane placed in the (x-y) plane, with surface density σ and surface pressure P. The displacement field $\boldsymbol{\xi}(x, y)$ can be decomposed as

$$\boldsymbol{\xi} = \xi_x \hat{e}_x + \xi_y \hat{e}_y + \xi_z \hat{e}_z. \tag{1}$$

We use δ to denote Eulerian perturbations and Δ to denote Lagrangian perturbations. For example, the Eulerian density fluctuation is given by $\delta\sigma = -\nabla_\parallel \cdot (\sigma\boldsymbol{\xi})$, where $\nabla_\parallel$ here operates on the two horizontal directions, and the Lagrangian density perturbation is given by $\Delta\sigma = \delta\sigma + \boldsymbol{\xi} \cdot \nabla_\parallel \sigma$. For the purpose of this analysis, we only need to consider the case with ξ_z nonzero, in which case the local area change of mass elements is second order in ξ, i.e., $\delta\sigma = \Delta\sigma = 0$ at linear order.

The equation of motion for *three-dimensional* fluid elements, in terms of Lagrangian variables, can be written as [31]

$$\rho_0 \left(\frac{\partial^2 \boldsymbol{\xi}}{\partial t^2} + \nabla \Delta U - (\nabla \cdot \boldsymbol{\xi}) \nabla U_0 \right) = \nabla \cdot \Delta \boldsymbol{t}, \tag{2}$$

where ρ_0 is the unperturbed mass density, U_0 is the unperturbed gravitational potential, and $\boldsymbol{t}$ is the stress tensor so that the right hand side represents the hydrodynamical force acting on the fluid element. In other words, the left hand side of the equation is the kinetic term and the right hand side of the equation represents the external force. Similarly, for a mass element on a two-dimensional disk, we can write down the equation of motion as

$$\sigma_0 \left(\frac{\partial^2 \boldsymbol{\xi}}{\partial t^2} + \nabla_\parallel \Delta U - (\nabla_\parallel \cdot \boldsymbol{\xi}) \nabla U_0 \right) = \boldsymbol{F}_{\text{disk-in}} + \boldsymbol{F}_{\text{disk-out}}, \tag{3}$$

where σ_0 is the unperturbed surface mass density and ΔU is the Lagrangian potential perturbation. Since we only consider the vertical displacement, $\boldsymbol{\xi}$ is divergence-free $\nabla \cdot \boldsymbol{\xi} = 0$, i.e., there are no density perturbations. In the equilibrium case, U_0 satisfies $\nabla^2 U_0 = 0$ except at the disk plane, where the vertical derivative is discontinuous:

$$\left. \frac{\partial U_0}{\partial z} \right|_+ - \left. \frac{\partial U_0}{\partial z} \right|_- = 4\pi\sigma_0. \tag{4}$$

The right hand side of Eq. (3) can be obtained by integrating Eq. (2) across the membrane (for details, see Supplemental Material [32]). It can also be derived directly from the membrane configuration—as we motivate here—since it physically represents external forces. The external force is given by two components of disk forces. The in-plane component is generated by the pressure variation and the tilt of the disk plane:

$$\boldsymbol{F}_{\text{disk-in}} = -\nabla_\parallel \Delta P + (\nabla_\parallel P \cdot \Delta \mathbf{n}) \hat{e}_z, \tag{5}$$

where $\mathbf{n} = \hat{e}_z - \partial_x \xi_z \hat{e}_x - \partial_y \xi_z \hat{e}_y = \hat{e}_z + \Delta\mathbf{n}$ is the normal vector to the disk. The pressure perturbation is related to the density perturbation through the disk equation of state: $\Delta P/P = \Gamma_1 \Delta\sigma/\sigma$ [33], where Γ_1 depends on the equation of state and the nature of the perturbation (e.g., adiabatic or isothermal). Therefore, the Lagrangian pressure perturbation is zero for vanishing $\Delta\sigma$. The off-plane disk force is due to the warping of the disk. If we imagine the local disk surface has a radius of curvature R, then the magnitude of out-of-plane force is just $2P/R$. For general mean curvature κ, we have

$$\boldsymbol{F}_{\text{disk-out}} = -P\kappa\mathbf{n}, \quad \text{with} \quad \kappa = \frac{\partial^2 \xi_z}{\partial x^2} + \frac{\partial^2 \xi_z}{\partial y^2}. \tag{6}$$

In order to compute the potential perturbation ΔU, in particular, its value and derivatives on the disk plane, we make a coordinate transformation so that $z' = z - \xi_z$, with x, y coordinates unchanged. The disk is mapped to the $z' = 0$ plane in this new coordinate system, which is more convenient for solving the boundary value problem. Using

$$\frac{\partial}{\partial x'} = \frac{\partial}{\partial x} + \partial_x \xi_z \frac{\partial}{\partial z}, \qquad \frac{\partial}{\partial y'} = \frac{\partial}{\partial y} + \partial_y \xi_z \frac{\partial}{\partial z}, \qquad \frac{\partial}{\partial z'} = \frac{\partial}{\partial z}, \tag{7}$$

the original Laplace equation $\nabla^2 U = 0$ (for $z \neq \xi_z$) becomes

$$\nabla'^2 U = 2\partial_{x'}\xi_z \frac{\partial^2 U}{\partial x' \partial z'} + 2\partial_{y'}\xi_z \frac{\partial^2 U}{\partial y' \partial z'} + (\partial^2_{x'}\xi_z + \partial^2_{y'}\xi_z)\frac{\partial U}{\partial z'} \tag{8}$$

with the matching conditions that $\partial_{z'} U|_+ - \partial_{z'} U|_- = 4\pi\sigma$ and $U|_+ = U|_-$. Because $\boldsymbol{\xi}$ is an infinitesimal displacement, we can write U as $U_0 + U_1$, with U_0 satisfying $\nabla'^2 U_0 = 0$ together with $\partial_{z'} U_0|_+ - \partial_{z'} U_0|_- = 4\pi\sigma$ and $U_0|_+ = U_0|_-$. The solution of U_0 is obviously known, and U_1 may be obtained by solving

$$\nabla'^2 U_1 = 2\partial_{x'}\xi_z \frac{\partial^2 U_0}{\partial x' \partial z'} + 2\partial_{y'}\xi_z \frac{\partial^2 U_0}{\partial y' \partial z'} + (\partial^2_{x'}\xi_z + \partial^2_{y'}\xi_z)\frac{\partial U_0}{\partial z'}, \tag{9}$$

with $\partial_{z'} U_1|_+ - \partial_{z'} U_1|_- = 0$ and $U_1|_+ = U_1|_-$, so that U_1 is completely regular in the entire spacetime. In particular, U_1 evaluated on the disk surface can be mapped back to ΔU with $\Delta U \coloneqq U(z') - U_0(z) = U_0(z') + U_1(z') - U_0(z)$, and $\nabla \Delta U$ is the gravitational backreaction described in Eq. (3).

At this point, we consider a planar mode with $\xi \propto e^{i\mathbf{k}\cdot\mathbf{x}}$ in the eikonal limit, that is, $|k| \gg 1$. The right hand side of Eq. (3) is dominated by $\mathbf{F}_{\text{disk-out}}$, which is proportional to k^2. On the other hand, as U_1 is also proportional to $e^{i\mathbf{k}\cdot\mathbf{x}}$ and the source term for U_1 in Eq. (9) is dominated by the term proportional to k^2, we have $U_1 \propto k^0$, $\nabla' U_1 \propto k$, and $\partial_{z'} U_1 \propto k$ ($\partial_{z'} \propto k$ as $1/k$ is the only length scale in the problem). So the gravitational restoring force is subdominant compared to the antirestoring force by the warping disk. The dispersion relation is approximately (with $\partial_t \to -i\omega$)

$$\omega^2 \approx -P/\sigma_0 k^2, \tag{10}$$

which leads to exponential mode growth if $P/\sigma_0 > 0$.

Relativistic case.—In the relativistic setting, we consider a model problem for compact objects with a membrane: an infinite membrane with surface mass density σ and surface pressure P, which is a good approximation for perturbations of (spherical) compact objects in the eikonal limit. We discuss all the steps of the derivation of the equation of motion of the membrane perturbations. Detailed manipulations are relegated to Supplemental Material [32].

If we consider the spacetime of a gravastar or a thin-shell wormhole, the metric can be expressed as diag$[-f(r), 1/h(r), r^2, r^2\sin^2\theta]$, with different prescriptions for $f(r)$ and $h(r)$. As we focus on perturbations of small wavelength, we can look closer at the neighborhood of any point on the membrane, and rewrite the metric as

$$\begin{aligned} ds^2 &= g^{(0)}_{\mu\nu} dx^\mu dx^\nu \\ &= -U(z)dt^2 + U_z(z)dz^2 + U_p(z)(dx^2 + dy^2), \end{aligned} \tag{11}$$

where $x = \theta\cos\phi$ and $y = \theta\sin\phi$. This local representation of the membrane metric is generic. The Israel boundary conditions on the membrane relate the extrinsic curvature K_{ij} to the surface-layer property by [34] ($d\tau = \sqrt{U}dt$)

$$\begin{aligned} K^x_x|^+_- = K^y_y|^+_- &= \frac{1}{\sqrt{U_z}} \frac{U'_p}{2U_p}\bigg|^+_- = -4\pi\sigma, \\ K^\tau_\tau|^+_- &= \frac{1}{\sqrt{U_z}} \frac{U'}{2U}\bigg|^+_- = 8\pi(P + \sigma/2), \end{aligned} \tag{12}$$

where $|^+_-$ indicates the difference between 0_+ and 0_- of the membrane in the z direction. Since we can always rescale z in the vertical and radial direction, in the rest of the discussion we shall set $U_z = 1$.

Let us now assume the membrane is perturbed with vertical displacement $\xi_z = \xi(x, y, t)$. The membrane stress energy tensor is given by

$$\tau^{\mu\nu} = \delta(z - \xi)[(\sigma + P)u^\mu u^\nu + P(g^{\mu\nu} - n^\mu n^\nu)], \tag{13}$$

where $\mathbf{n}$ is the normal vector of the membrane. It is given by $e_z(1 - h_{zz}/2) - \sum_{\alpha=t,x,y}(\xi_{,\alpha} + h_{\alpha z})e_\alpha/g^{(0)}_{\alpha\alpha}$ and e_x is given by $(\partial/\partial x)$ (similarly for e_y and e_z), where $h_{\mu\nu}$ is sourced by the membrane motion [compare with the right hand side of Eq. (8)]. In order to derive the equation of motion for ξ_z, we transform to the coordinate system with $z' = z - \xi$, $t' = t$, $x' = x$, $y' = y$, such that the membrane is mapped back to the "equatorial" plane in the new coordinates. The spacetime metric in the new coordinates can be written as $g_{\mu\nu} = g^{(0)}_{\mu\nu} + \xi_{\mu|\nu} + \xi_{\nu|\mu} + h_{\mu\nu} = g^{(0)}_{\mu\nu} + \tilde{h}_{\mu\nu}$, where $|$ represents the covariant derivative with respect to $g^{(0)}_{\mu\nu}$. The gravitational perturbation is more conveniently computed in the original (t, z, x, y) coordinate system:

$$\bar{h}^{\alpha}_{\mu\nu|\alpha} + 2R_{\alpha\mu\beta\nu}\bar{h}^{\alpha\beta} = 0, \tag{14}$$

with the trace-reversed $\bar{h}_{\mu\nu} \coloneqq h_{\mu\nu} - \frac{1}{2}hg^{(0)}_{\mu\nu}$ and assuming the Lorenz gauge condition $\bar{h}^{|\alpha}_{\mu\alpha} = 0$, which is preserved along the evolution driven by the wave equations if it is initially satisfied. The waves should be outgoing at infinity and the matching condition at the membrane leads to

$$\int_{-}^{+} dz\bar{h}^{\alpha}_{\mu\nu|\alpha} = \bar{h}_{\mu\nu,z}|^{+}_{-} = 8\pi\delta\tau_{\mu\nu} \tag{15}$$

with $\delta\tau^{xz} = P\partial_x\xi/U_p$, $\delta\tau^{yz} = P\partial_y\xi/U_p$, $\delta\tau^{tz} = \sigma\dot{\xi}/U$. The metric functions are continuous across the membrane, and only $\partial_z h_{\mu\nu}$ may be discontinuous (we shall assume a simple setup with reflection symmetry, where $\partial_z h_{\mu\nu}|_- + \partial_z h_{\mu\nu}|_+ = 0$, but the final result does not rely on this assumption). In the eikonal limit, $\partial_t, \partial_x, \partial_y, \partial_z$ all scale as k, which suggests that the boundary value for $\bar{h}_{\mu\nu} = \mathcal{O}(k)^0$ and $\partial\bar{h}_{\mu\nu} = \mathcal{O}(k)$. Their interior value should have similar scaling laws following the wave equation in Eq. (14). (Such coupled wave equations in Lorenz gauge can be solved numerically in Schwarzschild spacetime [35], or perturbatively with the WKB method because the separation of scales in $1/k$ and the curvature radius of the background spacetime $\sim U/U'$.)

The equation of motion for ξ is given by $T^{z'\nu}_{;\nu} = 0$. We integrate it from the lower side to the upper side of the membrane ($z' = 0_- \to 0_+$), which becomes (evaluated at $z' = 0$)

$$(\sigma + P)(u^{z'}u^{\nu})_{;\nu} = P(n^{z'}n^{\nu})_{;\nu} \tag{16}$$

or, more explicitly,

$$\frac{\sigma + P}{2U}(2\tilde{h}_{tz',t} - \tilde{h}_{tt,z'}) = -\frac{P}{2U}\tilde{h}_{tt,z'} + \frac{P}{2U_p}(\tilde{h}_{xx,z'} + \tilde{h}_{yy,z'}) + \frac{P\tilde{h}_{tz',t}}{U} - \frac{P\tilde{h}_{xz',x}}{U_p} - \frac{P\tilde{h}_{yz',y}}{U_p}. \tag{17}$$

By noticing that $\tilde{h}_{\mu\nu} = h_{\mu\nu} + \xi_{\mu|\nu} + \xi_{\nu|\mu}$ and $h_{\mu\nu,z'}|_+ + h_{\mu\nu,z'}|_- = 0$, the equation reduces to

$$\frac{\sigma}{U}\xi_{,tt} + \frac{P}{U_p}(\xi_{,xx} + \xi_{,yy}) = -\frac{\sigma}{U}h_{tz,t} - \frac{Ph_{xz,x}}{U_p} - \frac{Ph_{yz,y}}{U_p}. \tag{18}$$

It is clear that the $\xi_{,xx} + \xi_{,yy}$ terms here provide the antispring force that potentially drives the instability. However, to fully address the mode dispersion relation, we also need to account for the gravitational backreaction. The relevant terms in the eikonal limit are described by the terms on the right-hand side, which all scale as k according to the discussion under Eq. (15). Therefore, similar to the Newtonian case, the relativistic antispring force effect scales as k^2 and gravitational backreaction scales as k. In the eikonal limit, we therefore find

$$\omega^2 \approx -\frac{U(0)P}{U_p(0)\sigma}k^2, \tag{19}$$

which signals an instability if $P/\sigma > 0$. This result can be straightforwardly extended to cases for which the upper and lower spacetime have different cosmological constants.

Gravastars.—A gravastar can be modeled by a spherical membrane separating an inner de Sitter spacetime and an outer Schwarzschild spacetime. If the inner region is an anti–de Sitter (AdS) spacetime, it is usually called an AdS bubble [36,37]. Defining ρ as the "energy density" or cosmological constant in the inner space, a as the radius of the membrane, σ as the membrane surface energy density, and P as its pressure, the total mass M of the spacetime is (following the notation in [38])

$$M = M_v + M_s\sqrt{1 - \frac{2M_v}{a}} + \frac{M_s^2}{2a}, \tag{20}$$

where $M_s = 4\pi a^2\sigma$ is the thin-shell mass and $M_v = 4\pi\rho a^3/3$ is the volume energy within the shell. The pressure within the shell is related to these masses through

$$\begin{aligned} P &= \frac{1}{8\pi a}\left[-\frac{1 - 4M_v/a}{\sqrt{1 - 2M_v/a}} + \frac{1 - M/a}{\sqrt{1 - 2M/a}}\right] \\ &= \frac{1}{8\pi a}\left[\frac{3M_v/a}{\sqrt{1 - 2M_v/a}} - \frac{1 - M_v/a}{\sqrt{1 - 2M_v/a}} + \frac{1 - M/a}{\sqrt{1 - 2M/a}}\right]. \end{aligned} \tag{21}$$

To ensure meaningful values for P, we require that $M/a \leq 1/2$ and $M_v/a \leq 1/2$. If the gravastar satisfies the weak energy condition, the surface density σ and M_s are both positive. We notice that $M \geq M_v > 0$ according to Eq. (20) [note $(1 - x)/\sqrt{1 - 2x}$ is a monotonically increasing for $0 \leq x \leq 1/2$]. From the second line of Eq. (21), it is straightforward to see that the pressure is always positive. Intuitively, it can be viewed as a consequence of the outer spacetime squeezing the inner spacetime, as the outer spacetime has larger effective pressure than the inner spacetime (also with the self-gravitation of the membrane). Although the analysis in the previous section was with topology $\mathbb{R}^2$ while the membrane of gravastars has topology $\mathbb{S}^2$, this distinction is irrelevant as we consider local perturbations in the eikonal limit. This simple observation, together with the analysis of the warping mode instabilities, immediately suggests that gravastars satisfying the weak energy condition are unstable. The instability timescale is

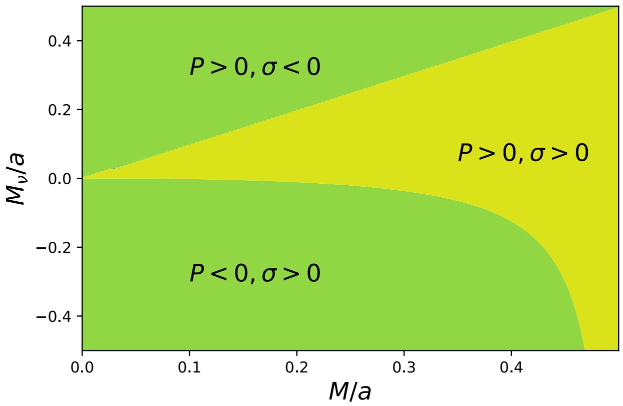

FIG. 2. The "phase diagram" of gravastars (with $M_v > 0$) and AdS bubbles (with $M_v < 0$). The regime with positive pressure and density is associated with the warping instability.

determined by Eq. (19) and depends on the prescription for P and σ.

In the more general setting, as we consider both de Sitter and AdS interiors and surface density with arbitrary sign, the warping instability applies part of the parameter space of gravastars and AdS bubbles, as shown in Fig. 2.

The modal stability of gravastars was initially studied in [38], which explicitly computed the quasinormal mode frequency for $\ell = 2$ axial and polar perturbations. However, the analysis in [38] treats the membrane as the provider of the matching condition between the inner and outer spacetime, in the same spirit as Eq. (4), but did not incorporate the membrane oscillations into the coupled mode equations. An explicit discussion of the gravastar mode analysis is included in Supplemental Material [32]. It is indeed the membrane modes that destabilize the whole system in the eikonal limit.

Thin-shell wormholes.—There are other horizonless compact objects generally considered in the literature as BH mimickers, or as candidates sourcing gravitational wave echoes. For example, thin-shell wormholes are commonly studied objects with compactness arbitrarily close to a BH. Consider two Schwarzschild solutions of the same mass M attached at radius r_0 [34,39], the corresponding thin-shell pressure and density at the wormhole throat are

$$P = \frac{1}{4\pi r_0}\frac{1 - M/r_0}{\sqrt{1-2M/r_0}}, \quad \sigma = -\frac{1}{2\pi r_0}\sqrt{1-2M/r_0} \tag{22}$$

so that the pressure is positive and the density is negative, which implies that static thin-shell wormholes are free from the warping instability. On the other hand, the empty shell models (which have positive surface energy density) as considered in [40,41] naturally require positive in-shell pressure to support against gravity, and are all unstable to warping perturbations in the eikonal limit [42].

Discussion.—We have discovered a generic instability for thin-layer structures in general relativity, if $P/\sigma > 0$, with applications highlighted in exotic compact objects with membranes. One may imagine various ways to "cure" these systems so that they are free from warping instabilities. One possible way is to add additional rigidity against warping for the membrane, e.g., a new term in the action with

$$S = \alpha \int d^3\xi K^{ij} K_{ij}, \tag{23}$$

where ξ is the parametrization for the "world tube" of the membrane, K^{ij} is the extrinsic curvature, and α is a positive constant characterizing the rigidity. A possible caveat is that such an additional term in the action may lead to higher-order derivative terms in the equation of motion, which may raise concerns regarding well-posedness of the problem. Moreover, adding dissipation to the system does not cure the instability. This is because the antispring force causes runaway behavior of the displacement instead of oscillations. If the displacement were to saturate at some value, the dissipation becomes zero as there is zero velocity, but the antispring force continues to drive the displacement to larger values, i.e., there is no saturation point. On the other hand, if we replace the membrane with a shell of matter of thickness d, this can remove the instability. The thickness essentially adds a spatial frequency cutoff $k \sim 1/d$ in the above analysis. The caveat is that d has to be sufficiently large so that the antispring in Eq. (10) becomes subdominant. Note that in this case the description for the dynamic behavior of matter with anisotropic stress is highly nontrivial and currently unknown.

A membrane with density and pressure having the same sign generically prefers configurations with higher surface curvature as they are associated with a lower energy state, if gravitational backreaction is neglected. For example, a membrane with an ellipsoidal shape has lower potential energy than that with a spherical shape. Mathematically, the potential energy is $\propto Q_{ij}^2/(2\lambda)$ where Q_{ij} is the mass quadrupole moment and λ the tidal Love number. Negative potential energy means that λ is negative. Even with gravitational backreaction included, if it is weaker than the antispring force such that the potential energy is still negative, the Love number λ will also be negative [43]. Therefore the warping instability is connected to the negativity of tidal Love numbers, which applies to generic deformations with any $\ell \geq 2$ [44]. In the eikonal limit, the tidal Love number λ_ℓ has to be negative if $P/\sigma > 0$.

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