POLARIZATION OF THE MUON FROM $K_{\mu 3}{}^{+}$ DECAY*

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The principal difficulty in extending the universal (V-A) Fermi interaction to strange-particle decays lies in determining the form factors of the strangeness-changing currents. Because one set of form factors is sufficient to determine all of the detailed properties (rates, spectra, polarization, etc.) of the decay, the consistency in measurements of these form factors based on different properties is a test of the theory.

For a universal vector coupling, the matrix element for K_{l3} decay can be written in terms of the pion and kaon four-momenta as[1]

$$M=[\tfrac{1}{2}f_+(P_K+P_\pi)_\lambda+\tfrac{1}{2}f_-(P_K-P_\pi)_\lambda][\bar{u}_\nu\gamma_\lambda(1+\gamma_5)u_e].$$

Because terms containing f_- are proportional to $(m_l/M_K)^2$ only, the form factor f_+ can be determined from a study of K_{e3} decay. The form factor f_-, or alternatively the ratio $f_-/f_+=\xi$, must be determined from the $K_{\mu 3}$ decay. The first information on ξ came from measurements of the ratio of the decay rates into the K_{e3} and $K_{\mu 3}$ modes,[2] but this ratio has an inherent quadratic ambiguity in ξ. Early measurements of the muon spectrum,[3-5] although free from this ambiguity in interpretation, reached conflicting conclusions as to its resolution. More recent measurements[6,7] and crude polarization measurements[8,9] have helped to resolve the ambiguity, but the latter have not been sufficiently precise to constitute an independent determination of the form factors. We report here a measurement of ξ from the longitudinal polarization of the muon in $K_{\mu 3}$ decay, finding $\xi=-0.15\pm 0.90$.

A separated beam of K^+ mesons from the Bevatron was stopped in the Berkeley 30-in. heavy-liquid bubble chamber filled with C_3F_8 in a magnetic field of about 15 kG. The liquid has a density of 1.22 and a radiation length of 28 cm. When a muon from $K_{\mu 3}$ decay comes to rest in the chamber, its spin component along the magnetic field is conserved so that the electron asymmetry about this direction measures the muon polarization. In the absence of any depolarization we expect an electron angular distribution of the form

$$1+(aP_\mu\cos\theta_{\mu B})\cos\theta_{eB}, \qquad (1)$$

where a is the μ-e decay asymmetry parameter, P_μ is the muon longitudinal polarization, and $\theta_{\mu B}$ and θ_{eB} are the observed angles between the muon and electron tracks and the magnetic field vector at the point where the muon decays.

From a sample of about 12 000 measured $K_{\mu 3}$ decays with a stopping muon, found in a scan of about 150 000 pictures, we selected 2988 events which met the following criteria:

(1) The muon range was between 7 and 28 cm ($38<T_\mu<96$ MeV). This excludes background from $K_{\pi 2}$ and $K_{\tau'}$ decays in which the short (1.5-mm) muon track from π-μ decay was unobservable.

(2) The muon decay vertex was more than 4 cm from the top or bottom of the chamber. This eliminated a region in which an electron with dip angle of the same sign as the muon dip would leave the chamber and thus be more easily overlooked.

(3) $|\cos\theta_{\mu B}|>0.2$. This eliminated a group of events with little or no analyzing power.

(4) No "kink" in the muon track greater than 5° in any view except in the last 5 cm of track.

Most of the expected background or scanning biases in this experiment favor negative or zero polarization. Possible sources of background include (a) $K_{\mu 2}$ decays in flight, (b) $K_{\pi 2}$ and τ' decays in flight, in which the π-μ decay is missed, and (c) $K_{\pi 2}$ decays at rest followed by in-flight decay of the π^+, which must be backward in the π^+ center of mass to satisfy our range criteria. Of these three sources, (a) would give a polarization of nearly -1, and are eliminated mainly on the basis of the K-track ionization. Those that remain must come from a fairly small backward cone in order to satisfy our range criteria, and this cone is partially eliminated by our dip criterion. We estimate this background to be <2%; the estimated background from (b) is <1%. Only (c) favors positive polarization. Without criterion (4) above, about 20% of our events would come from this source, and would give a polarization of about +0.75. If careful examination of a track always reveals a 10° projected-angle kink, this background is reduced to 2%. Criterion (4) insures a rejection at least this good.

Since the scanner looks nearly parallel to the magnetic field, it is difficult for scanning bias to affect the polarization, except in the region eliminated by criterion (2) above, where the events indeed show a large negative polarization. No scanning bias in the horizontal plane should have a large effect on the measured polarization. This assumption has been tested by artificially biasing the final sample of events and observing that the polarization is unaffected.

The asymmetry parameter of muon decay ($a = 0.33$) must be corrected for a bias against short electron tracks and for the ±12° average measurement errors in electron dip angle, becoming $a = 0.35$. As a rough check of the asymmetry parameter, and to detect any gross biases or depolarization that might arise, we have measured a for 800 $K_{\mu 2}$ decays in flight. In the observed K-momentum region of 250 to 500 MeV/c, backward decays in the center of mass produce laboratory muon momenta low enough to give a sample of steep stopping muons with known polarization. We obtain $a = +0.49 \pm 0.13$, assuming the $K_{\mu 2}{}^{+}$ polarization is -1.0. This value and the high value we obtain for $\langle P_{\mu} \rangle$ (see below) indicate that no substantial depolarization is likely. A magnetic field of 15 000 G is ordinarily more than adequate to prevent depolarization.

Using expression (1) to form a likelihood function, we obtain an average $K_{\mu 3}$ longitudinal polarization over our entire energy interval

$$\langle P_{\mu} \rangle = +0.74 \pm 0.16.$$

The factor $\cos\theta_{\mu B}$ can be considered as the analyzing power of each event and for our sample $\langle \cos\theta_{\mu B} \rangle = 0.54$.

Another method of obtaining the polarization is to form an angular distribution of the electrons about the magnetic field, weighting each event

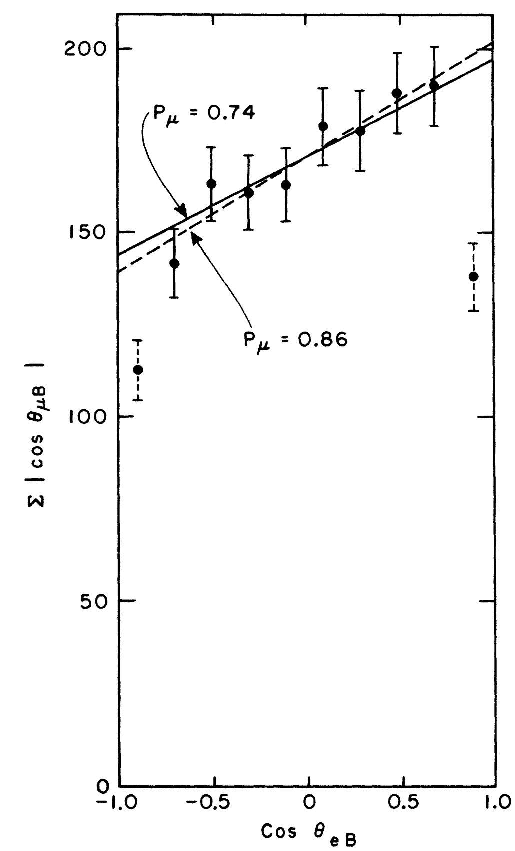

FIG. 1. Weighted angular distribution of μ-decay electrons about the magnetic field. The slope of the solid line is based on the P_{μ} obtained by a maximum-liklihood calculation, which covers all values of θ_{eB}. The broken line is a least-squares fit for the region $|\cos\theta_{eB}| \leq 0.8$. The positive abscissa means projections of mu and electron tracks on the magnetic field have the same sign.

by $\cos\theta_{\mu B}$. This is illustrated in Fig. 1. A least-squares fit to the distribution gives $P_{\mu} = 0.72 \pm 0.17$. The angular distribution clearly shows that we miss a large fraction of the very steep electrons, but since this bias is independent of the muon direction, it does not affect the polarization measurement. Refitting with the two extreme points omitted gives $P_{\mu} = 0.86 \pm 0.23$.

Even over our somewhat restricted range of energies, the energy dependence of the polarization provides information about the form factors. To utilize this information efficiently, we must fit for ξ directly. The formula for P_{μ} as a function of the kinetic energy of the muon and ξ for constant form factors has been obtained by Brene, Egardt, and Qvist.[1] Using this function $P_{\mu}(T_{\mu}, \xi)$ in expression (1) gives us a likelihood

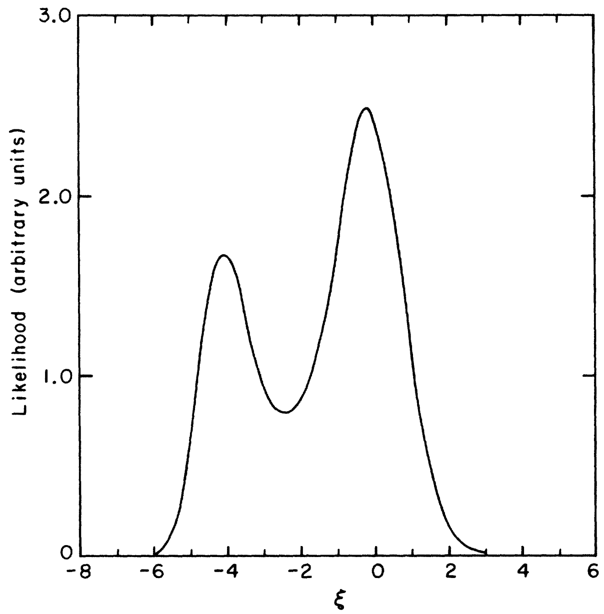

FIG. 2. Likelihood function obtained by fitting directly for ξ; $L = \Pi[1 + 0.35P(\xi, T_\mu)\cos\theta_{\mu B}\cos\theta_{eB}]$.

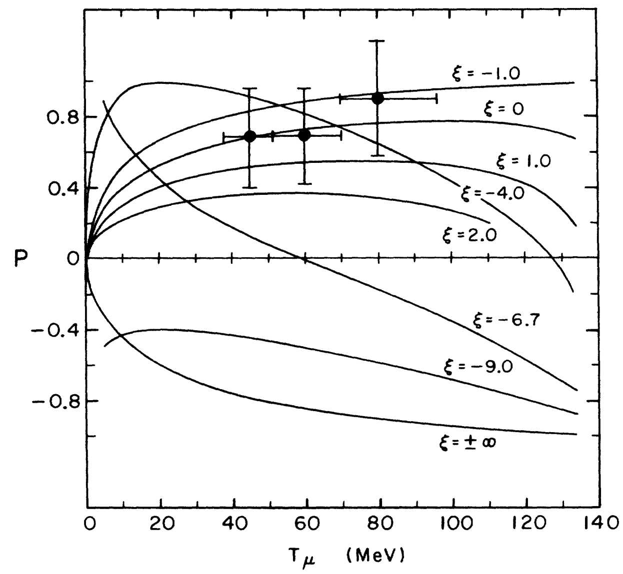

FIG. 3. Comparison of measured polarization with predictions for various values of ξ.

function for ξ, which is plotted in Fig. 2 and yields $\xi = -0.15 \pm 0.90$. The second peak at $\xi = -4.05 \pm 0.75$ is lower and predicts a branching ratio for $K_{\mu 3}$ decay of $(2.33 \pm 0.17)\%$, which is over three standard deviations below the lowest quoted value for this ratio.[7] In Fig. 3 we show the energy dependence of P_μ compared with the predictions for various values of ξ. The energy intervals 38 to 51 MeV, 51 to 70 MeV, and 70 to 96 MeV were chosen to contain roughly equal numbers of events. Our two solutions correspond to equal average values of P_μ in our energy interval but differ in the energy dependence of P_μ. The ambiguity is not inherent in the method; a more accurate measurement or one covering a wider energy range would resolve it. The geometry of our chamber precludes extending our measurement into the region above $T = 100$ MeV. The ambiguity does not overlap the inherent ambiguity of branching-ratio experiments, whose negative solutions are in the range $\xi = -6$ through -9, excluded by our data.

Our most probable value of ξ agrees with the recent work on the spectrum by the Turin group,[6] the recent lower value for the $K_{\mu 3}/K_{e3}$ branching ratio reported by the Michigan group,[7] and the earlier spectrum and angular-correlation data on the Michigan and Berkeley groups.[3,7] Work on the K_2^0 decay spectra and branching ratios by the Brookhaven[10] and Illinois[11] groups give similar values for ξ. Table I gives the current status of measurements of ξ.

Table I. Recent measurements of ξ in K^+ lepton decays.

Source	Method	ξ
This experiment	Polarization	-0.15 ± 0.90 (or -4.05 ± 0.75)
Turin[a]	Branching ratio[b]	$+0.3 \pm 0.8$ (or -7.1 ± 0.8)
	Muon spectrum	>-3
Michigan[c]	Branching ratio	-0.2 ± 0.8 (or -6.5 ± 0.8)
	Spectra and angular correlation	$+0.6 \pm 2.0$
	Combined result	-0.08 ± 0.70
Berkeley-Michigan[d]	Combined result	$+1.8 \pm 0.6$

[a]See reference 6.
[b]Computed from the measured $K_{\mu 3}$ branching ratio, assuming a K_{e3} branching ratio of $(5.0 \pm 0.5)\%$.
[c]See reference 7.
[d]See reference 3.

The simplest theoretical model predicts $\xi = 0$.[12] The effects of an intermediate vector boson in the weak interactions and a narrow P-wave $K\pi$ resonance (K^*) are indistinguishable, both shifting ξ slightly negative by an amount $(M_K^2 - M^2)/M(K^* \text{ or } B)^2$, the momentum dependence introduced being negligible.[1,13] Thus, for such models, values from $\xi = -0.1$ through $\xi = -0.5$ are reasonable. Calculations of the effect of an S-wave $K\pi$ phase shift on the form factors predict small positive values of ξ.[11,13] Determinations of ξ at the present level of accuracy cannot distinguish among the predictions of these and other detailed models.[14] A recent attempt by Schwinger[15] to explain the decay rate for $K^+ \to \pi^+ + \pi^0$ relative to that for $K_2^0 \to \pi^+ + \pi^-$ requires $\xi = -6.6$ which is inconsistent with our result.

Our data exclude a pure scalar current ($\xi = \pm\infty$). Pure tensor would give $\langle P_\mu \rangle = 0.40$ averaged over our sample. This is more than two standard deviations from our result, but we cannot exclude a mixture of vector with some tensor. However, agreement of our value of ξ with recent measurements by other methods suggests that a vector current is sufficient to account for the data.

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