unstable waves are the features of most general interest. A detailed account of the work will be published shortly.

*Work supported in part by National Science Foundation, Air Force, and TRW Space Technology Laboratories Independent Research Program.

†Permanent address: Rand Corporation, Santa Monica, California.

[1]H. Dreicer, Phys. Rev. 115, 238 (1959).

[2]T. Dupree, Phys. Fluids 6, 1714 (1963); Iu. Klimontovich, Zh. Eksperim. i Teor. Fiz. 33, 982 (1957) [translation: Soviet Phys.–JETP 6, 753 (1958)].

[3]N. Rostoker and M. Rosenbluth, Phys. Fluids 3, 1 (1960).

[4]B. D. Fried and R. Gould, Phys. Fluids 4, 139 (1961).

[5]E. C. Field and B. D. Fried, Bull. Am. Phys. Soc. 9, 329 (1964).

[6]G. J. Culler and B. D. Fried, Thompson-Ramo-Wooldridge Report No. M 19-3U3, 1963 (unpublished).

OPTICAL MIXING AS A PLASMA DENSITY PROBE*

Norman M. Kroll, Amiram Ron,† and Norman Rostoker
University of California, San Diego, La Jolla, California
(Received 11 May 1964)

It is difficult to stimulate plasma oscillations with a single light wave because a light wave is transverse and plasma oscillations are longitudinal. Moreover, the frequencies and wavelengths of the two kinds of waves in a plasma are usually quite different. However, if one considers two light beams of frequencies ω_1, ω_2 and wave vectors $\vec{k}_1, \vec{k}_2$ incident on a plasma, the plasma acts as a mixer and it is possible to satisfy the plasma dispersion relation by tuning the frequency ω_2 so that $\omega_2 - \omega_1 \cong \omega_p$, where $\omega_p = (4\pi e^2 n/m)^{1/2}$, the plasma-wave frequency; in addition, it is necessary to adjust the directions of $\vec{k}_1, \vec{k}_2$ so that $|\vec{k}_1 - \vec{k}_2| L_D < 1$, where $L_D = (k_B T/4\pi e^2 n)^{1/2}$ is the Debye length. In this case energy and momentum conservation can be satisfied, a resonance takes place, and the amplitude of the plasma wave is significant. The same light waves that produce the plasma wave will also scatter. Alternatively, it may be more convenient to introduce a third beam and measure the scattering of it by the stimulated plasma oscillations. The purpose of this paper is to estimate the scattering and show that the measurement is quite feasible with modern laser techniques.

The scattering cross section per unit frequency and per unit solid angle of a plasma electron is[1]

$$\frac{d\sigma}{d\omega'} = \frac{r_0^2}{2\pi} S\left(\vec{K} - \frac{\omega'}{c}\vec{1}, \omega' - \omega\right)(1 - \tfrac{1}{2}\sin^2\vartheta), \quad (1)$$

where $r_0 = e^2/mc^2$ is the classical electron radius and $\vec{K}$ is the wave vector at the incident light wave of frequency $\omega = Kc$. It is assumed that $\omega > \omega_p$, the plasma frequency. The scattered wave is of frequency ω' and is observed in a direction given by the unit vector $\vec{1}$, where $\cos\theta = \vec{1}\cdot\vec{K}/K$. The quantity $S(\vec{k}, \omega)$ is the spectral density of fluctuations of electron density, i.e.,

$$S(\vec{k}, \omega) = \lim_{V, T \to \infty} \frac{2|n(\vec{k}, \omega)|^2}{nVT};$$

$n(\vec{k}, \omega)$ is the Fourier component of the electron density.

If the energy flux of the incident beam is F_0, the energy flux per unit frequency scattered into the direction $\vec{1}$ is

$$dF_S/d\omega' = (NF_0/r^2)d\sigma/d\omega', \quad (2)$$

where N is the total number of electrons and r is the distance from the plasma to the detector.

This formula applies to the case where the scattered wave is detected in the "wave zone," i.e., for $r > a^2/\lambda$, where a is the diameter of the incident beam and λ is the wavelength. For a plasma in a state of thermal equilibrium, $d\sigma/d\omega'$ has sharp resonances at $\omega' = \omega \pm \omega_p$. The total contribution from one of these resonances is $\sigma_{\rm res} = r_0^2(kL_D)^2$. For illustrative purposes we assume a plasma of easily attained properties: density $n = 10^{14}$ cm^{-3}, temperature $k_B T = 10$ eV, and $\omega_p = 5.64\times10^{11}$ sec^{-1}. Consider the scattering of light from a ruby laser of wavelength $\lambda = 0.7\times10^{-4}$ cm. For the plasma resonance to be well defined we must have $kL_D < 1$, where k $|\vec{K} - \omega'\vec{1}/c| \cong K\theta$ which implies observation at an angle of $\theta \cong k/K$. For the plasma under consideration, $L_D = 3.16\times10^{-4}$ cm; we assume $kL_D = 0.2$ which implies that the scattered wave will be observed at an angle of $\theta = 0.57°$. The cross section for scattering by thermally excited plasma waves is $\sigma_{\rm res} = 0.32\times10^{-26}$ cm^2. Assume that the laser beam diameter is $a = 10^{-1}$ cm, that the plasma electrons which scatter occupy a volume a^3, and that the scattered signal is detected at a distance of $r = 100$ cm. Then $F_S/F_0 = 0.32\times10^{-19}$.

Since the observation of such a small scattered signal is quite difficult, it is desirable to consider methods for enhancing the density fluctuations beyond the thermal equilibrium value. One method is to make the plasma oscillations unstable by electron beams[2] or currents. Similar remarks apply to other modes that involve the electron density such as ion oscillations.[3] Scattering from stimulated ion oscillations in a discharge tube has recently been observed with microwaves.[4] It is, however, desirable to make measurements on a quiescent plasma. To this end several proposals have been advanced recently that involve production of plasma-wave excitation by externally applied electromagnetic fields. If the plasma is described by the Vlasov equations, then according to the linear theory an electromagnetic wave produces no density fluctuations. (The effects of inhomogeneities and anisotropy in velocity space are neglected.) However, if particle correlations are considered, there is a coupling between longitudinal and transverse waves which is proportional to the plasma parameter $1/nL_{\rm D}^3$. Thus density fluctuations in excess of the thermal value can be stimulated with a corresponding increase in the scattering. The plasma dispersion relation is $\omega \cong \omega_p[1 + \frac{3}{2}k^2 \times L_{\rm D}^2]$ for $kL_{\rm D} < 1$, so that if ω is only slightly greater than ω_p the process can be resonant as considered by Berk.[5] Considerable enhancement of the cross section can be obtained, but application is quite restricted due to the limitations on ω. Platzman and Tzoar[6] have recently considered light-off-light scattering for lasers where $\omega \gg \omega_p$ but find nothing larger than $\sigma \sim r_0^2$.

The purpose of this Letter is to consider the nonlinear coupling between longitudinal and transverse waves that can be described by the Vlasov equations. This leads to a considerably enhanced cross section.

Consider the Vlasov equations

$$\frac{\partial F}{\partial t} + \vec{v}\cdot\frac{\partial F}{\partial \vec{x}} - \frac{e}{m}\left[-\frac{\partial}{\partial \vec{x}}\Phi + \vec{E}(\vec{x},t) + \frac{\vec{v}}{c}\times\vec{B}(\vec{x}t)\right]\cdot\frac{\partial F}{\partial \vec{v}} = 0, \quad (3)$$

$$\frac{\partial^2}{\partial \vec{x}^2}\Phi = 4\pi en\left(\int F dv - 1\right), \quad (4)$$

with externally applied fields,

$$\vec{E}(\vec{x},t) = \vec{E}_1\cos(\omega_1 t - \vec{k}_1\cdot\vec{x}) + \vec{E}_2\cos(\omega_2 t - \vec{k}_2\cdot\vec{x} + \varphi),$$

$$\vec{B}(\vec{x},t) = -(\vec{k}_1/k_1)\times\vec{E}_1\cos(\omega_1 t - \vec{k}_1\cdot\vec{x})$$

$$-(\vec{k}_2/k_2)\vec{E}_2\cos(\omega_2 t - \vec{k}\cdot\vec{x} + \varphi),$$

corresponding to two light waves of frequencies ω_1 and $\omega_2 = \omega_1 + \Delta\omega$ and wave vectors $\vec{k}_1$ and $\vec{k}_2 = \vec{k}_1 + \Delta\vec{k}$, where $\Delta\omega \cong \omega_p$ and $\Delta kL_{\rm D} < 1$. Let $F = f_{\rm M} + f$ where $f_{\rm M}$ is the Maxwell distribution, and Fourier decompose Eqs. (3) and (4), i.e.,

$$f(\vec{x},\vec{v},t) = \frac{1}{VT}\sum_{\vec{k},\omega} e^{i(\omega t + \vec{k}\cdot\vec{x})} f(\vec{k},\vec{v},\omega),$$

and

$$n(\vec{k},\omega) = n\int d\vec{v} f(\vec{k},\vec{v},\omega).$$

According to the linear approximation $n^{(0)}(\vec{k},\omega) = 0$ and

$$f^{(0)}(\vec{k},\vec{v},\omega) = \frac{e}{m}\vec{E}(\vec{k},\omega)\cdot\frac{\partial f_{\rm M}(v)}{\partial \vec{v}}\frac{1}{i(\omega + \vec{k}\cdot\vec{v} - i\delta)}.$$

After carrying out the next iteration the result is

$$n^{(1)}(\vec{k},\omega) = -\frac{e^2 n}{m^2}\frac{1}{\epsilon(\vec{k},\omega)}\frac{1}{VT}$$

$$\times\sum_{\vec{k}',\omega'}\int d\vec{v}\left\{\frac{\vec{E}(\vec{k}',\omega')(\vec{v}/\omega')\times[\vec{k}'\times\vec{E}(\vec{k}',\omega')]}{\omega + \vec{k}\cdot\vec{v} - i\delta}\right\}\cdot\frac{\partial}{\partial \vec{v}}\left\{\frac{\vec{E}(\vec{k}-\vec{k}',\omega-\omega')}{(\omega-\omega') + (\vec{k}-\vec{k}')\cdot\vec{v} - i\delta}\cdot\frac{\partial f_{\rm M}}{\partial \vec{v}}\right\}, \quad (5)$$

where

$$\epsilon(\vec{k},\omega) = 1 - \frac{\omega_p^2}{k^2}\int d\vec{v}\,\vec{k}\cdot\frac{\partial}{\partial \vec{v}} f_{\rm M}(\vec{v})/(\omega + \vec{k}\cdot\vec{v} - i\delta)$$

is the longitudinal plasma dielectric constant, and

$$\vec{E}(\vec{k},\omega) = \tfrac{1}{2}VT\{\vec{E}_1[\delta_{\omega,\omega_1}\delta_{\vec{k},-\vec{k}_1} + \delta_{\omega,-\omega_1}\delta_{\vec{k},\vec{k}_1}] + \vec{E}_2[e^{i\varphi}\delta_{\omega,\omega_2}\delta_{\vec{k},-\vec{k}_2} + e^{-i\varphi}\delta_{\omega,-\omega_2}\delta_{\vec{k},\vec{k}_2}]\}.$$

The appearance of $\epsilon(\vec{k},\omega)$ in the denominator of Eq. (5) indicates the possibility of resonant processes when $\vec{k}=\pm\Delta\vec{k}$, $\omega=\pm\Delta\omega$. Only these terms are retained and additional approximations employed are $\omega_{1,2}\gg\vec{k}_{1,2}\cdot\vec{v}$, $|\Delta\omega|\gg|\Delta\vec{k}|v_{\text{th}}$, which lead to the result

$$\lim_{V,T\to\infty}\frac{2|n^{(1)}(\vec{k},\omega)|^2}{nVT}=\frac{1}{128\pi^2m^2n}\frac{k^4(\vec{E}_1\cdot\vec{E}_2)^2}{\omega_1^2\omega_2^2|\epsilon(\vec{k},\omega)|^2}[\delta(\omega+\Delta\omega)\delta(\vec{k}-\Delta\vec{k})+\delta(\omega-\Delta\omega)\delta(\vec{k}+\Delta\vec{k})]. \quad (6)$$

The two laser beams can be adjusted in direction so that $|\Delta\vec{k}|L_{\text{D}}<1$. It is assumed that one of the laser beams can be tuned so that $\Delta\omega\cong\omega_p$. Then

$$|\epsilon(\vec{k},\omega)|^2\cong[1-(\omega_p/\omega)^2]^2+\Gamma^2\to\Gamma^2, \quad (7)$$

where Γ is the larger of

$$(\tfrac{1}{2}\pi)^{1/2}\frac{\omega_p^{\;2}\omega}{(kv_{\text{th}})^3}\exp\left[-\frac{\omega^2}{2(kv_{\text{th}})^2}\right],$$

the Landau damping, or $\Gamma_c=2\pi\nu_c/\omega_p$, the collisional damping, where ν_c is the collision frequency.[7] For such a resonant process the density fluctuations can be very large. Scattering of either of the two light beams can be observed at $\omega_1-\omega_p$ or $\omega_1+2\omega_p$ and can be distinguished from the incident beams.[8] To compare the cross section with the Thomson cross section for thermal equilibrium, integrate over the resonance at $\omega'=\omega_1-\omega_p$, for example. Then

$$\frac{\sigma_{\text{res}}}{r_0^{\;2}(kL_{\text{D}})^2}=\frac{1}{32(2\pi)^3m^2n}\frac{k^4(\vec{E}_1\cdot\vec{E}_2)^2}{\omega_1^{\;2}\omega_2^{\;2}(kL_{\text{D}})^2|\epsilon(\vec{k},\omega)|^2}\times\delta(\vec{k}-\Delta\vec{k}). \quad (8)$$

To obtain a finite result we must consider the fact that the incident beams are finite with $\Delta k\sim 1/a$ where a is the beam diameter. This implies an angular spread $\delta\theta\sim\gamma/a$. We can account for this by averaging Eq. (8) over a Gaussian distribution for Δk with a dispersion $1/a^2$. This amounts to replacing $\delta(\vec{k}-\Delta\vec{k})$ by a^3.

To estimate Eq. (7) we make use of the plasma parameters previously assumed. In addition, it is necessary to make some assumptions about the electric fields $\vec{E}_1$ and $\vec{E}_2$. Present laser technology makes it possible to produce an electric field of 10^8 V/cm over a period of 10 nsec and since the two beams are nearly parallel $\vec{E}_1\cdot\vec{E}_2\sim E_1E_2$. The Landau damping is much smaller than the collisional damping so that we use $\Gamma_c\cong1.1\times10^{-3}$ corresponding to $n=10^{14}$ cm^{-3} and $k_BT=10$ eV. In making an estimate of $|\epsilon|^2$ it is assumed that the density and hence ω_p are constant over a volume of 10^{-3} cm^3. If the density is not constant $4[\Delta\omega_p/\omega_p]^2$ replaces Γ^2 in Eq. (7); the present estimate assumes $\delta n/n<10^{-3}$ over a distance of 10^{-1} cm. It is also assumed that the width of the laser output is sufficiently narrow to allow $\Delta\omega_p/\omega_p\sim10^{-3}$. While this is not typically the case for giant pulse laser oscillators it should be achievable with the use of an amplifier operating on the output of a lower power oscillator. With these conditions $\sigma_{\text{res}}/r_0^2k^2L_{\text{D}}^{\;2}=10^{13}$, and the ratio of scattered to incident energy flux is $F_s/F_0=0.3\times10^{-6}$. Measurement of this scattered flux should be possible, in view of the fact that Thomson scattering for $kL_{\text{D}}\sim1$, $\sigma\sim r_0^{\;2}$, and $F_s/F_0\sim10^{-11}$ has recently been measured.[9] Thus it should be possible to make a convenient density probe with two crossed laser beams that gives good resolution in both space and time without disturbing the plasma significantly.

*Work supported by the U. S. Atomic Energy Commission.

†Permanent address: Physics Department, Israel Institute of Technology, Haifa, Israel.

[1]M. N. Rosenbluth and N. Rostoker, Phys. Fluids 5, 776 (1962).

[2]W. E. Drummond, Phys. Fluids 5, 1133 (1962).

[3]S. Ichimaru, D. Pines, and N. Rostoker, Phys. Rev. Letters 8, 231 (1962).

[4]V. Arunasalam and S. C. Brown, Bull. Am. Phys. Soc. 9, 494 (1964).

[5]H. L. Berk, Princeton University, Plasma Physics Laboratory Report No. MATT-230, November 1963 (unpublished).

[6]P. M. Platzman and N. Tzoar, Bull. Am. Phys. Soc. 9, 495 (1964). See also Phys. Rev. Letters 12, 573 (1964).

[7]A. Ron, J. Dawson, and C. Oberman, Phys. Rev. 132, 497 (1963).

[8]To obtain the maximum effect it is required that the source velocities

$$v_s=(\omega_1-\omega_p)/|\vec{k}_1-\Delta\vec{k}| \text{ and } v_s'=(\omega_2+\omega_p)/|\vec{k}_2+\Delta\vec{k}|$$

differ fractionally from the velocity of light by no more than $1/ka$. One can show that, for small scattering angles θ, the fractional velocity error is just θ^2, and hence that this condition is satisfied for the parameters considered here. Proper velocity matching can always be achieved by projecting a third beam at frequency ω_3 with appropriately chosen $\vec{k}_3$ to yield a scattering beam at either $\omega_3 + \omega_p$, $\vec{k}_3 + \Delta\vec{k}$, or $\omega_3 - \omega_p$, $\vec{k}_3 - \Delta\vec{k}$.

[9]W. Ascoli-Bartoli, J. Katzenstein, and L. Lovisette, Bull. Am. Phys. Soc. 9, 495 (1964).

PULSE-SHAPE DISCRIMINATION ON THE GAMMA-RAY PULSES FROM $F^{19}(d,n\gamma)Ne^{20}$ OBSERVED WITH A LITHIUM-DRIFTED GERMANIUM GAMMA-RAY SPECTROMETER

T. K. Alexander, J. D. Pearson, A. E. Litherland, and C. Broude
Chalk River Nuclear Laboratories, Chalk River, Ontario, Canada
(Received 3 June 1964)

Lithium-drifted germanium gamma-ray spectrometers have been applied to the detection of high-energy gamma rays by Ewan and Tavendale.[1] They have found that the absorption of the gamma rays by pair production produces a very narrow peak ($\lesssim$0.2 % full width at half-maximum) at an energy (E_γ-$2m_0c^2$) that is superimposed on a background due to Compton scattering and electron escape. We are using one of these counters in a high-resolution study of the high-energy (5 MeV < E_γ < 12 MeV) gamma rays from the $F^{19}(d,n\gamma)Ne^{20}$ reaction[2] in order to obtain further information on the nuclear levels of Ne^{20} at high excitation energy.[3] In this Letter it is shown that the background under the (E_γ-$2m_0c^2$) peaks in a complex gamma-ray spectrum can be reduced considerably by pulse-shape discrimination against detector pulses containing a slow time-constant component. The resulting improvement in the quality of spectra will be advantageous in further studies of the $F^{19}(d,n\gamma)Ne^{20}$ reaction and is of general interest to experimenters using solid-state gamma-ray spectrometers.

The detector used for the measurements was a 5-mm deep by 19-mm diameter lithium-drifted germanium p-i-n diode fabricated by A. J. Tavendale. The compensated intrinsic region reached a thin aluminum layer on one face of the detector giving a thin window into the depleted region of the detector. The other face of the detector was over-compensated n-type germanium approximately 0.5 mm thick.

The pulse-shape discrimination tests were carried out with the circuit described by Alexander and Goulding[4] for pulse-shape discrimination of signals from an organic scintillator. The method is directly applicable to the solid-state detector, since the circuit is designed to accept pulses from the type of charge-sensitive preamplifier used with solid-state detectors. The geometry of the source and detector was such that the detector's edge was illuminated by a source of reaction gamma rays 4 mm in diameter and 9 cm away from the center of the detector. With this geometry, the (E_γ-$2m_0c^2$) peaks should be enhanced, since the high-energy electron-positron pairs recoil forward and have a greater chance of stopping completely in the depletion region.

Figure 1 compares the spectrum of high-energy gamma rays from the $F^{19}(d,n\gamma)Ne^{20}$ reaction with and without pulse-shape discrimination. The top curve (dots) is an ungated spectrum of pulses from the detector, whereas the lower curve (crosses) is the same spectrum, recorded simultaneously, but gated by pulse-shape selection. The background is reduced by a factor of approximately 2.5, whereas the prominent (E_γ-$2m_0c^2$) peaks are attenuated only by about 10 %. The slight shift in the peak positions is not due to pulse-shape selection but arises because the spectra were analyzed simultaneously in two analog-to-digital converters which had slightly different gains. The energy calibration shown on the right-hand side of the diagram applies only to the gated spectrum. These preliminary data indicate that a considerable improvement of the spectrum can be realized using a pulse-shape discriminator that rejects pulses having a slow time-constant component.

The data shown in Fig. 1 are not suitable for illustrating the detector resolution since the gamma rays are Doppler broadened due to the kinematics of the $F^{19}(d,n)Ne^{20*}$ reaction and the short lifetimes of the Ne^{20} states relative to the slowing down time of the recoiling Ne^{20} ions. Although the intrinsic resolution of the detector at a gamma-ray energy of 5.3 MeV is $\lesssim$10 keV (full width at half maximum), the widths of the peaks in Fig. 1 are typically 60 keV, showing that the gamma rays are not monoenergetic. The 6.13-MeV gamma ray in Fig. 1 follows the alpha decay of excited states of Ne^{20} and has less Doppler broadening (30 keV) because of the long