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$Y=2$ STATES IN SU(6) THEORY*

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(Received 30 November 1964)

Two-baryon states.—The SU(6) theory of strongly interacting particles[1,2] predicts a classification of two-baryon states into multiplets according to the scheme

$$\underline{56}\otimes\underline{56}=\underline{462}\oplus\underline{1050}\oplus\underline{1134}\oplus\underline{490}. \quad (1)$$

Because baryons are Fermions, only the two antisymmetric representations

$$\underline{1050}=[41000],\ \underline{490}=[03000] \quad (2)$$

can be realized in nature. The decomposition of these representations into [SU(3)⊗SU(2)] multiplets is as follows:

$$\underline{1050}=(\underline{28},\underline{5})\oplus(\underline{35},\underline{3}\oplus\underline{5}\oplus\underline{7})\oplus(\underline{27},\underline{1}\oplus\underline{3}\oplus\underline{5})\oplus(\underline{10}^*,\underline{3})\oplus(\underline{10},\underline{3}\oplus\underline{5})\oplus(\underline{8},\underline{1}\oplus\underline{3}), \quad (3)$$

$$\underline{490}=(\underline{28},\underline{1})\oplus(\underline{35},\underline{3})\oplus(\underline{27},\underline{1}\oplus\underline{5})\oplus(\underline{10}^*,\underline{3}\oplus\underline{7})\oplus(\underline{10},\underline{3})\oplus(\underline{8},\underline{3}\oplus\underline{5})\oplus(\underline{1},\underline{1}). \quad (4)$$

In each case the states of hypercharge $Y=2$ belong to a unique SU(4) multiplet, namely

$$\underline{1050}\to\underline{140}=(7,5)\oplus(5,3+5+7)\oplus(3,1+3+5)\oplus(1,3), \quad (5)$$

$$\underline{490}\to\underline{50}=(7,1)\oplus(5,3)\oplus(3,1+5)\oplus(1,3+7). \quad (6)$$

The symbols on the right of Eqs. (5) and (6) denote values of $(2T+1, 2J+1)$ of particles with $Y=2$ belonging to the corresponding multiplet.

We now propose the hypothesis that all low-lying resonant states of the two-baryon system belong to the $\underline{490}$ multiplet.[3] This means that six zero-strangeness states shown in Table I should be observed. In all these states odd T goes with even J and vice versa.

The main experimental support for our hypothesis is the fact that the triplet and singlet deuteron states [(3,1)⊕(1,3)] appear in the low-mass region only once. Therefore the splitting between $\underline{1050}$ and $\underline{490}$ must be large, and one of the two must be chosen for the low-lying states. We choose $\underline{490}$ because there is no evidence for the existence of a state (3,3) which should occur in $\underline{1050}$ at a mass within the range of accurate p-p scattering measurements.[4]

The states with $Y=2$ within the SU(6) multiplet $\underline{490}$ or $\underline{1050}$ all belong to SU(3) multiplets of the type $[2T, 3-T]$ and therefore have the SU(3) Casimir operator given by

$$C_2^{(3)}=12+2T(T+1). \quad (7)$$

They all have $S=0$, $N=J$, and their SU(4) Casimir operator $C_2^{(4)}$ reduces to a constant. The general mass formula Eq. (22) of Bég and Singh[5] thus collapses into

$$M=A'+B'T(T+1)+B''J(J+1). \quad (8)$$

Table I. $Y=2$ states with zero strangeness predicted by the $\underline{490}$ multiplet.

Particle	T	J	SU(3) multiplet	Comment	Predicted mass
D_{01}	0	1	$\underline{10}^*$	Deuteron	A
D_{10}	1	0	$\underline{27}$	Deuteron singlet state	A
D_{12}	1	2	$\underline{27}$	S-wave N-N^* resonance	$A+6B$
D_{21}	2	1	$\underline{35}$	Charge-3 resonance	$A+6B$
D_{03}	0	3	$\underline{10}^*$	S-wave N^*-N^* resonance	$A+10B$
D_{30}	3	0	$\underline{28}$	Charge-4 resonance	$A+10B$

The observed degeneracy of D_{01} and D_{10} gives

$$B'=B''. \tag{9}$$

Hence the mass formula may be written

$$M=A+B[T(T+1)+J(J+1)-2], \tag{10}$$

where $A=1876$ MeV is the deuteron mass. The value of B is unpredictable from group theory; however, it may be expected to be related to the coefficients in the mass formula for the baryon multiplet, which would give a value around 50 MeV. The masses predicted for the particles of Table I are then within an easily observable range.

The following are examples of some experiments in which the four newly predicted states should be found.

(a) D_{12} may be found in phase-shift analysis of p-p scattering.[6] The singlet D-wave resonance would be predominantly inelastic if $B > 50$ MeV, and would decay into N^*+N.

(a′) D_{12}^{++} may also be found in the reaction $p+p\rightarrow D_{12}^{++}+\pi^0$.

(b) D_{21}^{+++} may be found in four-prong events produced by the reaction

$$p+p\rightarrow D_{21}^{+++}+\pi^-,$$

$$D_{21}^{+++}\rightarrow N^{*+++}+p\rightarrow p+p+\pi^+.$$

(c) D_{03} may be found in phase-shift analysis of n-p scattering.[7] The triplet D-wave $J=3$ resonance would be predominantly inelastic if $B>60$ MeV and would decay into N^*+N^* (s wave).

(c′) D_{03}^{+} may also be observed in the reaction $p+p\rightarrow D_{03}^{+}+\pi^+$.

(d) D_{30}^{++++} may be found in six-prong events produced by the reaction

$$p+p\rightarrow D_{30}^{++++}+\pi^-+\pi^-,$$

$$D_{30}^{++++}\rightarrow N^{*++}+N^{*++}\rightarrow p+p+\pi^++\pi^+.$$

The fact that the D_{30}^{++++} has $J=0$ could here be checked by measuring the angular correlation between the two π^+, which should be approximately $(1+3\cos^2\theta)$ in the D_{30} rest system.

One-baryon states. —The one-meson one-baryon states are classified in SU(6) according to the scheme[2,8]

$$\underline{35}\otimes\underline{56}=\underline{56}\oplus\underline{70}\oplus\underline{700}\oplus\underline{1134}.$$

$Y=2$ states occur only in the representations $\underline{700}$ and $\underline{1134}$. The break-down of these states into $[\mathrm{SU}(4)\otimes\mathrm{SU}(2)]$ and $(2T+1, 2J+1)$ multiplets is as follows:

$$\underline{700}\rightarrow(\underline{35}\otimes\underline{2})=(5,4+6)\oplus(3,2+4)\oplus(1,2), \tag{11}$$

$$\underline{1134}\rightarrow(\underline{45}\otimes\underline{2})=(5,2+4)\oplus(3,2+2+4+4+6)\oplus(1,2+4). \tag{12}$$

In this case there is no experimental evidence favoring one representation rather than the other, but for the sake of simplicity we may hope that low-lying resonances belong to $\underline{700}$, in which T and J are always related[9] by $J=T\pm\frac{1}{2}$. The states of $\underline{700}$ with $Y=2$ all have $S=\frac{1}{2}$, $N=T$, and therefore their mass formula[5] again reduces to the form (8). For 1134 no such simple formula applies. Experimental evidence for these states may be sought in K^+-p and p-p interactions.

Zero-baryon states. —Two-meson states in SU(6) belong to the scheme[2]

$$\underline{35}\otimes\underline{35}=\underline{1}\oplus\underline{35}\oplus\underline{35}\oplus\underline{189}\oplus\underline{280}\oplus\underline{280}^*\oplus\underline{405},$$

only the symmetric representations $\underline{1}\oplus\underline{35}\oplus\underline{189}\oplus\underline{405}$ being allowed by Bose statistics. States with $Y=2$ occur in $\underline{189}$ and $\underline{405}$ according to the rules

$$\underline{189}\rightarrow(\underline{6}\otimes\underline{1})=(1,3)\oplus(3,1), \tag{13}$$

$$\underline{405}\rightarrow(\underline{10}\otimes\underline{3})=(1,3)\oplus(3,1+3+5). \tag{14}$$

A possible resonance[10] in the K^+K^+ system at a mass of 1250 MeV presumably belongs to the component (3, 1) and could lie in either 189 or 405. The $Y=2$ states of 405 have $S=1$, $N=T$, and therefore again obey a mass formula of the form (8). Experimental evidence for these states may be sought in reactions of the type

$$K^+ + p \to \Lambda + M_1^{++}, \quad K^+ + p \to \Sigma^+ + M_0^+.$$

One of us (F.D.) wishes to thank the Physics Department of the University of California, San Diego, for hospitality during the time this work was done.

Note added in proof.—We have found already existing experimental data[11] concerning the resonance D_{12} which is clearly seen in the reaction $\pi^+ + d \to p + p$. The mass is 2160 MeV and the full width at half maximum 120 MeV. The angular distribution of the protons is observed to be $[(0.31 \pm 0.03) + \cos^2\theta]$ at resonance, in agreement with the assignment $J = 2$. Mandelstam[12] obtained a theoretical description of this reaction in terms of compound states $(N + N^*)$; his theory is also consistent with the interpretation in terms of an SU(6) multiplet. There is a discrepancy between the SU(6) prediction and a phase-shift analysis[13] of p-p scattering at 660 MeV which shows a small $^1D_2$ phase shift; experiments at other energies are probably needed in order to resolve the discrepancy.

Assuming that the observed 2160 resonance is in fact D_{12}, the parameter B in the mass formula is 47 MeV, in agreement with the value derived from the single-baryon multiplet (56 multiplet). The states D_{03}, D_{30} are then predicted to lie at 2350 MeV.

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*Work done under the auspices of the U. S. Atomic Energy Commission.

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