results at 4.2°K show a net moment of 0.35 μ_B/Fe atom in a field of 30.8 kOe, which could result from a bending of moments away from the direction of antiferromagnetic alignment. His zero-field moment of 0.04 μ_B per Fe atom is also typical of weak ferromagnetism. Effects similar to this have been observed in weak ferromagnets such as NiF_2 and Fe_2O_3.[9] A curious feature of $Fe_{05}Au_{95}$ is the positive paramagnetic Curie temperature of +23°K deduced from the high-temperature susceptibility[1] which is in precise agreement with the transition temperature we find from our Mössbauer measurements.

In an effort to study the region of the magnetic transition we have performed measurements with and without external fields at temperatures near to and above the transition temperatures. External fields are found to induce magnetization even above the transition, yielding families of hyperfine splitting vs temperature in various fields which are similar to those found in ferromagnetic $Fe_{2.65}Pd_{97.35}$ by Segnan, Craig, and Perisho.[10] At temperatures above about 10°K the hyperfine spectra broaden and it becomes impossible to distinguish lines and hence to make polarization determinations. This broadening is indicative of a spectrum of Fe-Fe coupling strengths depending upon the number of Fe neighbors in the random alloy. In order to obtain more detailed information in the region of the transition we are preparing to employ higher fields and to investigate more dilute alloys, which are not expected to show such severe line broadening. In addition, neutron-diffraction studies are planned in order to more fully characterize the magnetic structure in this alloy system.

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VISCOUS FLOW OF FLUX IN TYPE-II SUPERCONDUCTORS*

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It has been shown[1,2] that when the Lorentz driving force $(\vec{J}\times\vec{\varphi}_0/c)$ on the flux quanta in a current-carrying superconductor in a transverse magnetic field exceeds the depinning threshold, a voltage appears along the superconductor. An incremental resistivity ρ_f may be defined, independent of the amount of pinning. This voltage arises by induction from the driven flow of quantized flux vortices across the sample, opposed by viscous forces, assumed of the form $-f\vec{v}_\varphi$, where $\vec{v}_\varphi$ is the velocity of the vortex with respect to the sample. In that case, it has been shown[1,2] that this "flow" resistivity is

$$\rho_f = dE/dJ = B\varphi_0 fc^2, \qquad (1)$$

which reduces the problem to analysis of the viscous drag coefficient f. The present note discusses the origin of f: the normal electron part, previously discussed by Volger, Staas, and van Vijfeijken,[3] as well as by Strnad, Hempstead, and Kim,[2] and a new mechanism which seems to dominate when $B \ll H_{c2}$.

Although (1) is conventionally derived for the geometry in which the applied field $\vec{H}_a$ is parallel to the surface of the strip, it is also val-

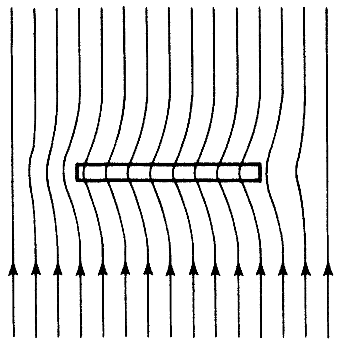

FIG. 1. Schematic cross-sectional diagram of field lines passing normally through a strip of type-II superconductor carrying uniform transport current into paper. Driving force tending to move flux lines arises from line tension trying to straighten lines rather than from gradient of line density.

id when $\vec{H}_a$ is perpendicular to the strip, the driving force then coming from the line tension $B\varphi_0/4\pi$ in the "bent" field lines rather than from a field gradient. This is illustrated in Fig. 1. We consider only the perpendicular case, since only in it are the experimental data free from complications due to the surface superconductivity of Saint James and de Gennes.[4] In this geometry, there are complications from the intermediate state at low fields, but for $H_a \gtrsim H_{c1}$ the sample will be essentially uniformly filled with fluxoid vortices. Since the current patterns start to overlap when $B \gtrsim H_{c1}$, the internal magnetic field will then be reasonably homogeneous even on a microscopic scale, and not too different from the applied field, for a thin sample.

Generalizing the treatment of the normal electron drag by Volger, Staas, and van Vijfeijken, we obtain

$$\left(\frac{\rho_f}{\rho_n}\right)_{\text{n.el.}} = \frac{B\varphi_0}{\int(\sigma_1/\sigma_n)H^2 dS} = \frac{\langle H\rangle^2}{\langle(\sigma_1/\sigma_n)H^2\rangle}, \tag{2}$$

where the first form is valid for isolated flux lines (the integration being over the fields of a single line), and the second is valid even if the vortices overlap. It is obtained from the first form by assigning the total drag equally to all overlapping vortices. We have also generalized their result by introducing σ_1 as the real part of the frequency-, temperature-, and field-dependent complex conductivity of the superconductor, while σ_n is the residual conductivity in the normal state. As usual, $B = \langle H\rangle$. We note at once that this expression reduces correctly to the normal resistivity at H_{c2}, where H becomes uniform and $\sigma_1 \to \sigma_n$.

Throughout the field region $B > H_{c1}$, the field is approximately uniform through the material, and (2) reduces approximately to $\langle\sigma_1/\sigma_n\rangle^{-1}$. This average is field dependent, since a fraction $\sim b \equiv B/H_{c2}$ of the material is in the vortex cores of radius $\sim\xi(T)$. In these cores, one expects $\sigma_1 \approx \sigma_n$, in view of the result of Caroli, de Gennes, and Matricon[5] that there is a density of localized low-lying states associated with each vortex of the order of the density of states of a cylinder of normal metal of radius ξ. Outside the core, we treat σ_1/σ_n as a temperature- and field-dependent constant $(\sigma_1/\sigma_n)_0$, which is expected to go to zero at $T = 0$ and to unity at T_c, varying approximately as T/T_c at intermediate temperatures. (It will probably exceed unity near the transition in some cases because of BCS coherence-factor effects.[6]) Thus, we expect

$$\left(\frac{\rho_f}{\rho_n}\right)_{\text{n.el.}} \cong \left\langle\frac{\sigma_1}{\sigma_n}\right\rangle^{-1} \approx \left[b + (1-b)\left(\frac{\sigma_1}{\sigma_n}\right)_0\right]^{-1}, \tag{3}$$

which will generally be greater than unity, contrary to the experimental fact that $\rho_f < \rho_n$. To correct this difficulty, we are led to seek another contribution to the viscous constant f in (1).

The new mechanism which we suggest is based on the assumption of an intrinsic relaxation time τ governing the approach of the superconducting wave function ψ toward its equilibrium form, when it is forced to vary in time by the motion of a vortex line. In a dirty superconductor, τ should be of the order of the time required for an electron to diffuse over a distance $\xi(T)$. This leads to

$$\tau \sim \hbar/\Delta_{00}(1-t), \tag{4}$$

where Δ_{00} is the gap parameter at $T = 0$ and zero field, and $t = T/T_c$. In the linear approximation, this τ should be identified with the characteristic time for spontaneous nucleation of the superconducting state, as discussed by de Gennes.[7] He has shown that $1/\tau$ is $\Delta_{00}/\hbar$ at $T = 0$, and that for $t > 0$ it falls according to his "universal function" $U(t)$ based on the difference of two digamma functions. For analytic convenience, it is worth noting that the temperature dependence of $1/\tau$ is approximated moderately well[8] by $(1-t^2)/(1+t^2)$. In any case, except very near T_c, $1/\tau$ is of the order of the gap frequency, so that $\omega\tau \approx \omega/\omega_g \ll 1$ for typical processes, and the departures from instantaneous equilibrium are expected to be small.

To make our assumption concrete, consider the leading term in the Ginzburg-Landau energy expression, $F = -\alpha|\psi|^2$. If ψ varies between ψ_0 and 0 at frequency ω, lagging by a time τ behind the instantaneous equilibrium value appropriate to the moving Abrikosov structure, then averaged over a cycle, the power dissipation will be of order $\langle F\rangle(\psi_0^{-1}d\psi/dt)^2\tau \approx \omega^2\tau\langle F\rangle$, for $\omega\tau \ll 1$. The physical interpretation of the $\omega^2\tau$ factor is that the electrons evaporate and condense from the ordered state $\omega/2\pi$ times per second, gaining and losing the condensation energy each time, and each time a net fraction $\omega\tau$ of this energy is dissipated to the production and/or heating of normal electrons because of the nonequilibrium situation required to produce a finite rate of change of ψ. One might say that the electrons are forced to pair and de-pair with respect to different pairing potentials, just as on the average the molecules of a liquid in near-equilibrium with its vapor evaporate and condense at different pressures during a cyclic volume change of the container, leading to an irreversible energy conversion.

In applying this model to moving flux lines, we approximate $\psi(\vec{r})$ by a constant value ψ_0 outside the core of radius ξ, within which ψ goes smoothly to zero at the center. If this core is moving with velocity v_φ, the characteristic frequency for the variation of ψ will be $\omega \approx v_\varphi/\xi$. The characteristic energy density will be

$$\alpha|\psi_0|^2 = \frac{H_{cb}^2(T)}{4\pi}\left|\frac{\psi_0(H,T)}{\psi_0(0,T)}\right|^2 \equiv \frac{H_{cb}^2(T)}{4\pi}\chi^2(H,T),$$

where $\chi^2(H,T)$ is expected to vanish linearly with field near H_{c2}. Because the drop in $\langle\psi^2\rangle$ at low fields is largely accomplished by bringing in more cores, rather than by lowering ψ_0, it seems reasonable to approximate $\chi^2(H,T)$ by $1-b^2$, rather than $1-b$. Since the cross sectional area of the core is $\sim\pi\xi^2$, the power dissipation per unit length of vortex is expected to be

$$P_\tau = a\frac{H_{cb}^2(T)}{4\pi}\chi^2(H,T)\frac{v_\varphi^2}{\xi^2}\tau(\pi\xi^2), \qquad (5)$$

where a is a numerical constant of order unity. Equating this dissipation to $f_\tau v_\varphi^2$, and inserting the resulting f_τ in (1), we obtain the flow resistivity if only this mechanism were present. To facilitate comparison, we transform this using theoretical relations appropriate for dirty superconductors:

$$\sigma_n = \frac{ne^2l}{mv_f} = \frac{c^2l}{4\pi v_F\lambda_L^2(0)} = \frac{c^2}{4\pi^2}\frac{\hbar}{\Delta_{00}\lambda^2(0)},$$

where $\lambda_L(0)$ is the London parameter at $T=0$, $(mc^2/4\pi ne^2)^{1/2}$, while λ is the actual penetration depth in the dirty material. We also use the relation from Ginzburg-Landau theory that $H_{c2}(T) = 4\pi H_{cb}^2(T)\lambda^2(T)/\varphi_0$, with the result that

$$\left(\frac{\rho_f}{\rho_n}\right)_\tau = \frac{4}{a\pi}\left(\frac{B}{H_{cb}(T)}\right)\left(\frac{\hbar}{\Delta_{00}\tau(T)}\right)\frac{\lambda^2(T)}{\lambda^2(0)}\frac{1}{\chi^2(H,T)}. \qquad (6)$$

Inserting the analytic approximation given above for the temperature dependence of τ, approximating $\lambda^2(0)/\lambda^2(T)$ by the two-fluid expression $(1-t^4)$, and approximating $\chi^2(H,T)$ by $1-b^2$, where $b = B/H_{c2}$, we obtain

$$\left(\frac{\rho_f}{\rho_n}\right)_\tau = \frac{4}{a\pi}\frac{1}{(1+t^2)^2}\frac{b}{(1-b^2)}. \qquad (7)$$

We note at once that this result has several very desirable features: (1) It is independent of κ, in qualitative agreement with experiment. (2) It correctly gives the order of magnitude unity for (ρ_f/ρ_n). (3) At H_{c2}, the new damping mechanism drops out smoothly because it depends on the presence of a superconducting condensate. This leaves only the normal electron contribution, which we have already seen accounts naturally for the behavior at H_{c2}.

We conclude by displaying representative curves obtained by combining the two mechanisms discussed above. In (7), let us simply set $(a\pi/4) = 1$. Noting that the total drag coefficient $f = f_{\text{n.el.}} + f_\tau$, we see that the flow <u>conductances</u> add. Combining the two mechanisms,

$$\sigma_f/\sigma_n = b + (1-b)(\sigma_1/\sigma_n)_0 + (1+t^2)^2(b^{-1}-b), \qquad (8)$$

and ρ_f/ρ_n is the reciprocal of this. As a crude estimate[9] for $(\sigma_1/\sigma_n)_0$ which should be valid at least at $t=0$ and $t=1$, we take $(\sigma_1/\sigma_n)_0 = t$. This leads to

$$\frac{\rho_f}{\rho_n} = \frac{b}{(1+t^2)^2(1-b^2)+b^2+(b-b^2)t}, \qquad (9)$$

which at $t=0$ reduces to simply $\rho_f/\rho_n = b \equiv B/$

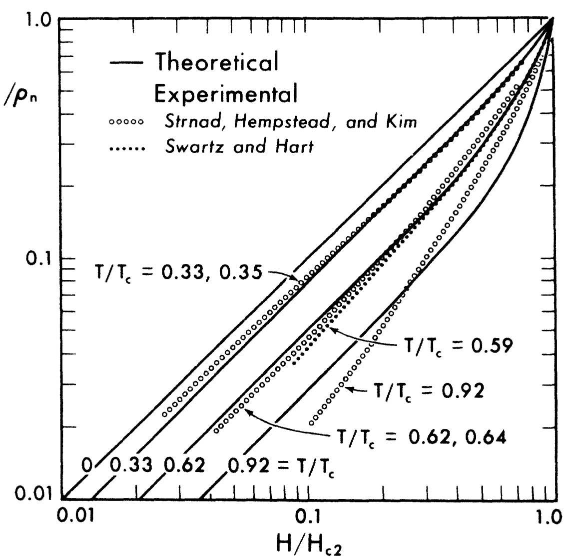

FIG. 2. Comparison of present theory [Eq. (9)] with experimental results for normalized flow resistivities from references 2 and 10. If only normal electron damping were present, (ρ_f/ρ_n) would rise above unity rather than falling below. Here we have taken $B = H$, a quite good approximation in this geometry. The data of Strnad, Hempstead, and Kim are on various Nb-Ta alloys; the data of Swartz and Hart are on a Pb-Tl alloy. Note that the experimental data do not show any strong dependence on κ or on the alloy system, in agreement with the present theory.

H_{c2}. The result (9) is compared with experimental data[2,10] on various alloys in Fig. 2. There is remarkably good agreement, considering the crudeness of the various estimates used above.

In closing it should be emphasized that the arguments in this paper are at best semiquantitative, and that more rigorous microscopic calculations along these lines will be required before any detailed agreement with experimental data can be expected. For example, the "two-fluid" analytic dependences should be replaced by appropriate results of microscopic theory. However, it is hoped that the ideas are qualitatively correct, and that the lack of mathematical complication will compensate for the lack of rigor.

Although we have discussed only a single application, it seems quite possible that the loss mechanism described here may also be important in other applications, such as microwave and other ac loss experiments, in the presence of either an applied magnetic field or trapped flux.

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