inversely proportional to the potential of G and C during flushout (between −20 and −100 V). The corresponding axial mobility of negative ions trapped in vortex lines is proportional to $\exp(\Delta/T)$ with $\Delta = 5.3 \pm 0.4°K$, and is perhaps one third of the free-negative-ion mobility at 1.2°K.

The absolute value of trapped-ion mobility cannot be accurately deduced by comparing free- and trapped-ion transit times to C' because free ions follow lines of force whereas trapped ions move only along vortex lines. Moreover, free and trapped ions may start upward from somewhat different heights between G and C. In equilibrium, the trapped ions will distribute themselves so that they are not subject to vertical fields. The location of the region of vanishing vertical field depends upon electrode potentials. The data points marked X, below the upper curve of Fig. 3, show the observed effect of a reduction in the potential V_C of collector C prior to flushup: The trapped ions float higher and their subsequent transit time to C' is reduced. For given electrode potentials, the transit times to C' are inversely proportional to mobilities, but the constants of proportionality are unknown and differ for free and trapped ions.

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[1]G. Careri, W. D. McCormick, and F. Scaramuzzi, Phys. Letters 1, 61 (1962).

[2]D. J. Tanner and R. J. Donnelly, Proceedings of the Ninth International Conference on Low Temperature Physics, Columbus, Ohio, 1964 (to be published).

[3]G. W. Rayfield and F. Reif, Phys. Rev. Letters 11, 303 (1963).

[4]F. Reif and Lothar Meyer, Phys. Rev. 119, 1164 (1960).

DISSIPATIVE MECHANISM IN TYPE-II SUPERCONDUCTORS

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Basic to the theory of type-II superconductors is Abrikosov's notion of the quantized flux lines, or vortices of superelectrons.[1] Such concepts as flux pinning, flux creep, and flux flow, that have been very useful in the understanding of hard superconductivity,[2] all envision the motion of Abrikosov flux lines. However, no satisfactory theoretical explanation has yet been advanced to account for the dissipative mechanism associated with the motion of flux lines. In this report we wish to direct theoretical interest to the simple, universal facts that have emerged from a large body of experimental data on the flux-flow state of type-II superconducting alloys.

Resistance in the mixed state has been measured with specimen geometry shown in Fig. 1. The upper critical field H_{c2} is determined resistively by orienting the externally applied magnetic field H parallel to the broad surfaces of the sample.[3] The resistance data reported here are, however, taken with H perpendicular to the sample surface. At a fixed field H, the voltage appearing in the direction of the current (longitudinal voltage) is measured as a function of current I. A detectable voltage appears when I exceeds the critical current; V then increases rapidly and becomes nearly linear in I. Although the critical currents vary considerably depending on the defect structure of the specimen, the slope $\Delta V/\Delta I$ in the linear region is found to be structure insensitive. In the upper figure are shown $V(I)$'s for two $Nb_{0.5}Ta_{0.5}$ samples containing different amounts of defects. From this observation, one would infer the $V(I)$ of a defect-free specimen to be the straight line passing through the origin. It is difficult in practice to prepare a sample in this ideal state, but its resistance can readily be determined from $\Delta V/\Delta I$ even in the presence of defects. We call the resistivity determined in this way the flow resistivity ρ_f. The structure-insensitive nature of ρ_f is further emphasized in the lower figure for two $Pb_{0.83}In_{0.17}$ samples. In one sample the copper plating has altered flux-pinning conditions (presumably on the surfaces) so as to increase the critical current at low fields but to decrease it at high fields. Note, however, that the flow resistivity at given field strength is practically

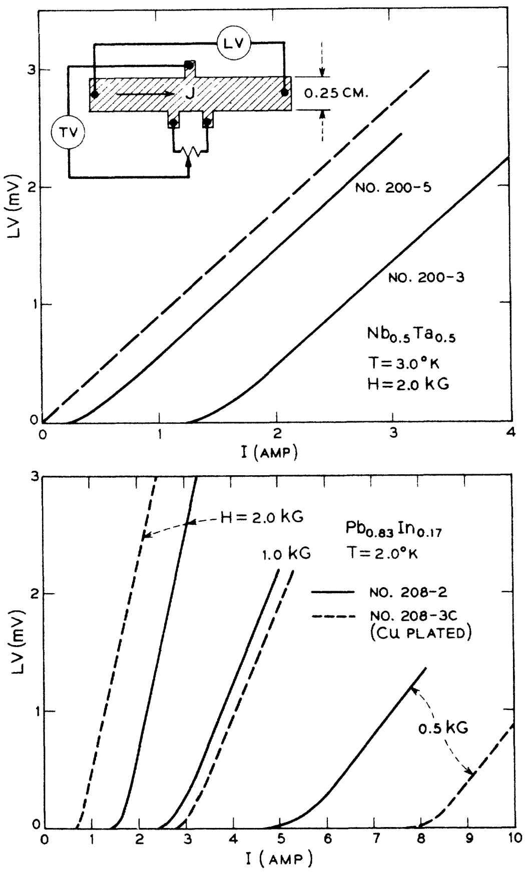

FIG. 1. Voltage vs current characteristics. The insert of the upper figure shows the geometry of a sheet sample on which voltages in two directions ($\parallel J$ and $\perp J$) are measured. The solid lines are $V(I)$ characteristics of two Nb-Ta samples containing different amounts of defects. Although the critical currents are different for these two samples, the slopes $\Delta V/\Delta I$ in the linear region are the same. The dashed line is the $V(I)$ expected for a defect-free sample. In the lower figure are shown $V(I)$'s for two Pb-In samples: one with copper plating on the surfaces and the other without.

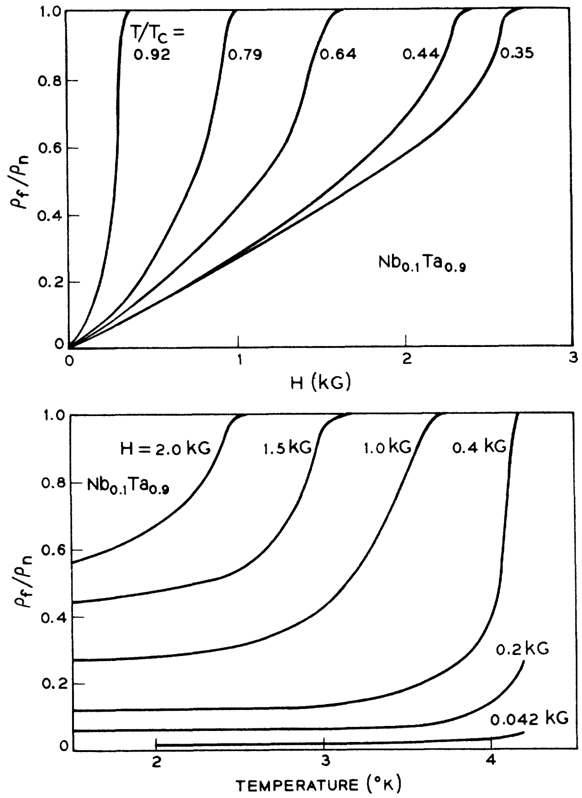

FIG. 2. Flow resistivity vs H and T. The flow resistivity ρ_f is determined from the slope $\Delta V/\Delta I$. The ratio ρ_f/ρ_n is displayed as a function of H at various values of T/T_c and as a function of T at various values of H.

unchanged by the plating.

The flow resistivity thus reflects the bulk properties of a type-II superconductor, and its dependence on H and T is of great interest in studying the dissipative mechanism in the flow region. Measurements have been made on many different compositions of Nb-Ta and Pb-In alloys. In Fig. 2 we show as a representative case the data obtained on $Nb_{0.1}Ta_{0.9}$ in the form of $\rho_f/\rho_n(H, T)$, ρ_n being the normal resistivity. The most important features common to type-II superconductors are the following: (a) At $T \ll T_c$ the flow resistivity increases linearly in H, and (b) in the presence of magnetic fields the flow resistivity persists down to zero temperature. It is also significant to note that an approximate law of corresponding states holds, i.e., $\rho_f/\rho_n(H/H_{c2}, T/T_c)$ is independent of alloy composition. This is shown in Fig. 3 for Nb-Ta alloys. As the reduced temperature T/T_c approaches zero, we obtain the simple result

$$\rho_f/\rho_n = H/H_{c2}. \tag{1}$$

Originally[3,4] the flux-flow state was characterized by the relation

$$F = J\varphi_0/c = \eta v, \tag{2}$$

where F is the Lorentz force on a flux line of unit length, $\varphi_0 = hc/2e$ is the flux quantum, and $\eta(H, T)$ is the viscosity coefficient characterizing the bulk superconducting properties of the specimens. The power dissipation den-

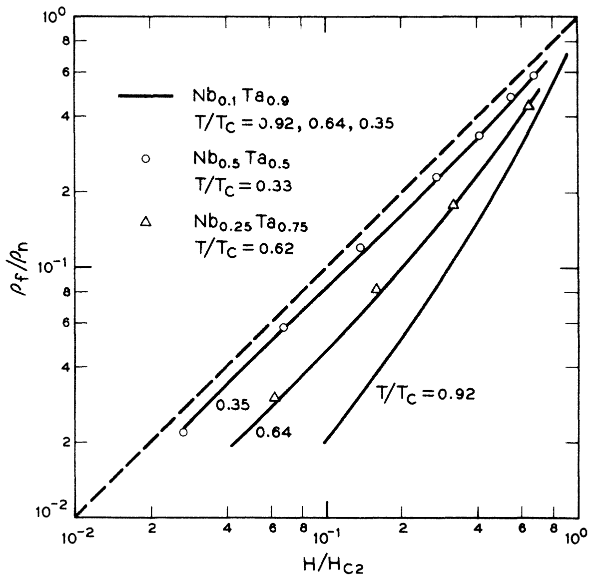

FIG. 3. Flow resistivity vs H/H_{c2} and T/T_c. ρ_f/ρ_n is displayed as a function of H/H_{c2} at given values of T/T_c. The solid lines are for the $Nb_{0.9}Ta_{0.1}$ sample of Fig. 2. The circles and triangles represent the data for two other Nb-Ta compositions.

sity arising from the viscous flow is

$$P=nFv=(\varphi_0/\eta c^2)(n\varphi_0)J^2, \tag{3}$$

where n is the density of flux lines and $n\varphi_0 = B \simeq H$. If this is the sole source of power dissipation, it should be equated to the measured power:

$$E_0 J = \rho_f J^2 = (\varphi_0/\eta c^2)BJ^2;$$

or

$$\rho_f = (\varphi_0/\eta c^2)B \simeq (\varphi_0/\eta c^2)H. \tag{4}$$

In this formulation the mean electric field $E_0 = vB/c$ is regarded as arising from the motion of flux lines. However, the viscosity coefficient η is left as a free parameter and the correct damping mechanism should lead to the empirical relation [from (1) and (4)]

$$\eta \simeq \varphi_0 H_{c2}/\rho_n c^2. \tag{5}$$

In relating η to ρ_n, Volger, Staas, and Van Vijfeijken[5] assumed that the electric field associated with moving vortices, $E=vH/c$, generates normal eddy currents. The dissipation arising from this mechanism is

$$P=n\int(E^2/\rho_n)dxdy = (nv^2/\rho_n c^2)\int H^2 dxdy,$$

and it leads to

$$\eta = (\rho_n c^2)^{-1}\int H^2 dxdy, \tag{6}$$

where the integration covers the extension of one flux line. The order of magnitude of this expression may be estimated by considering the line energy[1]

$$\epsilon = \int(H^2/8\pi + E_k)dxdy \simeq (\varphi_0 H_{c2}/16\pi^2)(\ln\kappa)/\kappa^2, \tag{7}$$

an approximate expression valid for $\kappa = \lambda/\xi \gg 1$. Clearly,

$$\int H^2 dxdy < 8\pi\epsilon = \varphi_0 H_{c2}(\ln\kappa)/2\pi\kappa^2,$$

and η given by (6) is smaller at least by a factor $(\ln\kappa)/2\pi\kappa^2$ than the empirical relation (5). Actually, at $T \ll T_c$ the normal carriers are confined into the vortex core of area $\sim\xi^2$, reducing η by another factor κ^2. Thus the damping due to normal eddy currents as considered here is far too small to account for the observed dissipation.

In type-II superconductors the reduced variable H/H_{c2} can be related to a simple geometrical relation

$$H/H_{c2} \simeq \xi^2/d^2, \tag{8}$$

where ξ is the coherence length and d is the distance between the flux lines. This follows from $H \simeq B = n\varphi_0 = \varphi_0/d^2$, and $H_{c2} \simeq \varphi_0/\xi^2$. The meaning of (8) is particularly significant in light of the recent calculation by Caroli, De Gennes, and Matricon,[6] who have shown that at the central region of a flux line the energy gap has nearly vanished and the vortex core is practically equivalent to a cylinder of normal metal of radius ξ. The ratio (8) thus represents the fraction of normal volume occupied by the vortex cores. It is then tempting to attribute the empirical relation (1) to a simple configuration effect—the electrical resistance is contributed only by the normal cores. In fact, Rosenblum and Cardona[7] have emphasized this point in their interpretation of the microwave surface resistance of type-II superconductors. At microwave frequencies, they find the resistance to be structure insensitive and its dependence on H and T quite similar to that shown in Fig. 2. Thus the relation (1) appears to manifest a very basic dissipative mechanism in type-II superconductors.

This remarkably simple result could be understood if the current density were approximately constant throughout the superconductor, including the vortex-core regions. This would obviously not be possible if the vortex cores were stationary normal regions, since then the current would bypass them entirely. In fact, the condition for equilibrium of a vortex structure is precisely that the current vanish at the vortex core, since $F = J \times H$; thus pinned vortices will not need to have any current flow in the cores. No such condition is valid for flowing vortices since they must sustain a net Lorentz force. The assumption of uniform current flow also requires that the electric field be concentrated in the vortex-core regions and its magnitude be much larger than $(v/c)\times H$. Anderson[8] has pointed out that there must exist electric fields additional to $(v/c)\times H$ since this field in the presence of the inhomogeneous normal conductivity will not satisfy current conservation, so that space charges must build up; but it is by no means obvious why these might lead to approximate homogeneity of current. The only presently understandable region is that near H_{c2} at reasonable T. At this point uniform E and J must be expected and are not incompatible; the sharp drop in resistivity is probably explained by Tinkham's[9] observation that ρ_n actually drops at the transition point, rather than rising, because of the density-of-states peak.

We finally comment on the question of Hall voltages in type-II superconductors. Inasmuch as the flux lines are described as vortices of superconducting electrons, the lines may move not in the direction of the applied force, but perpendicular to the force.[10] If so, one expects a large Hall voltage in the flux-flow state. In many of our samples we do observe transverse voltages, but they are due primarily to surface impurities and thermal effects. Although some components of these transverse voltages could be interpreted as Hall-type emf, the magnitude is very much smaller than that expected from the vortices moving perpendicular to the force.[11] Our observations, however, do not as yet exclude the possibility of these effects existing in pure crystals.

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[1]A. A. Abrikosov, Zh. Eksperim. i Teor. Fiz. 32, 1442 (1957) [translation: Soviet Phys. –JETP 5, 1174 (1957)].

[2]P. W. Anderson and Y. B. Kim, Rev. Mod. Phys. 36, 39 (1964).

[3]C. F. Hempstead and Y. B. Kim, Phys. Rev. Letters 12, 145 (1964).

[4]Y. B. Kim, C. F. Hempstead, and A. R. Strnad, Rev. Mod. Phys. 36, 43 (1964).

[5]J. Volger, F. A. Staas, and A. G. Van Vijfeijken, Phys. Letters 9, 303 (1964).

[6]C. Caroli, P. G. de Gennes, and J. Matricon, Phys. Letters 9, 307 (1964).

[7]B. Rosenblum and M. Cardona, Phys. Rev. Letters 12, 657 (1964).

[8]We are indebted to P. W. Anderson for the theoretical considerations mentioned here.

[9]M. Tinkham, private discussion.

[10]W. F. Vinen, Rev. Mod. Phys. 36, 48 (1964); P. H. Borcherds, C. E. Gough, W. F. Vinen, and A. C. Warren, Phil. Mag. 10, 349 (1964).

[11]The absence of Hall voltages in type-II superconductors was reported earlier by A. I. Schindler and D. J. Gillespie, Phys. Rev. 130, 953 (1963); C. F. Hempstead and Y. B. Kim, Bull. Am. Phys. Soc. 9, 29 (1964).

WEAK TIME DEPENDENCE IN PURE SUPERCONDUCTORS*

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In recent years the equilibrium properties of type-II superconductors have been successfully investigated by means of the Ginzburg-Landau theory.[1] In this theory the state of the superconductor is characterized by an order parameter (energy gap), and an electromagnetic field, both slowly varying with position. One expects a similar characterization to hold also for certain restricted types of time variations, especially if these are slow enough to preclude breakup of Cooper pairs. (The motion of fluxoids through the superconductor should be a case in point.) Because of the time dependence, there appears to be no obvious way of basing such a theory on thermodynamics alone. We therefore use the BCS model[2] in the Gor'kov formulation[3] to derive the necessary time-dependent generalizations. In this paper we restrict ourselves to pure superconductors. We make the following assump-