BOOTSTRAP THEORY AND THE PARITY-NONCONSERVING DECAYS OF THE HYPERONS*

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In a recent series of publications,[1-4] a general "bootstrap" theory of octet enhancement in the strong, electromagnetic, and (parity-conserving) weak violations of SU(3) symmetry has been proposed. The theory was successfully applied[4] to the strong and electromagnetic mass splittings in the $\frac{1}{2}^+$ octet and the $\frac{3}{2}^+$ decuplet of baryons.

In the present Letter we apply similar bootstrap techniques to the separate problem of the parity-nonconserving weak interactions. The particular case considered here is nonleptonic hyperon decay. We find, neglecting strong and electromagnetic violations of SU(3), that the parity-nonconserving $\bar{B}B\Pi$ couplings (B is the baryon octet, Π is the pseudoscalar octet) satisfy

$$S(\Sigma_+{}^+) \approx 0,$$
$$S(\Xi_-{}^-) \approx \xi S(\Lambda_-), \quad S(\Sigma_-{}^-) \approx (\tfrac{2}{3})^{1/2}(2\xi - 1)S(\Lambda_-),$$
$$S(\Xi_0{}^0) \approx -\xi S(\Lambda_0), \quad S(\Sigma_0{}^+) \approx -(\tfrac{2}{3})^{1/2}(2\xi - 1)S(\Lambda_0); \quad (1)$$

where $S(\Sigma_+{}^+)$, for example, is the S-wave amplitude[5] for $\Sigma^+ \to n + \pi^+$ and $S(\Sigma_0{}^+)$ is the corresponding amplitude for $\Sigma^+ \to \pi^0 + p$. The coefficient ξ in (1) is a known function, to be determined below, which depends only on the F/D ratio λ of the strong $\bar{B}B\Pi$ coupling. A number of arguments suggest[6,7] $\lambda = \frac{1}{3}$ to $\frac{2}{3}$; with λ in this range we find that ξ varies smoothly between 1.5 and 1.8. These relations, which should hold to within ~20%, are independent of any assumptions (including CP invariance) about the transformation properties of the weak interactions. Moreover, if we assume that the parity-nonconserving weak interaction has only 1, 8, and 27 components, with charge conjugation properties correctly described by Cabibbo's theory[8] (note that this theory conserves CP), we find that the S-wave amplitudes transform mainly like an octet. This means, of course, that the amplitudes $S(\Lambda_0)$ and $S(\Lambda_-)$, $S(\Xi_-{}^-)$ and $S(\Xi_0{}^0)$, etc., are related by the $|\Delta I| = \frac{1}{2}$ rule.[9,10] Note that the $|\Delta I| = \frac{1}{2}$ rule adds only one relationship to the predictions of Eq. (1).

Using the currently available data on lifetimes and polarizations, Stevenson et al.[5] have deduced a tentative set of amplitudes for the nonleptonic hyperon decays. There are seven observable parity-nonconserving amplitudes. The $|\Delta I| = \frac{1}{2}$ rule provides three familiar relations among these quantities. In Eq. (1) we have predictions for three further ratios among the four remaining independent amplitudes. Our predictions are in good agreement with the S-wave amplitudes of Stevenson et al. The prediction of Eq. (1) that the decay $\Sigma^+ \to n + \pi^+$ is pure P wave (rather than pure S) has not yet been tested directly, but polarization measurements may settle this question in the near future.

Now let us see how these results are derived. To see what bootstrap theory has to do with weak interactions, consider the nucleon as a pion-nucleon bound state. Neglecting the parity-nonconserving weak interaction, the nucleon would be a purely P-wave πN state, but when the weak interaction is included the nucleon picks up a small S-wave component. The amount of S wave in the nucleon state then gives the parity-nonconserving part of the πNN coupling. Now in a theory where the nucleon is composite, this S-wave piece of the nucleon state is determined by the parity-nonconserving part of the "potential" which acts between a pion and a nucleon. Since nucleon exchange is an important piece of this potential, the parity-nonconserving πNN coupling will react on itself, so the problem has a self-consistency or bootstrap aspect. However, the bootstrap is not the whole story here. There are some direct weak-interaction effects, such as intermediate vector boson exchange (if it exists), which also contribute to the parity-nonconserving part of the πN potential. These parts of the potential are not determined by bootstrap requirements and must be taken as given—we call them the driving terms, D. It is the driving terms which determine the overall scale of the parity-nonconserving effects.

In general, if we take the coupling at the vertex $\bar{B}^i B^j \Pi^k$, $i,j,k=1,\cdots,8$, to be $(\gamma_5 g_{ijk} + \delta g_{ijk})$, where g is the strong SU(3)-symmetric[11] coupling and δg is a small parity-nonconserving piece, then, working to first order in the parity-nonconserving weak interaction, the bootstrap requirements will provide relations like

$$\delta g_{ijk} = \sum_{i'j'k'} A_{ijk,i'j'k'} \delta g_{i'j'k'} + \cdots + D_{ijk}. \quad (2)$$

Equation (2) is simpler than the corresponding relations for parity-conserving coupling shifts in several important respects: (i) Since, as in Schrödinger theory, the parity-nonconserving weak interaction is "off-diagonal" and can produce no mass shifts in lowest order,[2-4] there are no mass-shift terms on the right side of Eq. (2). (ii) From more detailed dynamical considerations, to be discussed below, we find that the shifts in $\bar{B}B\Pi$ couplings are very nearly decoupled from the other parity-nonconserving couplings between strongly interacting particles.

Equation (2) can be greatly simplified if we use the fact that $A_{ijk,i'j'k}$ is invariant under SU(3). Following the general methods given in reference 3, we first note that in SU(3) δg_{ijk} belongs to the product representation $\underline{8} \otimes \underline{8} \otimes \underline{8}$. Thus we can relabel δg by $\delta g_{N,n}{}^{\beta}$ where $\underline{N}=\underline{1}$, $\underline{8}, \underline{27}, \cdots$, runs over all the distinct irreducible representations contained in $\underline{8} \otimes \underline{8} \otimes \underline{8}$, n is the component of the representation, and β is an index (to be specified later) which distinguishes between different representations with the same dimension. Similarly, we change the labels on the driving term to $D_{N,n}{}^{\beta}$. Then, since the A matrix conserves SU(3), Eq. (2) becomes[1-4]

$$\delta g_{N,n}{}^{\beta} = \sum_{\beta'} A_N{}^{\beta\beta'} \delta g_{N,n}{}^{\beta'} + D_{N,n}{}^{\beta}, \quad (3)$$

or

$$\delta g_{N,n}{}^{\beta} = \sum_{\beta'} (1-A_N)_{\beta\beta'}{}^{-1} D_{N,n}{}^{\beta'},$$

where $A_N{}^{\beta\beta'}$ is independent of n. Note that this equation is similar to those which appeared in previous work on the baryon and decuplet mass splittings[2,4]—again we look for eigenvalues of A_N which are near one. Clearly, if A_N has an eigenvalue near unity, the component of $\delta g_{N,n}{}^{\beta}$ lying along the associated eigenvector will be preferentially enhanced. By studying the enhanced eigenvector, we can find ratios among the amplitudes without having to explicitly calculate the hard-to-treat driving terms.

The A matrix can be calculated with the S-matrix perturbation theory which has been discussed elsewhere.[3] In outline, the calculation proceeds as follows. We consider baryons as bound states in the ΠB channel. The parity-nonconserving $\bar{B}B\Pi$ coupling δg will then appear as a factor in the residue of the baryon poles in the (parity-nonconserving) partial-wave amplitude $\delta T_{ij,kl}$ for the scattering process $J=\frac{1}{2}^+$ (P wave) $B^i\Pi^j \to J=\frac{1}{2}^-$ (S wave) $B^k\Pi^l$. Using the partial-wave expansion given by Singh,[12] one finds, with a suitable normalization for δT, that the direct channel baryon pole has the form $-\sum_x g_{ixj}\delta g_{xkl}/(W-M)$, where W is the total c.m. energy and M is the baryon mass. Application of the methods of reference 3 then gives

$$\delta g_{ijk} = -\frac{1}{2\pi i g^2} \sum_{xyzu} \int_L \frac{[X_{i,xy}(W')\delta T_{xy,zu}(W')Y_{zu,jk}(W')]}{W'-M} dW',$$

$$g^2 = \sum_{jk} (g_{ijk})^2,$$

$$Y_{zu,jk}(W') = [D_-(W')D_-{}^{-1}(M)]_{zu,jk},$$

$$X_{i,xy}(W') = \lim_{W\to M} \sum_{lm} g_{ilm}(W-M)[D_+(W')D_+{}^{-1}(W)]_{xy,lm}, \quad (4)$$

where $D_\pm$ is the strong-interaction denominator matrix[13] for ΠB scattering in the $J=\frac{1}{2}^\pm$ state (note that $D_+{}^{-1}$ is singular at $W=M$) and the contour L runs clockwise around the left cuts in δT. Note that since g_{ijk} is SU(3) symmetric, g^2 is independent of i. Now, according to reference 4, if the baryons are in fact composite objects,[14] then the dispersion integral in Eq. (4) should converge rapidly and we need only keep the nearest singularities in δT. Since we are only interested in the singularities of δT that lie near $W=M$, we can set $D_-(W')\approx D_-(M)$ and make a linear approximation[15] for D_+ which yields

$$Y_{zu,jk}\approx\delta_{zj}\delta_{uk},\quad X_{i,xy}(W')\approx(W'-M)g_{ixy}. \tag{5}$$

Finally, to obtain the A matrix, we need the nearby ("shortcut") singularity in δT associated with baryon exchange, and a simple calculation shows that this can be approximated by the pole $\sum_v \delta g_{xvu} g_{zvy}/(W-M)$. Then, inserting our B-exchange pole into (4) and using (5), one finds

$$A_{ijk,i'j'k'}\approx\delta_{kk'}g^{-2}\sum_x g_{ii'x}g_{jj'x}. \tag{6}$$

Before proceeding with the analysis of Eq. (6), let us recall our previous remark that the other parity-nonconserving corrections to the couplings of strongly interacting particles will have a small effect on δg. To see why this is so, we note that in the low-energy region where static kinematics are valid, the s-channel scattering $\cos\theta_s$ is equal to the u-channel angle $\cos\theta_u$,[16] which implies that a P- to S-wave transition in the u channel crosses only to a P- to S-wave transition in the s channel. Thus, only u-channel processes which connect P to S waves can contribute to the nearby part of the left-hand cut in our problem. This means that B exchange will contribute but $\frac{3}{2}^+$ decuplet exchange, which appears as a P- to D-wave transition, will not.[17]

Returning to the analysis of Eq. (6), we first switch to the N, n, β representation of Eq. (3). We choose the previously unspecified index β such that for a given SU(3) symmetry violation N,n at the $\bar{B}B\Pi$ vertex, β runs over the independent irreducible representations of $\bar{B}B$. For example, for $N=1$, β runs over the $\underline{8}_s$ and $\underline{8}_a$ $\bar{B}B$ states since only these states can combine with the Π octet to make an SU(3) singlet, whereas for $N=8$, β runs over all the $\bar{B}B$ states since any one of them can combine with the Π supermultiplet to make an octet. The results of evaluating Eq. (6) in this representation are listed in Table I.[18] Note the remarkable features that $A_N^{\beta\beta'}$ is essentially diagonal in β and β' and is independent[19] of N (recall, however, that for a given N, only certain β are allowed by SU(3) considerations. As we shall see below, charge conjugation and SU(3) properties of certain theories may further restrict the β's which appear in the driving term).

One feature of Table I is that the diagonal element (eigenvalue) A^{11} is (in this approximation) always equal to unity. However, a coupling $\delta g_{N,n}{}^1$ cannot contribute to the strangeness-changing decays, so we need not consider this particular eigenvalue. For λ in the usual range $\lambda=\frac{1}{2}$ to $\frac{2}{3}$, the only other eigenvalue which is close to one comes from the two-by-two matrix connecting $\underline{8}_s$ and $\underline{8}_a$. This particular eigenvalue is

$$e(\lambda)=\frac{9\lambda^2+1+(16+180\lambda^2)^{1/2}}{10+18\lambda^2},$$

which varies from 0.67 to 0.82 as λ goes from $\frac{1}{3}$ to $\frac{2}{3}$. The associated eigenvector is $\delta g_{N,n}{}^{8s}=R(\lambda)\delta g_{N,n}{}^{8a}$ where $R(\lambda)=[\frac{1}{2}-e(\lambda)](5+9\lambda^2)\times(3\lambda\sqrt{5})^{-1}$. The enhancement of nonleptonic decays, caused by the factor $[(1-e(\lambda)]^{-1}$, is evidently of order 3 to 5 in the amplitude, or 10 to 25 in the decay rate.

Since this particular eigenvalue remains close to unity and the others are so far from unity, we expect that for a given N or n, the dominant terms in δg will be $\delta g_{N,n}{}^{8s}$ and $\delta g_{N,n}{}^{8a}$ and the ratio between these terms will be roughly that given above. A straightforward calculation[20] then gives the result of Eq. (1) with $\xi=[5^{1/2}-R(\lambda)]/[5^{1/2}+R(\lambda)]$.

So far we have shown that $\delta g_{N,n}{}^8$ is enhanced, where N can be any of the SU(3) symmetry violations $\underline{1}$, $\underline{8}_a$, $\underline{8}_s$, $\underline{10}$, $\underline{10}^*$, or $\underline{27}$. The question of which N's actually appear in the driving terms can only be answered in a more specific theory. To see how this goes, let us examine some SU(3)-charge conjugation properties of

Table I. This table gives the $\beta\beta'$ dependence of the matrix $A_N^{\beta\beta'}$ defined in the text. In this particular case, the A matrix is independent of N and λ is the F/D ratio for the strong interaction $\bar{B}B\Pi$ couplings.

	$\underline{1}$	$\underline{8}_s$	$\underline{8}_a$	$\underline{10}$	$\underline{10}^*$	$\underline{27}$
$\underline{1}$	1	0	0	0	0	0
$\underline{8}_s$	0	$-\frac{3+9\lambda^2}{10+18\lambda^2}$	$-\frac{3\sqrt{5}\lambda}{5+9\lambda^2}$	0	0	0
$\underline{8}_a$	0	$-\frac{3\sqrt{5}\lambda}{5+9\lambda^2}$	$\frac{1}{2}$	0	0	0
$\underline{10}$	0	0	0	$-\frac{1}{5+9\lambda^2}$	0	0
$\underline{10}^*$	0	0	0	0	$-\frac{1}{5+9\lambda^2}$	0
$\underline{27}$	0	0	0	0	0	$\frac{1-3\lambda^2}{5+9\lambda^2}$

the terms $D_{N,n}{}^{8a}$ and $D_{N,n}{}^{8s}$ which drive the enhanced eigenvector. The $\bar{B}B$-octet states $\underline{8}_a$ and $\underline{8}_s$ with $J=0$ have $\mathcal{C}=+1$.[21] The Π octet also has $\mathcal{C}=+1$. The coupling of Π to the enhanced $\bar{B}B$ octet then has $\mathcal{C}=+1$ for the symmetric combinations $N=\underline{1}$, $\underline{8}_s$, and $\underline{27}$, and $\mathcal{C}=-1$ for the antisymmetric combination $\underline{8}_a$, while $\underline{10}$ and $\underline{10}^*$ can be combined into two orthogonal states with $\mathcal{C}=+1$ and -1, respectively. Thus for example, in Cabibbo's theory where the parity-nonconserving driving terms have $N=\underline{1}$, $\underline{8}$, and $\underline{27}$ and $\mathcal{C}=-1$, only the octet violation $N=\underline{8}_a$ can drive the enhanced eigenvector, and the resulting hyperon decay amplitudes will obey the $|\Delta I|=\frac{1}{2}$ rule.

In conclusion, let us turn from strangeness-changing to strangeness-conserving couplings, which, while not observed via baryon decays, may be of interest in connection with parity-nonconserving forces in nuclear physics.[1] The strangeness-conserving coupling $\delta g_{8,3}{}^{8}$ will be enhanced by the same factor $[1-e(\lambda)]^{-1}$ that enhanced the strangeness-changing coupling $\delta g_{8,6}{}^{8}$. The strangeness-conserving coupling $\delta g_{8,3}{}^{1}$ is also associated with an eigenvalue near one, but the driving term for this coupling requires a parity nonconservation with $N=\underline{8}$, $\mathcal{C}=+1$ which is not present in the Cabibbo theory. More detailed implications of octet enhancement for parity-nonconserving forces in nuclear physics have been discussed in reference 1.

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[9]Our bootstrap equations predict a $|\Delta I|=\frac{1}{2}$ rule which should hold to within about 20% in the amplitude. If the basic weak interaction responsible for nonleptonic decays turns out to be pure octet, then the $|\Delta I|=\frac{1}{2}$ rule will be more exact.

[10]M. Gell-Mann [Phys. Rev. Letters 12, 155 (1964)] has shown that octet dominance and the charge conjugation properties of Cabibbo's theory are enough to give the sum rule $2S(\Xi_-{}^-)=S(\Lambda_-)+\sqrt{3}S(\Sigma_0{}^+)$. Since we have here assumed the same charge-conjugation properties, and deduced octet dominance by bootstrap methods, our relations naturally include this sum rule. Note that we are not making the further assumptions of R invariance or a new octet of pseudoscalar mesons which appeared in the various justifications of this sum rule given by Lee, Sugawara, Coleman, and Glashow.

[11]Throughout this Letter we neglect strong and electromagnetic violations of SU(3).

[12]V. Singh, Phys. Rev. 129, 1889 (1963).

[13]$D_{ij,kl}$ is considered as a matrix connecting channel $B^i\Pi^j$ to channel $B^k\Pi^l$.

[14]Here, the essential difference between elementary and composite objects lies in the expected asymptotic behavior of the denominator functions.

[15]In fact, to obtain Eq. (6), we only need $Y(M)$ and $dX(W)/dW|_{W=M}$, and (5) gives these quantities exactly. Note that the curvature of D does not introduce any uncertainty here.

[16]To see this, note that $t=-2q_s{}^2(1-\cos\theta_s)=-2q_u{}^2\times(1-\cos\theta_u)$. Now, in crossing from the s to the u channel, t does not change, and in the static region $q_u{}^2\approx q_s{}^2$, which gives $\cos\theta_s\approx\cos\theta_u$.

[17]We have also considered Π exchange, which can only proceed through a lack of parity conservation in the t channel. One finds that the integrated contribution of this cut is small.

[18]Techniques for finding the values of A in the N, n, β representation are given in reference 3 [see in particular Eqs. (36) to (38).

[19]As Eq. (6) shows explicitly, A depends nontrivially only on the SU(3) indices (i,j) and (i',j') of $\bar{B}B$, whereas N refers to the SU(3) violation in the $(\bar{B}B\Pi)$ vertices. The circumstance that A_N is actually independent of N and has a particularly simple form in the β representation is peculiar to this problem. The A matrix which appears in the equations for the weak parity-conserving corrections to couplings is much more complicated.

[20]In the second line of Eq. (1), for example, $|\Delta Y|=1$ couplings of $\bar{B}B$ to π^- are compared. Here we know that $\bar{B}B$ is in an octet state with $Y=-1$, $I=\frac{1}{2}$, and 8_s to 8_a ratio $R(\lambda)$, and it is easy to pick out the ratios of $\bar{\Lambda}\Xi^-$, $\bar{p}\Lambda$, and $\bar{n}\Sigma^-$ which appear in this state. The phase conventions used in Eq. (1) are those of J. J. de Swart, Rev. Mod. Phys. 35, 916 (1963), and the couplings $\delta g_{N,n}{}^{8s}$ and $\delta g_{N,n}{}^{8a}$ refer to $\underline{8}_s$ and $\underline{8}_a$ states which have been normalized to one.

[21]M. Gell-Mann, Phys. Rev. Letters 12, 155 (1964). For a self-charge conjugate representation, $\mathcal{C}$ is equal to the charge conjugation parity of the $I=0$, $Y=0$ member of the representation.