charge parities, which may be chosen as

$$A \text{ with } N = 0 \text{ eigenstates,}$$
$$G \text{ with } Y = 0 \text{ eigenstates.} \qquad (12)$$

Experimental confirmation of Eq. (10) is in a somewhat preliminary stage. According to the interpretation[5] of Σ/Λ ratios in K^- capture,[6] $f_{\Sigma\Sigma}/f_{\Lambda\Sigma} \approx 0$; and the $Y_1^*(1385)$ is observed[7] to have a decay ratio $\Sigma\pi/\Lambda\pi \approx 4 \pm 4\%$. The $Y_1^*(1660)$ is an enigma and appears to show both $Y^*(+)$ and $Y^*(-)$ decay characteristics[8]; but its other properties are also confused, and it is tempting to suggest that it comprises at least two unresolved resonances.[9]

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SU(6) AND WEAK INTERACTIONS*

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Several authors[1,2] have extended Wigner's supermultiplet theory of the nucleus[3] to include symmetry.[4] They take the underlying symmetry of strong interactions to be SU(6) and assume that the intrinsic and unitary spins of elementary particles are generated by an SU(2) ⊗ SU(3) subgroup. Pseudoscalar and vector mesons are assigned to the adjoint representation of SU(6), i.e., the $\underline{35}$, and the $J = \frac{1}{2}^+$ baryon octet is combined with the $J = \frac{3}{2}^+$ decuplet to form a 56-plet.[1] Other consequences, including a mass formula[5,6] and relations among the electromagnetic properties of baryons,[7,8] have also been derived. Here we wish to examine weak interactions in the light of SU(6).

Our principal interest lies in the transformation properties of the weak Hamiltonian H_W. Their most general form is very complicated, but if our experience with isotopic spin and unitary spin is any guide, we can expect a considerable simplification to occur in practice. Strangeness-changing decays, for example, may transform in isospace as an admixture of $T = \frac{1}{2}$, $\frac{3}{2}$, $\frac{5}{2}$, and $\frac{7}{2}$, but experiment shows that the $T = \frac{1}{2}$ representation dominates this admixture.[9] Similarly, in unitary-spin space, the dominant transformation properties of H_W appear to be those of an octet rather than the 10- and 27-fold representations.[10,11] In each of these examples, it is the lowest available representation that actually determines the properties of H_W. We may therefore ask whether the same is true in SU(6).

In the nonrelativistic limit, which is used throughout this discussion, leptonic decays of hyperons are described by an admixture of Fermi and Gamow-Teller interactions[12] and nonleptonic decays by S- and P-wave interactions.[13] H_W is therefore a sum of two distinct parts: In one the intrinsic spins of the hadrons are coupled to a resultant zero, and in the other they are coupled to unity. Since the minimal unitary spin of H_W is that of an octet, we must assign it to an SU(6) representation containing at least one octet with spin zero and another with spin one.[14] The lowest such representation is the $\underline{35}$ and so we ask whether the assignment of H_W to a 35-plet is consistent with experiment.

Notice that this assignment does not allow H_W to belong to any SU(3) multiplet other than an octet. The octet, in turn, restricts strangeness-changing decays to isospin $T = \frac{1}{2}$. In other words, minimal transformation properties beget minimal transformation properties.

We begin the answer to our question with leptonic decays. In the direct product, $\underline{56}^* \otimes \underline{56}$, of antibaryon and baryon multiplets, the $\underline{35}$ appears only once and so the D/F ratios of weak currents are fixed. Fermi decays are pure F type,[8] exactly as in Cabibbo's theory,[10] and Gamow-Teller decays have a D/F ratio of $\frac{3}{2}$.[8] Since this latter ratio agrees quite well with the value of 1.7 obtained in a recent analysis of experimental data,[15] we conclude that the dominance of the $\underline{35}$ is a reasonable assumption for leptonic decays.

Next we turn to nonleptonic hyperon decay. To determine the structure of the effective Hamiltonian, we decompose $\underline{56}^*\otimes \underline{56}$ into irreducible representations and then combine each of them with the meson $\underline{35}$ to form an overall $\underline{35}$. There are three possible coupling schemes,

$$H_0 \sim [(\underline{56}^*\otimes \underline{56})_{\underline{1}}\otimes \underline{35}]_{\underline{35}},$$

$$H_1 \sim [(\underline{56}^*\otimes \underline{56})_{\underline{35}}\otimes \underline{35}]_{\underline{35}},$$

$$H_2 \sim [(\underline{56}^*\otimes \underline{56})_{\underline{405}}\otimes \underline{35}]_{\underline{35}}, \qquad (1)$$

but the first does not give rise to observable decays; moreover, the two types of H_1 coupling are Hermitian conjugates of one another. Therefore the effective Hamiltonian for observable decays contains only two independent $\underline{35}$'s, one from H_1 and the other from H_2.

The S-wave amplitudes arising from H_1 and H_2 are denoted by S_1 and S_2, respectively, and the corresponding P-wave amplitudes by P_1 and P_2. Matrix elements for processes of interest are given by

$$\langle \Lambda | p\pi^- \rangle = \sqrt{3}(S_1' - P_1'),$$

$$\langle \Sigma^+ | p\pi^0 \rangle = (S_1' + \tfrac{1}{3}P_1') - 2(S_2 + P_2),$$

$$\langle \Xi^- | \Lambda\pi^- \rangle = \sqrt{3}(S_1' - \tfrac{1}{3}P_1') - \sqrt{3}(S_2 - P_2), \qquad (2)$$

and

$$\langle \Sigma^+ | n\pi^+ \rangle = \sqrt{2}(S_2 + P_2), \qquad (3)$$

where

$$S_1' = S_1 - \tfrac{1}{8}S_2, \quad P_1' = P_1' = P_1 - \tfrac{3}{8}P_2.$$

It can be seen immediately that the S-wave amplitudes in (2) satisfy Lee's relation[11]

$$\sqrt{3}\langle \Sigma^+ | p\pi^0 \rangle + \langle \Lambda | p\pi^- \rangle = 2\langle \Xi^- | \Lambda\pi^- \rangle \qquad (4)$$

for all values of S_1 and S_2. The P-wave amplitudes, however, will not satisfy (4) unless

$$P_2 = 0. \qquad (5)$$

Since this condition also implies that $\Sigma^+ \to n + \pi^+$ is pure S wave, i.e.,

$$\alpha(\Sigma^+ \to n + \pi^+) = 0, \qquad (6)$$

it would appear that the $\underline{35}$ provides a good description of nonleptonic decay. Unfortunately this is not the case because (2) and (3) possess two features which contradict experiment.

Suppose first that P_2 is taken to be zero and S_1 and S_2 are chosen such that

$$\alpha_0 \alpha_\Lambda < 0; \qquad (7)$$

it then follows from (2) and (5) that

$$\alpha_\Lambda \alpha_\Xi > 0. \qquad (8)$$

Similarly, if $\alpha_\Lambda \alpha_\Xi$ is made negative, $\alpha_0 \alpha_\Lambda$ will be positive. Thus (2) and (3) are not consistent with the relative signs of α_0, α_Λ, and α_Ξ as observed experimentally.[9,16] Second, if the matrix element for $\Sigma^- \to n + \pi^-$ is calculated from the $\Delta T = \frac{1}{2}$ prediction

$$\sqrt{2}\langle \Sigma^+ | p\pi^0 \rangle = \langle \Sigma^- | n\pi^- \rangle - \langle \Sigma^+ | n\pi^+ \rangle \qquad (9)$$

together with (2) and (3), we find[17]

$$\sqrt{3}\langle \Sigma^- | n\pi^- \rangle_S = +\sqrt{2}\langle \Xi^- | \Lambda\pi^- \rangle_S,$$

$$\sqrt{3}\langle \Sigma^- | n\pi^- \rangle_P = -\sqrt{2}\langle \Xi^- | \Lambda\pi^- \rangle_P, \qquad (10)$$

which implies

$$\alpha(\Sigma^- \to n + \pi^-) = -\alpha_\Xi. \qquad (11)$$

Although the experimental situation is somewhat ambiguous at present,[9] it does not seem likely that (11) is satisfied. Equations (10) and (11) hold for all values of P_2 and therefore constitute a deeper contradiction of experiment than (7) and (8).

Despite these unsatisfactory features, it is noteworthy that the assignment of H_W to a 35-plet comes so close to predicting Lee's relation [Eq. (4)]. To see why this happens, analyze the SU(3) structure of H_1 and H_2. In H_1, the spin-zero and spin-one combinations of baryons and antibaryons are both unitary octets; therefore, the observable parts[18] of H_1 transform in unitary-spin space like

$$[(\bar{B}\times B)_{\underline{8}} \times M]_{\underline{8}}, \qquad (12)$$

where M denotes the pseudoscalar-meson octet. Similarly, the observable S-wave part of H_2 transforms as an admixture of[19]

$$[(\bar{B}\times B)_{\underline{8}} \times M]_{\underline{8}} \text{ and } [(\bar{B}\times B)_{\underline{27}} \times M]_{\underline{8}}, \qquad (13)$$

while the P-wave part includes terms like (13) together with

$$[(\overline{B}\times B)_n \times M]_{\underline{8}}, \quad n=\underline{10}, \underline{10}^*. \tag{14}$$

Now, as will be shown elsewhere,[20] Lee's relation is automatically satisfied whenever the baryon-antibaryon coupling involves only the 8- and 27-fold representations of SU(3) [see Eqs. (12) and (13)]; but, if the $\underline{10}$ and $\underline{10}^*$ are present, as in Eq. (14), the relation does not generally hold. It is for this reason that the amplitudes S_1, P_1, and S_2 satisfy (4), while P_2 does not.

Another point to notice is that $\Sigma^+\to n+\pi^+$ is engendered by H_2 and not by H_1. The reason for this is that the combination $(\overline{\Sigma}^-n)$ has isotopic spin $T=\frac{3}{2}$ and can appear in the 10- and 27-fold factors of Eqs. (13) and (14), but not in the 8-fold factor of (12).[21]

It is now evident that weak interactions give rise to an interesting predicament in SU(6); namely, that the assignment of H_W to the lowest available representation is a reasonable assumption for leptonic decays, but not for nonleptonic decays. One way out of it is to assume that (i) H_W is a current $\times$ current interaction[22] with weak currents belonging to a 35-plet; and (ii) that comparable contributions to nonleptonic decay are obtained from the $\underline{35}$ and one other representation in the product

$$\underline{35}\otimes\underline{35}=\underline{1}\oplus 2\times\underline{35}\oplus\underline{189}\oplus\underline{280}\oplus\underline{280}^*\oplus\underline{405},$$

The question arises as to which of these representations will remove the discrepancies noted above while preserving Eqs. (4) and (6). Although a detailed answer cannot be given at this point, there are qualitative arguments that suggest a plausible one.

The effective Hamiltonian for nonleptonic hyperon decay will include admixtures[23] of $\underline{189}$, $\underline{280}$, and $\underline{405}$ via the following coupling schemes:

$$H_X\sim[(\underline{56}^*\otimes\underline{56})_{\underline{35}}\otimes\underline{35}]_X, \quad X=\underline{189}, \underline{280}, \underline{405}; \tag{15}$$

$$H_Y'\sim[(\underline{56}^*\otimes 56)_{\underline{405}}\otimes 35]_Y, \quad Y=\underline{280}, \underline{405}; \tag{16}$$

$$H_{\underline{405}}''\sim[(56^*\otimes 56)_{\underline{2695}}\otimes\underline{35}]_{\underline{405}}. \tag{17}$$

It is important to note that the baryon-antibaryon couplings in H_X and H_Y' are exactly the same as those in H_1 and H_2, respectively [see Eq. (1)]. The coupling in (17) is quite distinct and will not be needed in our discussion. We shall assume that only the SU(3)-octet parts of H_X and H_Y' contribute to nonleptonic decays.

As a consequence of this assumption, the observable part[18] of H_X must behave in unitary-spin space exactly like Eq. (12) for all values of X. Now it is not difficult to show that the direct product $(\underline{56}^*\otimes\underline{56})_{\underline{35}}\otimes\underline{35}$ contains only two observable terms like (12); one is pure S wave and the other pure P wave. These terms have already appeared in H_1, and so the contribution from H_X to the matrix elements in Eq. (2) must be directly proportional to the amplitudes S_1 and P_1. Nothing will therefore be gained by incluing H_X in the effective Hamiltonian.

In H_Y' the observable S-wave term is a linear combination of the terms in (13), and it will automatically satisfy the Lee relation [see the discussion immediately below Eq. (14)]. Since there are two independent couplings in (13), the S-wave part of $H_{\underline{405}}'$ is independent of H_2, and hence its contribution to the matrix elements of (2) and (3) are not proportional to S_2. On the other hand, the S-wave part of $H_{\underline{280}}'$ is linearly dependent upon H_2 and $H_{\underline{405}}'$ and can therefore be omitted from our considerations.

The observable P-wave parts of $H_{\underline{405}}'$, like that of H_2, include all four couplings in (13) and (14) and are independent of H_2. Individually they do not satisfy Lee's relation due to the presence of terms like (14)[20]; however, we can always choose a linear combination that does.

On the basis of these arguments, we would like to suggest that the SU(6) behavior of nonleptonic decays is determined by an admixture of the $\underline{35}$ and $\underline{405}$ representations:

$$H_{\rm NL}\sim H(\underline{35})+H(\underline{405}). \tag{18}$$

In the case of hyperon decays, $H(\underline{35})$ is a combination of H_1 and H_2 [see Eq. (1)], and $H(\underline{405})$ is identical to $H_{\underline{405}}'$ [see Eq. (16)]. The S-wave amplitudes will satisfy Lee's relation [Eq. (4)] automatically, and so will the P-wave amplitudes if the admixture of H_2 and $H_{\underline{405}}'$ is choosen correctly. The decay mode $\Sigma^+\to n+\pi^+$ is engendered only by these latter terms [see paragraph below Eq. (14)], and if it turns out to be pure S wave, $H_{\rm NL}$ may take a relatively simple form:

$$H_{\rm NL}\sim H_1+H_2+H_{\underline{405}}' \quad \text{for } S \text{ waves},$$
$$\sim H_1 \quad \text{for } P \text{ waves}. \tag{19}$$

If it turns out to be pure P wave, then

$$H_{NL} \sim H_1 + H_2 + H_{\underline{405}}' \quad \text{for both } S \text{ and } P \text{ waves.} \qquad (20)$$

In either case, the addition of $H(\underline{405})$ to H_{NL} should provide enough arbitrary amplitudes for us to obtain the correct relative signs[9] of α_0, α_Λ, and α_Ξ.

The full consequences of (19) and (20) are presently being studied and will be reported in another publication. It should be borne in mind that while they have the attractive features outlined above, they may also have unattractive ones, such as Eq. (10). Their success, or failure, may also hinge upon their consequences for Ω^- decay. (Notice, by the way, that the absence of the 405 term would force

$$\Omega^- \to \Xi + \pi$$
$$\to \Lambda + K$$

to be pure P wave, as suggested in a recent dynamical model[24]; its presence leaves this as an open question.)

In conclusion, we note that the proposed transformation properties of H_{NL} [see (18)] are very similar to those attributed by Bég and Singh[6] to symmetry-breaking strong interactions; they also appear in Sakita's electromagnetic mass formula.[8]

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