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[1]G. Hägg, Nature 135, 874 (1935); G. Hägg, Z. Physik. Chem. 29B, 192 (1935).

[2]J. H. Ingold and R. C. DeVries, Acta Met. 6, 736 (1958).

[3]A. Magneli and B. Blomberg, Acta Chem. Scand. 5, 372 (1951).

[4]P. W. Anderson, to be published.

[5]C. S. Barrett and B. W. Batterman, Phys. Rev. Letters 13, 390 (1964).

[6]J. W. Gibson and R. A. Hein, Phys. Rev. Letters 12, 688 (1964).

[7]W. R. Gardner and G. C. Danielson, Phys. Rev. 93, 46 (1954).

[8]We wish to thank Dr. D. L. Nash of Bell Telephone Laboratories, Murray Hill, New Jersey, for the spectrographic analysis.

MOTION OF QUANTIZED FLUX LINES IN SUPERCONDUCTORS*

J. Bardeen
Department of Physics and Materials Research Laboratory, University of Illinois, Urbana, Illinois
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It has been argued[1] that in a superconductor a flux line moving with velocity $\vec{v}_L$ should be subject to a force $\vec{F}_L = Ne\Phi(\vec{v}_L \times \vec{n})$, where N is the electron concentration, Φ is the total flux, $\vec{n}$ is a unit vector along its direction, and we take units such that $c=1$. This force is presumed to be analogous to the Magnus force acting on a vortex line moving in a liquid. Such a force would give rise to vibrational modes, a large Hall angle for current flow in type-II superconductors, and other effects. There is considerable experimental evidence that no such force exists.[2] The purpose of this note is to point out that the theoretical arguments given for such a force are in error. In the absence of current flow, the only force to be expected is a viscous drag in a direction opposite to the motion of the line. If there is a current density, $\vec{J}=N(-e)\vec{v}_0$, superimposed on that of the flux line, the only driving force is the force $\vec{J}\times\vec{n}$ regardless of the velocity of the line. There are also forces between lines which result from an interaction between the current distributions. When a flux line moves in a neutral fluid at rest there are no currents except those associated with the flux line, and thus no force on the core except the viscous drag.

The only limitation is that the supercurrent flow be large compared with the normal currents generated by the motion, and this will be true except for very high velocities of the line. The motion is essentially adiabatic; the current flow pattern follows the motion. One may calculate the overall force by Ampere's rule, and this leads to $\vec{F}=\Phi\vec{J}\times\vec{n}$. There is, of course, an electric field generated by the vortex motion, but this gives no net force on a neutral system; the force on the electrons is balanced by that on the positive background. The electric field does give rise to a normal current flow (resulting from a nonthermal equilibrium electron distribution), and this in turn gives a frictional drag on the line.

In the bulk of a superconductor, Magnus forces associated with superfluid flow are balanced by the Lorentz force from the magnetic field. How this occurs when the local London theory applies has been described by London.[3] The stresses corresponding to the integrated Lorentz force act at the boundary. The absence of internal stresses follows more generally from the Meissner effect; the supercurrents represent the thermal equilibrium situation in the presence of the magnetic field.

The errors in previous treatments appear to arise from neglect of the forces from the compensating positive background. Suppose that $\vec{v}_s(\vec{r})$ is the velocity field of the flux line at rest. In uniform motion it is approximately $\vec{v}_s(\vec{r}-\vec{v}_L t)$, giving an inertial force $\partial\vec{v}_s/\partial t = -(\vec{v}_L\cdot\nabla)\vec{v}_s$. Part of this, $-\vec{v}_L\times\nabla\times\vec{v}_s$, contributes to the Magnus force. The magnetic field $H(\vec{r}-\vec{v}_L t)$ depends in a similar way on time. From the time-varying field, there is an electric field $\vec{E}=-\vec{v}_L\times\vec{H}+\nabla\varphi$, where φ is such that div $\vec{E}=0$. This electric field acts both on the electrons and on the positive background. The total force density is zero, so that there is no net force of reaction on the core resulting from its motion.

If we consider the force on the electrons alone, and combine it with the Magnus force, the total force per unit volume is

$$-Nm\vec{v}_L\times\{\nabla\times\vec{v}_s-\frac{e}{m}\vec{H}\}-Ne\nabla\varphi.$$

The term in curly braces vanishes from the London equation except at the core, and may be replaced by $-(e/m)\varphi\vec{n}\delta(\vec{r}-\vec{v}_Lt)$. The net force on the core from the reaction to the above is $-Ne\varphi\vec{v}_L\times\vec{n}$, which is the negative of the integral of the Lorentz force. In a frame moving with the vortex line, it represents the net force from the electron current moving with drift velocity $-\vec{v}_L$. However, as discussed above, this force is cancelled by the reaction from the corresponding force on the positive background, so that the total force due to motion of the line vanishes. This result is contrary to the case of a vortex line moving in liquid helium at rest where there is a net Magnus force associated with motion relative to the fluid.

The normal current density may be estimated roughly as to order of magnitude from $\vec{J}_n=\sigma_n\vec{E}$ and compared with J_S. From the above expression for E, we have

$$J_n\sim Ne^2\tau v_LH/m.$$

An estimate of J_S from the London equation is

$$J_s\sim\lambda_LNe^2H/m,$$

where λ_L is the penetration depth. The ratio is

$$J_n/J_s\sim\tau v_L/\lambda_L\sim\omega\tau,$$

where $\omega=v_L/\lambda_L$ corresponds roughly to the frequency of the disturbance resulting from the motion. Thus, the condition that $J_n\ll J_S$ is equivalent to $\omega\tau\ll 1$. For example, for $\tau\sim 5\times10^{-9}$ sec, $\lambda_L\sim5\times10^{-6}$ cm, and $v_L\sim 1$ cm/sec, J_n/J_S is of the order 10^{-3}. This value of τ corresponds to a mean free path of almost a centimeter. Thus the normal-current flow should be very small under nearly all conditions.

The frictional drag may be estimated from the integral of σ_nE^2, which gives the power lost as heat. If there is a driving force on the line from uniform current $J=N(-e)v_0$ of electrons moving with drift velocity v_0, the line will move in a direction perpendicular to both n and v_0. In most cases, the motion is probably retarded by trapping of the line at imperfections, as discussed by Anderson and Kim.[4]

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[1]J. Friedel, P. G. de Gennes, and J. Matricon, Appl. Phys. Letters 2, 119 (1963); P. G. de Gennes and J. Matricon, Rev. Mod. Phys. 36, 45 (1964).

[2]P. W. Borcherds, C. E. Gough, W. F. Vinen, and A. C. Warren, Phil. Mag. 10, 349 (1964); C. F. Hempstead and Y. B. Kim, Bull. Am. Phys. Soc. 9, 29 (1964).

[3]F. London, Superfluids (John Wiley & Sons, Inc., New York, 1950), Vol. I, p. 69

[4]P. W. Anderson and Y. B. Kim, Rev. Mod. Phys. 36, 39 (1964).