guide, the production cross section above threshold might be expected to be at most of the order of 0.1 mb. Because the accelerator results[3,4] imply high mass for a hypothetical quark, and because its fractional charge cannot be taken away in collisions with nuclear matter, it seems reasonable to assume that the effective cross section for removal of quarks by energy degradation is less than 15 mb. With these assumptions, a strongly interacting, long-lived, charge-$\frac{1}{3}e$ quark must, if it exists, have a mass such that $M_Q \gtrsim 16$ BeV/c^2. Reducing the assumed quark-production cross section to 0.01 mb, $M_Q \gtrsim 7$ BeV/c^2.

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[1]M. Gell-Mann, Phys. Letters 8, 214 (1964).

[2]G. Zweig, CERN Reports No. 8182/TH401 and No. 8419/TH412, 1964 (unpublished).

[3]L. B. Leipuner, W. T. Chu, R. C. Larsen, and R. K. Adair, Phys. Rev. Letters 12, 423 (1964); V. Hagopian, W. Selove, R. Ehrlich, E. LeBoy, R. Lanza, D. Rahm, and M. Webster, Phys. Rev. Letters 13, 280 (1964).

[4]D. R. O. Morrison, Phys. Letters 9, 199 (1964); H. H. Bingham et al., Phys. Letters 9, 201 (1964); W. Blum et al., Phys. Rev. Letters 13, 353a (1964).

[5]A. W. Sunyar, A. Z. Schwarzschild, and P. I. Connors, Phys. Rev. 136, B1157 (1964).

[6]The photocathode masks consisted of a large number of apertures distributed uniformly over the photosensitive surface; therefore, variations in photocathode efficiency should not have affected the pulse-size attenuation factor. As well as reducing the scintillator light by a factor of 9, the width of the size distribution of light scintillations is narrowed by a factor of 9 by the masks. This rather closely simulates the scintillations of $\frac{1}{3}e$ particles, since the most probable energy loss is a factor $\frac{1}{9}(20-\ln 9)/20 \approx \frac{1}{10}$ less than for charge e particles, and the width of the energy-loss distribution is narrowed by a factor of $\frac{1}{9}$ relative to the distribution for charge-e particles. [See B. Rossi, High Energy Particles (Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1952), pp. 29-35.]

[7]This primary proton spectrum was fitted to the following results: F. B. McDonald and W. R. Webber, Phys. Rev. 115, 194 (1959) for 5 BeV and 18 BeV; D. Lal, Proc. Indian Acad. Sci. A38, 93 (1953) for 1.5×10^3 BeV; M. F. Kaplon, B. Peters, H. L. Reynolds, and D. M. Ritson, Phys. Rev. 85, 295 (1952) for 5×10^3 BeV; and J. Outhie, C. M. Fisher, P. H. Fowler, A. Kaddoura, D. H. Perkins, K. Pinkau, and W. Walter, Phil. Mag. 6, 89 (1961) for $(6 \text{ to } 40)\times10^3$ BeV.

ELECTROMAGNETIC MASS CORRECTIONS IN THE SU(6) SYMMETRY*

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Recently Gürsey and Radicati[1] and Pais[2] have proposed an interesting new theory, SU(6) symmetry, for the strongly interacting particles. In their theory spin and unitary spin are incorporated in the same scheme. This is because the group SU(6) contains both SU(2) and SU(3) as its subgroups. In particular, they have assigned the pseudoscalar octet and the vector nonet to the 35-dimensional representation and the baryon octet and spin-$\frac{3}{2}$ decuplet to the 56-dimensional representation. Some consequences of the SU(6)-symmetry scheme have been investigated,[3,4] and the predictions of the new theory seem to be in agreement with experiments. The aim of this Letter is to obtain further a set of mass relations between the members of various isomultiplets in the limit where the SU(6) symmetry is broken by electromagnetism only. Similar work[5] has, in fact, already been done with the SU(3) group for the octet and decuplet where the electromagnetic mass splittings in these two representations are not related. Since the SU(6) group contains both the octet and the decuplet in its 56-dimensional representation, we expect that the assumption of SU(6) symmetry would relate the electromagnetic mass splittings of the octet and the decuplet and might put further restrictions on the mass relations. These mass relations could then be compared with experimental results, when such information is available. We should mention that the approach of the present work is similar to that of the recent work of Bég, Lee, and Pais,[4] where they have uniquely expressed the magnetic moments of all members of the baryon octet and the spin-$\frac{3}{2}$ decuplet in terms of the proton magnetic moment.

The states of the 56-dimensional representation of SU(6) are described by the completely symmetric tensor $B^{\alpha\beta\gamma}(\vec{q})$, where $\alpha,\beta,\gamma = 1, 2, \cdots, 6$, and $\vec{q}$ is the given momentum. This tensor is reducible under the group SU(3) $\otimes$SU(2)$_{\vec{q}}$; in the rest frame ($\vec{q}=0$) the explicit reduction is as follows[6]:

$$B^{\alpha\beta\gamma} = \chi^{ijk} d^{ABC} + (3\sqrt{2})^{-1}\{\epsilon^{ij}\chi^{k}\epsilon^{ABD} b_D^{\ C} + \epsilon^{jk}\chi^{i}\epsilon^{BCD} b_D^{\ A} + \epsilon^{ki}\chi^{j}\epsilon^{CAD} b_D^{\ B}\}, \quad (1)$$

where $i,j,k=1,2$; $A,B,C,\cdots=1,2,3$; ϵ^{ij} and ϵ^{ABC} are the Levi-Civita symbols in two and three dimensions, respectively. χ^i is the usual Pauli spinor and χ^{ijk} are the spin-$\frac{3}{2}$ wave functions, both normalized to unity. $b_B^{\ A}$ is the baryon-octet tensor, and d^{ABC} is the symmetric decuplet tensor of SU(3).

We now consider the electromagnetic mass corrections of the various isomultiplets within the $\underline{56}$ representation. We shall assume that the electromagnetic mass-splitting operator (denoted by C) transforms like a spin singlet in the spin space and like the charge operator, Q, in the unitary spin space. Then in the absence of the principal mass-breaking interaction,[3] but in the presence of all other symmetric strong interactions, the following is the most general expression[7] for the mass term:

$$\langle 56|M|56\rangle = B_{\alpha\beta\gamma}^{\dagger}[m_0\delta_\mu^{\ \alpha}\delta_\nu^{\ \beta}\delta_\lambda^{\ \gamma} + 3m_1\delta_\mu^{\ \alpha}\delta_\nu^{\ \beta}C_\lambda^{\ \gamma} + 9m_2\delta_\mu^{\ \alpha}C_\nu^{\ \beta}C_\lambda^{\ \gamma} + 27m_3C_\mu^{\ \alpha}C_\nu^{\ \beta}C_\lambda^{\ \gamma}]B^{\mu\nu\lambda}, \quad (2)$$

where $C_\alpha^{\ \beta} = \delta_i^{\ j}Q_A^{\ B}$.

In our convention the charge operator Q is

$$Q = (F_3 + 3^{-1/2}F_8) = \tfrac{1}{3}\begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad (3)$$

$$b = \begin{pmatrix} (\Sigma^0/\sqrt{2}) + (\Lambda^0/\sqrt{6}) & \Sigma^+ & p \\ \Sigma^- & -(\Sigma^0/\sqrt{2}) + (\Lambda^0/\sqrt{6}) & n \\ \Xi^- & \Xi^0 & -(2\Lambda^0/\sqrt{6}) \end{pmatrix}, \quad (4)$$

and

$$d^{111} = N^{*++}, \quad d^{222} = N^{*-}, \quad d^{333} = \Omega^-,$$
$$d^{113} = 3^{-1/2}Y_1^{*+}, \quad d^{123} = 6^{-1/2}Y_1^{*0},$$
$$d^{233} = 3^{-1/2}\Xi^{*-}, \text{ etc.} \quad (5)$$

Substituting expressions for $B^\dagger$, B, and Q in the right-hand side of (2), we obtain the following mass relations for various isomultiplets:

(a) Baryon octet.–

$$\Sigma^+ - \Sigma^0 = m_1 + m_2 - 6m_3,$$
$$\Sigma^0 - \Sigma^- = m_1 - 2m_2 + 3m_3,$$
$$\Xi^0 - \Xi^- = m_1 - 2m_2 + 3m_3,$$
$$p - n = m_1 + m_2 - 6m_3. \quad (6)$$

We thus obtain

$$\Sigma^- - \Sigma^+ = (n-p) - (\Xi^0 - \Xi^-), \quad (7)$$

and

$$\Sigma^- - \Sigma^0 = \Xi^- - \Xi^0. \quad (8)$$

The mass relation (7) is the same as in SU(3), which is known to be in good agreement with the experimental results, while relation (8) is a new prediction.[8] Taking the values of the masses from Rosenfeld et al.[9] we obtain for the left-hand side of (8) 4.75 ± 0.10 MeV, while for the right-hand side 6.5 ± 1.0 MeV. Thus the new mass relation (8) is also in good agreement with experimental values.

(b) Spin-$\frac{3}{2}$ decuplet.–

$$N^{*++} - N^{*+} = m_1 + 4m_2 + 12m_3,$$
$$N^{*+} - N^{*0} = m_1 + m_2 - 6m_3,$$
$$N^{*0} - N^{*-} = m_1 - 2m_2 + 3m_3,$$
$$Y_1^{*+} - Y_1^{*0} = m_1 + m_2 - 6m_3,$$
$$Y_1^{*0} - Y_1^{*-} = m_1 - 2m_2 + 3m_3,$$
$$\Xi^{*0} - \Xi^{*-} = m_1 - 2m_2 + 3m_3. \quad (9)$$

From (9) we obtain[10]

$$N^{*+} - N^{*0} = Y_1^{*+} - Y_1^{*0}, \quad (10)$$

$$N^{*0} - N^{*-} = Y_1^{*0} - Y_1^{*-} = \Xi^{*0} - \Xi^{*-}. \quad (11)$$

Among 18 particle states of the octet and the decuplet there are eight isomultiplets. Their mass splittings are given by 10 equations (6) and (9), depending upon three parameters, m_1, m_2, and m_3. We, therefore, have the following seven mass relations in SU(6)-symmetry

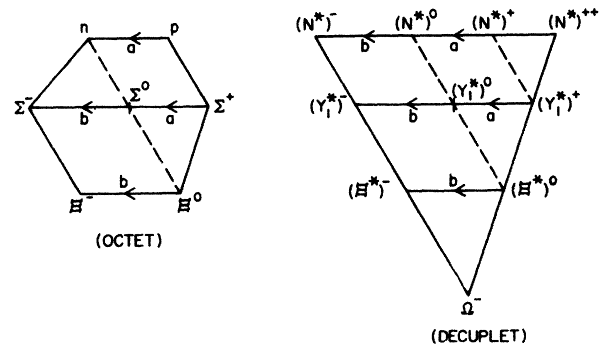

FIG. 1. Diagram showing the relations of the electromagnetic mass splittings of the members of the octet isomultiplets with those of the members of the decuplet isomultiplets.

scheme:

$$\Sigma^+-\Sigma^0=p-n=N^{*+}-N^{*0}=Y_1^{*+}-Y_1^{*0}; \qquad (12a)$$

$$\Sigma^0-\Sigma^-=\Xi^0-\Xi^-=N^{*0}-N^{*-}=Y_1^{*0}-Y_1^{*-}=\Xi^{*0}-\Xi^{*-}. \qquad (12b)$$

Equations (12a) and (12b) relate the mass splitting in the decuplet to that in the octet in an extremely simple manner. This is shown as in the diagram of Fig. 1. In particular, the theoretical predictions are $N^{*-}>N^{*0}>N^{*+}$, $Y_1^{*-}>Y_1^{*0}>Y_1^{*+}$, and $\Xi^{*-}>\Xi^{*0}$. At present the following experimental results are available to us:

(i) the mass difference in the N^* isomultiplet[11],

$$N^{*-}-N^{*++}=0.6\pm 5.0 \text{ MeV}.$$

(ii) the mass difference in the Y_1^* isomultiplet,

$$Y_1^{*-}-Y_1^{*+}=17\pm 7 \text{ MeV}$$

(Cooper et al.[12]), and

$$Y_1^{*-}-Y_1^{*+}=4.3\pm 2.2 \text{ MeV}$$

(Huwe et al.[13]). Although the above experimental results have large uncertainties, they are in qualitative agreement with the prediction from relation (12). Further experiments to determine the above mass differences with better accuracy would be encouraging.

Due to the fact that the width of Ξ^* ($\Gamma=7.5$ MeV) is much smaller than that of N^* ($\Gamma=125$ MeV), the determination of the mass difference between Ξ^{*0} and Ξ^{*-} would be more accurate than that between the members of the N^* isomultiplet (considering that the number of events is the same in each case). It would, therefore, be very interesting to know this difference; its magnitude and sign can be easily compared with the above predictions as another simple test of SU(6) symmetry.

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[1]F. Gürsey and L. A. Radicati, Phys. Rev. Letters 13, 173 (1964).

[2]A. Pais, Phys. Rev. Letters 13, 175 (1964).

[3]F. Gürsey, A. Pais, and L. A. Radicati, Phys. Rev. Letters 13, 299 (1964); T. K. Kuo and T. Yao, Phys. Rev. Letters 13, 415 (1964); M. A. B. Bég and V. Singh, Phys. Rev. Letters 13, 418, 509 (1964).

[4]M. A. B. Bég, B. W. Lee, and A. Pais, Phys. Rev. Letters 13, 514 (1964).

[5]S. Coleman and S. L. Glashow, Phys. Rev. Letters 6, 423 (1961). S. Okubo, Phys. Letters 4, 14 (1963). S. L. Glashow and J. J. Sakurai, Nuovo Cimento 25, 337 (1962); 26, 622 (1962). S. P. Rosen, Phys. Rev. Letters 11, 100 (1963).

[6]The reduction form [Eq. (1)] of $B^{\alpha\beta\gamma}$ has been written in a different form from that given by Bég, Lee, and Pais,[4] but it can easily be shown that they are identical.

[7]Terms involving $(C^2)_\beta{}^\alpha$, $(C^3)_\beta{}^\alpha, \cdots$, do not appear explicitly in (2) because they can be eliminated by the relation $9C^2=2+3C$.

[8]We obtain one extra relation because in SU(6) there is no F-D mixture as in SU(3).

[9]A. H. Rosenfeld et al., Rev. Mod. Phys. 36, 977 (1964).

[10]We obtain one relation less than those obtained by Rosen[5]; namely, $N^{*++}-N^{*-}=3(Y_1^{*+}-Y_1^{*0})$. This is because in his work Rosen considered only up to second-order perturbation of the electromagnetic mass-splitting interaction. Had we assumed $m_3=0$, we would recapture his results.

[11]R. Birge et al., Proceedings of the Dubna Conference on High-Energy Physics, August 1964 (to be published).

[12]W. A. Copper, H. Filthuth, A. Fridman, E. Malamud, E. S. Gelsema, J. C. Kluyver, and A. G. Tenner, Phys. Letters 8, 365 (1964).

[13]D. O. Huwe et al., University of California Lawrence Radiation Laboratory Report No. 11291, 1964 (unpublished).