for the case of a disk.

Pellam's experiment provides evidence for hitherto unobserved phenomena relating to the fundamental properties of He II. Despite the fact that the flow pattern about the Rayleigh disk is not well understood, Pellam's qualitative results are sufficiently clear-cut to demand further study. In interpreting such studies it is now evident that Helmholz flow plays an important role and must be taken into account.

The author wishes to thank Dr. P. Bendt for re-emphasizing the possible relevance of the suggestion on Helmholz flow made in reference 17.

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[1]J. R. Pellam, Proceedings of the Ninth International Conference on Low Temperature Physics, Columbus, Ohio, 31 August-4 September 1964 (unpublished), paper CB.8.

[2]J. R. Pellam, Phys. Rev. Letters 5, 189 (1960).

[3]J. R. Reppy, D. Depatie, and C. T. Lane, Phys. Rev. Letters 5, 541 (1960).

[4]F. London, Superfluids (John Wiley & Sons, Inc., New York, 1954), Vol. II.

[5]K. R. Atkins, Liquid Helium (Cambridge University Press, New York, 1959).

[6]Classical applications of the Rayleigh disk are discussed by Leo L. Beranek, Acoustic Measurements (John Wiley & Sons, Inc., New York, 1949), pp. 148-158. The complexity of the flow pattern in sound excitation of a disk is illustrated in Figs. 4 and 17, p. 153.

[7]J. R. Pellam and W. Hanson, Phys. Rev. 85, 216 (1952).

[8]J. R. Pellam, Progress in Low Temperature Physics, edited by C. J. Gorter (North-Holland Publishing Company, Amsterdam, 1955), Vol. I, p. 54.

[9]H. Schlichting, Boundary Layer Theory (McGraw-Hill Book Company, Inc., New York, 1960).

[10]Arnold Sommerfeld, in Mechanics of Deformable Bodies (Academic Press, Inc., New York, 1950), Vol. II. A remarkable photograph on p. 211, Fig. 47, illustrates Helmholz flow in steady state.

[11]B. H. Wick, Natl. Advisory Comm. Aeron., Tech. Note No. 3221 (1954).

[12]A. Fage and F. C. Johansen, Proc. Roy. Soc. (London) A116, 170 (1927); Phil. Mag. 5, 417 (1928).

[13]R. Von Mises and K. O. Friedrichs, Fluid Dynamics, Lecture Notes, Brown University, Providence, Rhode Island, 1941 (unpublished).

[14]R. Von Mises, Theory of Flight (McGraw-Hill Book Company, Inc., New York, 1945).

[15]Thomas R. Koehler and John R. Pellam, Phys. Rev. 125, 791 (1962).

[16]Paul P. Craig and John R. Pellam, Phys. Rev. 108, 1109 (1957).

[17]P. P. Craig, Phys. Rev. Letters 7, 331 (1961).

[18]R. P. Feynman, Progress in Low Temperature Physics, edited by C. J. Gorter (North-Holland Publishing Company, Amsterdam, 1955), Vol. I, p. 17.

ALFVÉN-WAVE PROPAGATION IN PYROLYTIC AND SINGLE-CRYSTAL GRAPHITE*

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Alfvén-wave propagation has recently been observed in bismuth[1] and antimony.[2] We have observed and studied the phenomenon in pyrolytic graphite (PG), which afforded the possibility of investigating the unique behavior of Alfvén waves in an almost two-dimensional plasma. The experiments were carried out at 4.2°K with a 35-Gc/sec absorption cavity spectrometer. The results are in good agreement with the anisotropy of the Fermi surface of graphite. The carrier density, estimated on the basis of these measurements using Shubnikov-de Haas effective masses for the basal plane, is $3.3\times10^{24}/\mathrm{m}^3$ for each carrier, in satisfactory agreement with the measurements by other experimental techniques. The Alfvén-wave behavior in PG is similar to that observed in single-crystal graphite in our more recent experiments. The latter measurements do indicate, however, certain details of structure which are not resolved in the PG data. In the low-field region, cyclotron resonance was observed in both PG and single-crystal graphite.

Alfvén-wave propagation occurs in compensated conductors in the range $\omega_p \gg \omega_c \gg (\omega, \tau^{-1})$, where ω_p, ω_c, ω, and τ^{-1} are the plasma, cyclotron, signal, and collision frequencies, respectively. In this region the effective permittivity is given by[1]

$$\epsilon_{\mathrm{eff}} = \frac{1}{B^2}\sum_i \int_{\mathrm{FS}} n_i(k_z) m_i^*(k_z) dk_z = \frac{D(n, m^*)}{B^2} \text{ (mks units)}, \quad (1)$$

where B is the magnetic field, z is the field direction, the subscript i refers to the carrier type, $m_i^*(k_z)$ is the cyclotron effective mass, $n_i(k_z)$ is the density of states along k_z, and the notation "FS" means that the integration is carried out over the Fermi surface. Since graphite is characterized by a distribution of masses in k space,[3] the carrier mass-density function D is written in the integral form in place of the usual $n(m_h^*+m_e^*)$.[1]

For electromagnetic propagation through flat specimens, the B dependence of ϵ_{eff} leads to an interference pattern periodic in B^{-1}, and the period thus provides a measure of $D(n,m^*)$. The experiments were carried out at 4.2°K in a cylindrical TE_{112} absorption cavity at 35 Gc/sec using a modified Varian epr spectrometer. The PG samples were in the form of thin flakes with parallel c-plane faces, the face area being less than 1 mm². Six highly annealed samples were investigated, with thicknesses ranging from ≈0.05 to ≈0.20 mm. The samples were placed at the center of the bottom plate of the cavity, at right angle to the bottom, i.e., with the c axis perpendicular to the cavity axis. In this configuration the electromagnetic field is present at both sample faces, and the boundary conditions at the faces reproduce only when the number of wavelengths λ inside the sample changes by an integer. The corresponding interference condition is thus $N\lambda = d$, where N is an integer, and d is the sample thickness. The applied dc magnetic field B could be rotated in the plane perpendicular to the cavity axis. The carrier mass-density function D is given in terms of the period of the interference pattern $\Delta(1/B)$ as

$$D(n,m^*) = 10^{-7}\pi[\omega d\Delta(1/B)]^{-2}\ \text{kg/m}^3.$$

Figure 1 shows the actual recorder traces for two samples of different thicknesses obtained for B parallel to the c axis. Note that the period varies inversely with the field. Note further that the interference pattern terminates when λ is about equal to d, as seen in the bottom curve at about 7 kG. The low-field structure corresponds closely to the cyclotron resonance data observed by Galt, Yager, and Dail in single-crystal graphite.[4]

Figure 2 shows the behavior of microwave absorption in a single sample as a function of the angle α between field B and the c axis. The interference pattern manifests a $B\cos\alpha$ dependence, in agreement with the quasi-two-dimensional nature of the material.

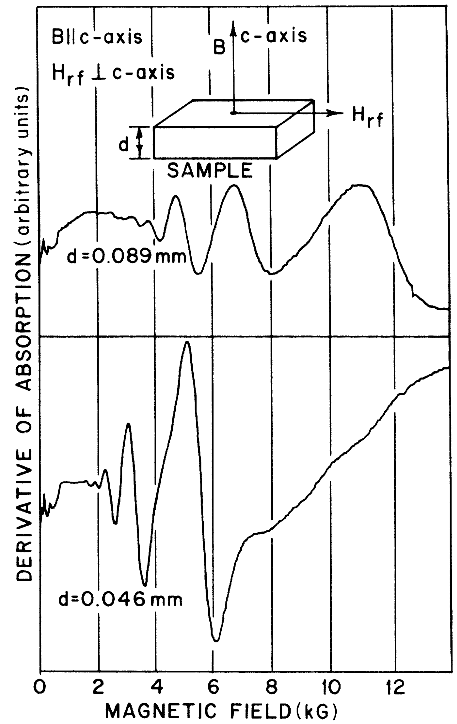

FIG. 1. Recorder trace of the derivative of absorption vs magnetic field for two PG samples of different thickness. The large oscillations represent the Alfvén-wave interference pattern, and cyclotron resonance appears in the low-field region of the curve. The sharp spike at 12.5 kG is an epr marker.

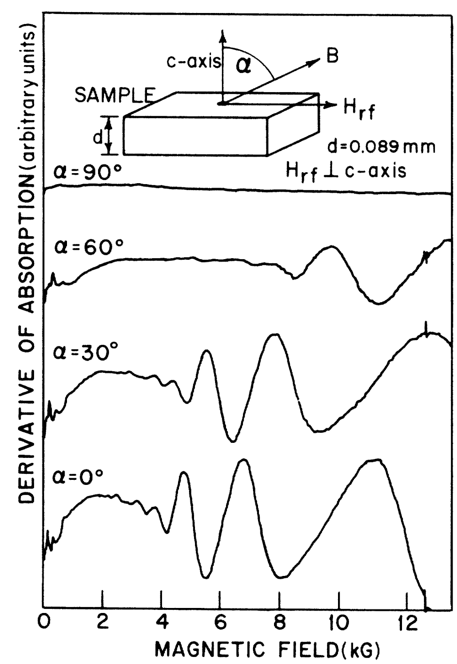

FIG. 2. Anisotropy of the Alfvén propagation in PG. The experimental configuration is indicated at the top. Note that the pattern is essentially determined by the projection of B on the c axis.

Measurements obtained on six samples of varying thicknesses were analyzed in terms of Eq. (2), yielding the carrier mass density function $D=(3.0\pm0.3)\times10^{-7}$ kg/m^3 for B parallel to the c axis. A calculation of D, carried out via Eq. (1) neglecting trigonal warping,[5] gives $D=3.25\times10^{-7}$ kg/m^3. Furthermore, if one uses the values of 0.057 and 0.039 for the hole and electron masses in the basal plane obtained from the Shubnikov–de Haas experiment[6] as average masses, the concentration $n_e=n_h\approx3.3\times10^{24}$/m^3 is estimated, in satisfactory agreement with the concentration obtained by other methods.[6,7]

Preliminary measurements were also carried out on purified natural single-crystal graphite. Typical results are shown in Fig. 3

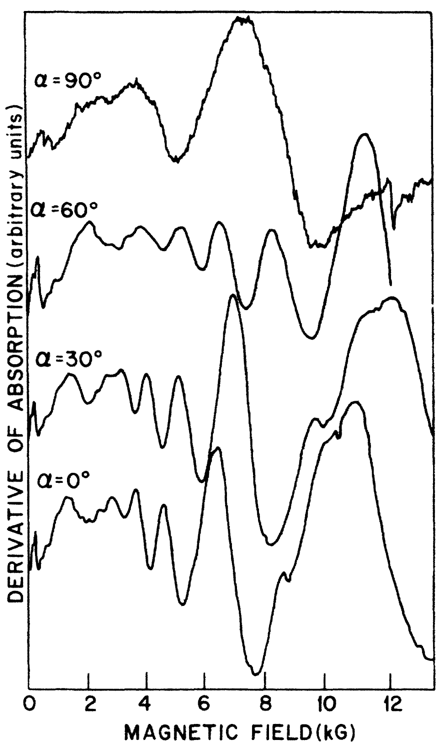

FIG. 3. Preliminary results obtained on single-crystal graphite. The configuration is as indicated in Fig. 2. Gross features of the behavior are strikingly similar to that of PG. Note, however, the behavior of absorption in the top trace (amplified five times relative to remaining curves), and the sharp kinks superimposed on the smooth interference pattern, seen in the lower curves.

for various values of α. The Alfvén-wave and cyclotron-resonance behavior resembles that observed in PG. Note, however, the complicated behavior in the $\alpha=90°$ curve (shown amplified), which was not observed in PG. No comparable resolution is, of course, expected in PG in this configuration due to lack of long-range order in the basal plane. Another unique feature of the data in Fig. 3 is the sharp kinks (seen clearly at $B=8.8$, 10.5, and 12.9 kG at $\alpha=0°$), which may possibly be associated with the Shubnikov–de Haas effect, but which, again, were not detected in the PG results. Further experimental and theoretical investigation of the features reported here are in progress.

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[1]G. A. Williams, Bull. Am. Phys. Soc. 7, 409 (1962); J. Kirsch, Phys. Rev. 133, A1390; G. A. Williams and G. E. Smith, IBM J. Res. Develop. 8, 276 (1964).

[2]G. A. Williams, Bull. Am. Phys. Soc. 8, 353 (1963).

[3]P. Nozières, Phys. Rev. 109, 1510 (1958); M. S. Dresselhaus and J. G. Mavroides, IBM J. Res. Develop. 8, 262 (1964).

[4]J. K. Galt, W. A. Yager, and H. W. Dail, Jr., Phys. Rev. 103, 1586 (1956).

[5]M. S. Dresselhaus, private communciation.

[6]D. E. Soule, J. W. McClure, and L. B. Smith, Phys. Rev. 134, A453 (1964).

[7]C. A. Klein, J. Appl. Phys. 33, 3338 (1962).