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INTRINSICALLY BROKEN U(6)⊗U(6) SYMMETRY FOR STRONG INTERACTIONS

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With an ever increasing internal-symmetry group for hadrons, the possibility of combining internal symmetry and space-time symmetry has become all the more appealing. Recent attempts[1,2] in this direction have, however, been confined to essentially nonrelativistic situations where the spin degrees of freedom can be regarded as internal. The group that has emerged from these investigations as a likely global group is SU(6). Following up this line of thought, we wish to extend these results to relativistic quantum field theory. We shall produce a chain of symmetries culminating in $W_6 \equiv U(6) \otimes U(6)$ that arises naturally in this case. In contradistinction to symmetries previously considered in physics, the largest members of this chain are intrinsically broken. In other words, there does not exist a total Lagrangian that possesses W_6 symmetry, since the kinetic energy and mass terms will automatically break it. The symmetry will show up only in the interaction term and will consequently make sense in terms of a strong-coupling limit.

Let us consider a triplet of spin-$\frac{1}{2}$ fermions (quarks),[3] and let $\psi^i(x)$, $i=1,2,3$, be the corresponding Dirac fields.[4] Introducing their left- and right-handed Weyl components R and L,

$$\begin{pmatrix} L \\ 0 \end{pmatrix} = \frac{(1+\gamma_5)}{2}\psi, \quad \begin{pmatrix} 0 \\ R \end{pmatrix} = \frac{(1-\gamma_5)}{2}\psi, \qquad (1)$$

we consider the following 72-parameter group of transformations:

$$L \to (1 + ia_{i\mu}\sigma^\mu\lambda_i)L,$$

$$R \to (1 + ia_{i\mu}^{*}\sigma^\mu\lambda_i)R, \qquad (2)$$

where $\sigma^\mu = (1, \vec{\sigma})$; λ_i, $i = 0, 1, \cdots, 8$, are the 3×3 Hermitean matrices defined by Gell-Mann[5] $[\lambda_0 = (\frac{2}{3})^{1/2}1]$, and $a_{i\mu}$ are complex parameters. The generators of the transformations (2) can be written as

$$\lambda_i,\ \sigma_{\mu\nu}\lambda_i,\ i\gamma_5\lambda_i, \qquad (3)$$

in the space of Dirac spinors. The matrices in (3) are reducible; the irreducible components are

$$\sigma_\mu\lambda_i,\ \pm i\sigma_\mu\lambda_i. \qquad (4)$$

The generators with upper (lower) sign act on L (R). This makes clear that the transformation group (2) is the complexification of U(6)—the full linear group in six dimensions GL(6). Finite-dimensional representations of GL(6) of physical interest can be classified by a method similar to the "unitary trick" of Weyl for the Lorentz group.[6] We form out of the set (3) the generators

$$\tfrac{1}{2}(1\pm\gamma_5)\Sigma_\mu\lambda_i,\ \Sigma_\mu=(1,\sigma_{23},\sigma_{31},\sigma_{12}),\tag{5}$$

which span the group $W_6\equiv U(6)\otimes U(6)$. The generators in (5) induce on R and L the infinitesimal transformations

$$R\to(1+i\alpha_\mu{}^i\sigma^\mu\lambda_i)R,$$

$$L\to(1-i\beta_\mu{}^i\sigma^\mu\lambda_i)L,$$

where $\alpha_\mu{}^i$ and $\beta_\mu{}^i$ are real.

A parity-conserving, Lorentz-invariant four-Fermion interaction which is invariant under GL(6) is[7]

$$\mathcal{L}_I=g\{\tfrac{1}{2}\bar\psi\sigma_{\mu\nu}\lambda_i\psi\bar\psi\sigma^{\mu\nu}\lambda_i\psi+\bar\psi\lambda_i\psi\bar\psi\lambda_i\psi+\bar\psi\gamma_5\lambda_i\psi\bar\psi\gamma_5\lambda_i\psi\}$$

$$=2g\delta_{\mu\nu}\{R^+\sigma_\mu\lambda_iLR^+\sigma_\nu\lambda_iL$$

$$+L^+\sigma_\mu\lambda_iRL^+\sigma_\nu\lambda_iR\}.\tag{6}$$

The equal-time commutation relations satisfied by the field ψ are not in general invariant under GL(6), and therefore it cannot be an invariance of the underlying Hilbert space. We consider it rather to be a dynamical symmetry possessed by the interaction Lagrangian and physical matrix elements in some approximation.

Another interaction Lagrangian invariant under GL(6) is

$$\mathcal{L}_I'=g'\{\bar\psi\gamma_\mu\lambda_i\psi\bar\psi\gamma^\mu\lambda_i\psi-\bar\psi\gamma_\mu\gamma_5\lambda_i\psi\bar\psi\gamma^\mu\gamma_5\lambda_i\psi\}.\tag{7}$$

There is a U(6) subgroup of GL(6) obtained by taking the $a_{i\mu}$ real. This is the group considered in reference 1. Both interaction Lagrangians (6) and (7) are invariant under this subgroup, which leaves the canonical commutation relations invariant.

The complete Lagrangian is the sum of kinetic energy, mass, and interaction terms:

$$\mathcal{L}=\mathcal{L}_K+\mathcal{L}_M+\mathcal{L}_I,$$

$$\mathcal{L}_K=i\bar\psi\gamma\cdot\partial\psi,$$

$$\mathcal{L}_M=-m\bar\psi\psi.$$

Since the four-momentum has four components, while GL(6) [or, for that matter, any of its SU(6) subgroups] has no four-dimensional representation, it is clear that there is no possibility of making $\mathcal{L}_K$ GL(6)-invariant (a similar argument applies to $\mathcal{L}_M$). Thus the only term of $\mathcal{L}$ that is GL(6) invariant is $\mathcal{L}_I$. If in some strong-coupling sense $\mathcal{L}_I\gg\mathcal{L}_K+\mathcal{L}_M$, then it is fair to claim that $\mathcal{L}$ exhibits intrinsically broken GL(6) invariance. Before we analyze in more detail this intrinsic symmetry breaking, let us give the representations to which the quarks and their bilinear covariants belong. The quarks form the representation[8] (1, 6) ⊗(6, 1) of W_6, and the bilinear covariants $\bar\psi\gamma_\mu(1+\gamma_5)\lambda_i\psi$ and $\bar\psi\gamma_\mu(1-\gamma_5)\lambda_i\psi$ form, respectively, the representations (35, 1)⊕(1, 1) and (1, 35)⊕(1, 1). Finally, $\bar\psi\sigma_{\mu\nu}\lambda_i\psi\oplus\bar\psi\lambda_i\psi\oplus\bar\psi\gamma_5\lambda_i\psi$ together form (6*, 6)⊕(6, 6*). The particle contents of all these representations are given in Table I. It is important to remember that irreducible representations of W_6 do not, in general, have definite Lorentz-transformation properties. In order to build up objects with well-defined transformation properties under the Lorentz group, one must consider reducible representations of W_6. There is some latitude in choosing the representations

Table I. Spin–parity–unitary-spin content of representations of W_6.

Representation of W_6	Spin, parity, unitary-spin dimensionality (J^P, N_3)
(35, 1)⊕(1, 35)⊕(1, 1)⊕(1, 1)	$(1^+,8)\oplus(1^-,8)\oplus(1^+,1)\oplus(1^-,1)\oplus(0^+,8)\oplus(0^-,8)\oplus(0^+,1)\oplus(0^-,1)$
(6*, 6)⊕(6, 6*)	$(1^+,8)\oplus(1^-,8)\oplus(1^+,1)\oplus(1^-,1)\oplus(0^+,8)\oplus(0^-,8)\oplus(0^+,1)\oplus(0^-,1)$
(6, 1)⊕(1, 6)	$(\tfrac{1}{2}^+,3)$
(56, 1)⊕(1, 56)	$(\tfrac{1}{2}^+,8)\oplus(\tfrac{3}{2}^+,10)$

in which one places the baryons. One obvious candidate is $(56,1)\oplus(1,56)$.

We now return to the intrinsic symmetry breaking, coming from the kinetic energy and mass terms in $\mathfrak{L}$. Their transformation properties (as members of incomplete W_6 multiplets) are easily found to be $(35,1)\oplus(1,35)$ and $(6^*,6)\oplus(6,6^*)$. We are treating the kinetic energy and mass terms as perturbations on an otherwise symmetric Lagrangian. Such a procedure is nonconventional (i.e., not describable in terms of the usual Feynman diagrams), but if this feature is ignored, one can proceed formally with group-theoretical arguments. It is simplest to think of the symmetry-breaking terms in the language of spurions, with the spurions possessing the requisite W_6 transformation properties. Spurions can only contribute in pairs to self-mass terms, with the pairs necessarily possessing the quantum numbers of the vacuum. These spurion pairs are of particular interest when classified according to the U(6) subgroup of W_6 mentioned above ($a_{i\mu}$ real). For the breakdown of this U(6) symmetry our spurion-pair mechanism implies that to lowest order the symmetry-breaking terms in the mass formulas transform like members of the 35-, 189-, and 405-dimensional representation, as assumed by Bég and Singh.[9]

Because of this subgroup most of the nonrelativistic results[10] based on U(6) can be obtained from W_6. However, W_6 predicts a super-supermultiplet structure on top of U(6), most characteristically the axial-vector and scalar mesons listed in Table I. Correspondingly, more general mass formulas can be derived on the basis of W_6 symmetry.

It is of interest to find the "would-be-conserved" currents of our model, and to calculate their (nonvanishing) divergences.[11] For example, corresponding to the parameter a_{i2} there is a current

$$j_\mu^{\ i2}=\frac{\delta\mathfrak{L}}{\delta a_{i2,\mu}}=\frac{i}{2}\bar{\psi}[\gamma_\mu,\sigma_{31}]_+\left(\frac{1+\gamma_5}{2}\right)\lambda^i\psi,$$

and its divergence

$$\partial_\mu j_\mu^{\ i2}=\frac{\delta\mathfrak{L}}{\delta a_{i2}}=\bar{\psi}(\gamma_3\partial_1-\gamma_1\partial_3)\left(\frac{1+\gamma_5}{2}\right)\lambda^i\psi+m\bar{\psi}\sigma_{31}\gamma_5\lambda^i\psi.$$

We then ascribe in the sense of a Goldberger-Treiman argument particles to these nonvanishing divergences of currents. In this way we may introduce a nonet each of vector, axial-vector, and pseudoscalar (the last arising from the noninvariant mass term) mesons, that form together an incomplete W_6 multiplet. The coupling of these particles to other physical states may be viewed as an effect of W_6 breakdown.

In conclusion we wish to point out that W_6 seems to be the natural group of hadrons.[12] One crucial test of its approximate validity would be the experimental discovery of the 1^+ and 0^+ mesons it predicts.

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[4]$\psi(x)$ is a 12-component spinor.

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[7]We use four-Fermion interactions only as a guide in our search for symmetries.

[8]We label the representations of W_6 by the dimensionalities of the representations of its two commuting U(6) factors.

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[11]See, e.g., M. Gell-Mann and M. Lévy, Nuovo Cimento 16, 705 (1960).

[12]There is a larger contender for the title of symmetry group of hadrons, a 144-parameter group, which we do not discuss here.

About the strong-coupling limit on which our above discussions are based, we would still like to make the following remark: One possible way of handling this problem is to study the behavior of vertex functions in the limit $g \to \infty$. We have done such a study in the context of chain diagrams and found that for $g \to \infty$ the vertex functions exhibit W_6 symmetry.

ERRATUM

CALCULATION OF NUCLEON-DEUTERON SCATTERING AND THE TRITON BINDING ENERGY. R. Aaron, R. D. Amado, and Y. Y. Yam [Phys. Rev. Letters 13, 574 (1964)].

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In addition we would like to call attention to the neutron-deuteron scattering data at 2.45 MeV of J. D. Seagrave and L. Cranberg [Phys. Rev. 105, 1816 (1957)]. These data are more accurate and more extensive than the data with which we compare in Fig. 2. They agree essentially exactly with our theoretical curve marked $Z = 0.0488$ over the measured angles from 25 to 165 degrees.