lished with a considerable degree of confidence. Of course, we do not claim that our proof is rigorous, but it will be much better than simply assuming the Mandelstam representation.

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GROUP U(6)⊗U(6) GENERATED BY CURRENT COMPONENTS*

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It has been suggested[1-3] that the equal-time commutation rules of the time components of the vector and axial-vector current octets ($\mathfrak{F}_{i\alpha}$ and $\mathfrak{F}_{i\alpha}{}^5$, respectively) are the same as if these currents had the simple form $\mathcal{G}_{i\alpha}$ and $\mathcal{G}_{i\alpha}{}^5$, defined as follows:

$$\mathcal{G}_{i\alpha} = \tfrac{1}{2}i\bar{q}\lambda_i\gamma_\alpha q,$$

$$\mathcal{G}_{i\alpha}{}^5 = \tfrac{1}{2}i\bar{q}\lambda_i\gamma_\alpha\gamma_5 q, \qquad (1)$$

where q is an SU(3) triplet with spin $\tfrac{1}{2}$ – for example, the quarks[2] or aces.[4] Here the matrices λ_i ($i = 1, \cdots, 8$) are the SU(3) analogs of the Pauli matrices, as defined in reference 1. The operators

$$F_i(t) = -i \int d^3x\, \mathfrak{F}_{i4},$$

$$F_i{}^5(t) = -i \int d^3x\, \mathfrak{F}_{i4}{}^5, \qquad (2)$$

then generate at equal times the algebra of SU(3)⊗SU(3), which may be a very approximate symmetry of the strong interactions,[1,3] while the F_i generate a subalgebra corresponding to SU(3), which is a fairly good symmetry of the strong interactions.

We now propose to extend these considerations to the space components of the currents as well. First we define[1-3] a ninth λ matrix $\lambda_0 \equiv (\tfrac{2}{3})^{1/2}1$ and a corresponding ninth pair of currents $\mathfrak{F}_{0\alpha}$ and $\mathfrak{F}_{0\alpha}{}^5$ (where $\mathfrak{F}_{0\alpha}$ would be $\sqrt{6}$ times the baryon current in a true quark

or ace theory). We then assume that the equal-time commutation relations of all the 72 components of the $\mathfrak{F}_{i\alpha}$ and $\mathfrak{F}_{i\alpha}{}^5$ ($i = 0, \cdots, 8$) are the same as those of the $\mathcal{G}_{i\alpha}$ and $\mathcal{G}_{i\alpha}{}^5$, at least as far as terms proportional to the spatial δ function are concerned. (There are also, in general, terms[5] involving gradients of the δ function, which vanish on space integration and which we ignore here.) The system of $\mathcal{G}_{i\alpha}$ and $\mathcal{G}_{i\alpha}{}^5$ is closed under equal-time commutation, and the space integrals $\int \mathcal{G}_{i\alpha}d^3x$ and $\int \mathcal{G}_{i\alpha}{}^5 d^3x$ generate the algebra of U(6)⊗U(6). Our assumption thus implies that $\int \mathfrak{F}_{i\alpha}d^3x$ and $\int \mathfrak{F}_{i\alpha}{}^5 d^3x$ also generate the algebra of U(6)⊗U(6). We assume further that this algebra is a very approximate symmetry of the strong interactions.

We now exhibit some of the structure of the algebra by looking at the $\mathcal{G}_{i\alpha}$ and $\mathcal{G}_{i\alpha}{}^5$. We note that the space integrals of the densities

$$\bar{q}\lambda_i\gamma_4 q = q^\dagger\lambda_i q \quad (i = 0, \cdots, 8)$$

and

$$i\bar{q}\lambda_i\gamma_n\gamma_5 q = q^\dagger\lambda_i\sigma_n q \quad (n = 1, 2, 3)$$

generate the subalgebra corresponding to U(6); the same is then true of the corresponding components of the $\mathfrak{F}$'s. We may refer to the algebra of the space integrals of these $\mathfrak{F}$ components as the A spin, with generators A_r ($r = 0, 1, \cdots, 35$). Now the space integrals of

the densities $q^\dagger \lambda_i \frac{1}{2}(1+\gamma_5)q$ and $q^\dagger \lambda_i \sigma_n \frac{1}{2}(1+\gamma_5)q$ also generate a group U(6), and so do the corresponding terms with $\frac{1}{2}(1-\gamma_5)$. The corresponding integrals of $\mathfrak{F}$ components thus give a left-handed A spin A_γ^+ and a right-handed A spin A_γ^-, respectively, with

$$A_\gamma = A_\gamma^+ + A_\gamma^- \quad (r = 0, 1, \cdots, 35). \tag{3}$$

Those 36 components of $\mathfrak{F}_{i\alpha}$ and $\mathfrak{F}_{i\alpha}^5$ (out of a total of 72) that are the densities of the A_γ do not go just into themselves under Lorentz transformations, but yield instead the complete system of 72 components of the $\mathfrak{F}_{i\alpha}$ and $\mathfrak{F}_{i\alpha}^5$, which form the densities of A_γ^+ and A_γ^-.

We have assumed above that the A^+ and A^- spins are separately very approximate symmetries of the strong interactions. We may now add the further assumption that the total A spin is a good symmetry, nearly as good as the subset that constitutes the F spin. This approximate conservation of A spin is then our way of describing the success achieved by the SU(6) symmetry of Gürsey and Radicati,[6] Sakita,[7] and Zweig,[8] treated further in a series of recent Letters.[9-14] In reference 10, our interpretation of the symmetry is hinted at, but otherwise it is described in different language, which does not make clear the physical identification of the symmetry operators with integrals of components of the vector and axial-vector currents occurring in the weak and electromagnetic interactions. Also, the Lorentz-complete system, obeying the commutation rules of U(6)⊗U(6), is not given.

In a relativistic situation, where a state like ρ exists part of the time as 2π, part of the time as $N+\bar{N}$, part of the time as $\Delta+\bar{\Delta}$, etc., with a different set of channel spins in each case, it is evidently not sufficiently specific to talk of "spin independence" of strong interactions. In contrast, our statement in terms of the approximate conservation of the Gamow-Teller operator $\int \mathfrak{F}_{in}^5 d^3x$ $(n = 1, 2, 3)$ does have a definite meaning.

One set of consequences of our approach is that the Gamow-Teller matrix elements within an SU(6) supermultiplet can be exactly computed in the limit of SU(6) symmetry. We adopt the assignments of the $J^\pi = \frac{1}{2}^+$ baryon octet and $J^\pi = \frac{3}{2}^+$ baryon decimet to the SU(6) representation 56, and the assignment of the vector-meson octet and singlet and the pseudoscalar octet to the representation 35; these

assignments have explained at least six well-known facts.[15] The axial-vector strength, within the baryon octet, comes out to be $1(D) + \frac{2}{3}(F)$; for the nucleon, this gives $(-G_A/G_V) = 5/3$, as indicated in reference 10, to be compared with an observed value more like 1.2. The agreement is fair, as is the agreement of the D/F ratio with the results on leptonic hyperon decays. The matrix elements of the Gamow-Teller operator between octet and decimet are also exactly specified in the limit of SU(6) symmetry and can be checked by neutrino experiments.

Let us now go on to discuss the badly broken symmetry U(6)⊗U(6), which bears about the same relation to U(6) symmetry as the U(3) ⊗U(3) symmetry generated by the time components of vector and axial-vector currents[1,3] bears to the eightfold way. On the way from the full U(6)⊗U(6) down to U(3), we could pass through U(6) or through U(3)⊗U(3) symmetry as an intermediate stage; these are alternatives in somewhat the same way as are L-S and j-j coupling in atomic physics. It seems that the operators of U(6), all of which have nonrelativistic limits, form a much better symmetry system than those of U(3)⊗U(3); hence, the useful procedure is to go from U(6) ⊗U(6) to U(6), and then to U(3) and U(2). [Actually U(6) is not much worse than U(3).]

The baryons are presumed to have zero mass in the limit of U(6)⊗U(6) symmetry, as in the limit of U(3)⊗U(3) symmetry.[1,3] The perturbation that reduces the symmetry of U(6) is assumed to transform like $(\underline{6}, \underline{6}^*)$ and $(\underline{6}^*, \underline{6})$ under (A^+, A^-), and like $\underline{1}$ under A. Thus it transforms like a common quark mass term qq, which takes a left-handed q going like $(\underline{6}, \underline{1})$ into a right-handed q going like $(\underline{1}, \underline{6})$, and vice versa.[16] The $J^\pi = \frac{1}{2}^+$ octet and $J^\pi = \frac{3}{2}^+$ decimet belonging to 56 can be placed either in $(\underline{1}, 56)$ and $(56, \underline{1})$, or in $(\underline{6}, 21)$ and $(21, \underline{6})$, if we restrict ourselves to representations that transform like $3q$. The latter is very attractive, because it splits into a 56 and a 70, where the masses to first order in the perturbation are in the ratio 1:−2; as in reference 3, we must interpret negative mass as positive mass with negative parity, and so we are led to a 56 with unit mass and a 70 with opposite parity and roughly twice the mass. The 70 contains a $\frac{3}{2}^-$ octet, a $\frac{1}{2}^-$ singlet, a $\frac{1}{2}^-$ octet, and a $\frac{1}{2}^-$ decimet. Thus the prediction of reference 3 that the $\frac{1}{2}^+$ octet is accompanied by a $\frac{1}{2}^-$ sing-

let of roughly twice the mass is contained in our present result. The $\frac{3}{2}^-$ octet has probably been seen [including $N(1512)$], but the $\frac{1}{2}^-$ octet and decimet have not so far been identified.

In the limit of $U(6) \otimes U(6)$ symmetry, the vector and pseudoscalar mesons of the 35 can be put into either of two pairs of representations that transform like $q + \bar{q}$. The mesons could go like (35, 1) and (1, 35), or else like (6, 6*) and (6*, 6). If they belong to the adjoint representation pair (1, 35) and (35, 1), as the current components do, then the usual 35 is accompanied by another 35, consisting of a normal axial-vector octet and singlet and an abnormal scalar octet. [Here, "normal" means that the $Y = 0$, $I = 0$ member of an axial vector, scalar, or pseudoscalar SU(3) multiplet is even under charge conjugation; "abnormal" means it is odd.] If the mesons belong to (6, 6*) and (6*, 6), then the usual 35 is accompanied by a 1 (a normal pseudoscalar singlet), another 1 (a normal scalar singlet), and a 35 consisting of an abnormal axial-vector octet and singlet and a normal scalar octet. In either case, the perturbation that reduces $U(6) \otimes U(6)$ to $U(6)$ does not split the mesons into $U(6)$ multiplets in first order; in second order, they are split. The assignment to (6, 6*) and (6*, 6) is appealing because the pseudoscalar singlet could be identified with $\eta(960)$, the scalar octet may include $\kappa(725)$, and the abnormal axial octet may include the meson at about 1220 MeV with $I = 1$ that decays into $\pi + \omega$.

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[16]At the end of reference 3, it is suggested that perhaps the perturbation in the energy density that reduces $U(3) \otimes U(3)$ symmetry to $U(3)$ symmetry generates, together with the algebra of $U(3) \otimes U(3)$, a small algebra, which could be that of $U(6)$. Such a use of $U(6)$ is not the same as the use we are discussing in this Letter. However, we might consider the analogous possibility that the perturbation in the energy density that reduces $U(6) \otimes U(6)$ to $U(6)$ generates, together with the algebra of $U(6) \otimes U(6)$, the algebra of $U(12)$, corresponding to all unitary transformations on the four Dirac components and the three unitary-spin components of a quark field. Even if this is true, of course, $U(12)$ need not be a useful symmetry of strong interactions.