N^* PRODUCTION BY NEUTRINOS†

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Several striking features of the high-energy neutrino experiment in progress at CERN[1] are these: (a) The "inelastic" events number approximately as many as the "elastic" ones; (b) most of the "inelastic" events fall into the single-pion-production category; and (c) single-pion production,

$$\nu_l\ (\bar\nu_l)+N\to l\ (\bar l)+N+\pi, \qquad (1)$$

appears to proceed predominantly through the formation and subsequent decay of the (3,3) pion-nucleon isobar,

$$\nu_l\ (\bar\nu_l)+N\to l\ (\bar l)+N^*, \qquad (2a)$$

$$N^*\to N+\pi, \qquad (2b)$$

where l denotes a muon or an electron.

In this note we wish to study the direct N^*-production process, (2a).[2,3] If one assumes a local V-A interaction for the leptons, the dynamics of this reaction is entirely contained in the N-N^* transition vertex. A cursory glance at Fig. 1, where some of the intermediate states contributing to the vertex function are singled out, reveals that Landau singularities will be encountered in a dynamical calculation. These are of interest in their own right, but here we report only some results of a <u>phenomenological</u> study of this vertex. The analysis of the subsequent decay [Eq. (2b)] of the N^* will then reveal the nature of the vertex functions involved.

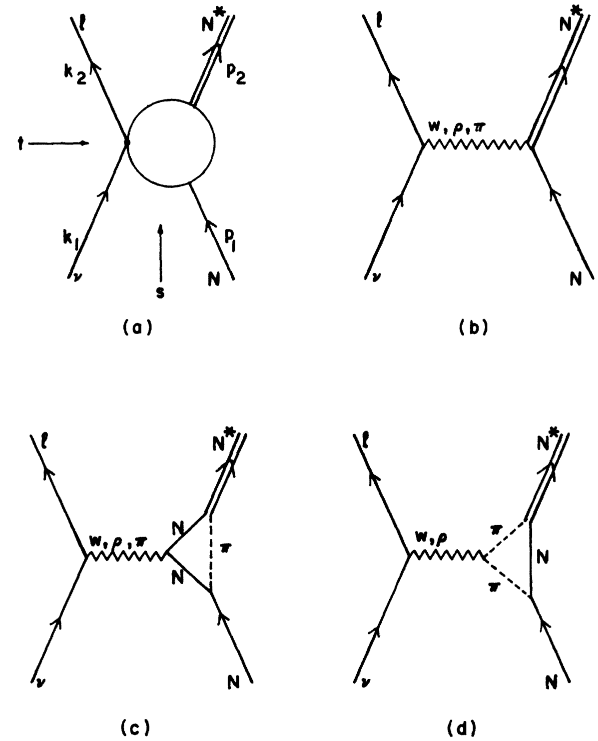

FIG. 1. Feynman diagrams for N^* production by neutrinos.

With the assumption of a point V-A interaction for the leptons, the most general matrix element for N^* production by neutrinos can be written as follows:

$$\mathfrak{M}=\frac{G}{\sqrt{2}}\bar\psi_\lambda\left[\delta_{\lambda\mu}(F_1^A+F_1^V\gamma_5)+\frac{ip_{1\lambda}\gamma_\mu(F_2^A+F_2^V\gamma_5)}{M_1}+\frac{p_{1\lambda}(p_1+p_2)_\mu(F_3^A+F_3^V\gamma_5)}{M_1^2}+\frac{p_{1\lambda}(p_1-p_2)_\mu(F_4^A+F_4^V\gamma_5)}{M_1^2}\right]u_N\bar u_l\gamma_\mu(1+\gamma_5)u_\nu. \qquad (3)$$

The Rarita-Schwinger representation[4] for the $\frac{3}{2}^+$ N^* is indicated by ψ_λ. We denote by k_1, p_1, k_2, and p_2 the four-momenta of the neutrino, nucleon of mass M_1, lepton of mass m_l, and isobar of mass M_2, respectively, as in Fig. 1(a). The transition form factors, $F_i^{V,A}$, are functions of momentum transfer t, where the above combination is selected for convenience. If one applies the conserved-vector-current (CVC) hypothesis[5] to this octet-decuplet transition, the vector form factors satisfy the following linear relationship:

$$F_1^V+\frac{(M_1+M_2)}{M_1}F_2^V+\frac{(M_2^2-M_1^2)}{M_1^2}F_3^V-\frac{t}{M_1^2}F_4^V=0. \qquad (4)$$

We prefer not to impose this restriction at the outset but shall discuss the consequences of

Eq. (4) in our numerical analysis.

The differential cross section for unpolarized production is calculated in closed form and can be summarized by

$$\frac{d\sigma}{dt}=\frac{G^2}{4\pi}\frac{T}{(s-M_1^2)^2}, \tag{5}$$

where

$$T=(M_1M_2m_lE_\nu)\overline{\sum}|\mathfrak{M}|^2$$

$$=\sum_{i=1}^{5}R_i(t)X_i(s,t); \tag{6}$$

$$s=-(p_1+k_1)^2,$$

$$t=-(k_2-k_1)^2.$$

Summation over the N^* and lepton spins and averaging over the initial nucleon spin are implied. The $X_i(s,t)$ can be simply expressed in terms of invariant quantities:

$$X_1=(p_2\cdot k_2)(p_1\cdot k_1)+(p_2\cdot k)(p_1\cdot k_2),$$
$$X_2=(p_2\cdot k_2)(p_1\cdot k_1)-(p_2\cdot k_1)(p_1\cdot k_2),$$
$$X_3=(p_2\cdot k_2)(p_2\cdot k_1),$$
$$X_4=(p_1\cdot k_2)(p_1\cdot k_1),$$
$$X_5=(k_1\cdot k_2). \tag{7}$$

The R_i are functions of the form factors and depend only on t. Since only X_2 is antisymmetric under k_1-k_2 interchange, only R_2 contains V-A interference terms; moreover, only $F_1^{V,A}$ and $F_2^{V,A}$ contribute to R_2. The V-A interference effect becomes less and less important at higher energies since X_2 depends linearly on s while X_1, X_3, and X_4 vary as s^2. For the corresponding antineutrino process, replacement of R_2 by $-R_2$ with a suitable reinterpretation of the form factors is required in the above expressions for the differential cross section.

In order to estimate the total cross section and its energy behavior for a specific charge channel of (2a), we postulate the following phenomenological form factors:

$$F_i^{V,A}(t)=a_i^{V,A}/(t-b_i^{V,A})^2, \tag{8}$$

for $i=1$, 2, and 4. The t dependence is analogous to that of the form factors[6] appearing in the "elastic" case, and has the desirable feature of insuring a finite asymptotic cross section.[7] For $F_3^{V,A}(t)$, on the other hand, the above form yields a logarithmically increasing cross section. Hence, $F_3^{V,A}$ must fall off faster than t^{-2}. In addition, the higher order induced nature of these terms in (3) results in a very slowly rising contribution to the total cross section above threshold. Thus we ignore these terms. Since the coefficients of $F_4^{V,A}$ are proportional to m_l, we also drop these terms.

Numerical results are presented in Table I for N^{*++} production by ν_μ.[8] We have retained $F_1^{V,A}$ and $F_2^{V,A}$ and have set $b_1^{V,A}=b_2^{V,A}=b$. Values chosen for the cutoff parameter b are 1, 20, 37.4, 60, and $120m_\pi^2$, where the third one coincides with that for the "elastic" process. Various cases are tabulated according to the choice of the parameters $a_i^{V,A}$. The front-to-back ratio, F/B, is defined for the outgoing muon relative to the incident neutrino in the center-of-mass system. The total cross sections for $b=37.4m_\pi^2$ are plotted against laboratory neutrino energy in Fig. 2 in order to illustrate the general behavior.

The results should be interpreted in the following sense: The universal four-fermion coupling constant $GM_1^2=1.02\times10^{-5}$ is adopted[9] for the $\Delta S=0$ process, $\nu_\mu+n\to N^{*++}+\mu^-$. The form factors selected for cases (a) through (d) are normalized at zero momentum transfer to unity for the vector ones and 1.2 for the axial-vector ones. Hence for these cases the numerical results demonstrate the relative effectiveness of the form factors in contributing to the cross section. For the remaining cases, the CVC hypothesis has been imposed. With $F_3^V=F_4^V=0$, the numerical values of $F_1^V(0)$ and $F_2^V(0)$ are related by Eq. (4) and for cases (e) through (g) are deduced to be 5.6 and −2.4, respectively, from data on N^* photoproduction;[10] for cases (e′) through (g′), the predictions[11] of SU(6) are adopted: $F_1^V(0)=3.5$ and $F_2^V=-1.5$.

We find the total cross section and front-to-back ratio are very sensitive to the types of form factors involved and the cutoff parameter b. In particular, we single out the following features:

(1) As b increases, σ increases while the ratio F/B decreases by orders of magnitude. The ratio F/B always remains relatively higher, however, for F_1^A [case (a)] than for the other pure cases [(b) through (d)].

(2) In general, the cross section rises rapidly and saturates within several BeV. This behavior is extremely striking for F_1^A.

Table I. Total cross section[a] and front-to-back ratio for the production process, $\nu_\mu + n \to N^{*+} + \mu^-$, with $M_1 = 939$ MeV, $M_2 = 1238$ MeV, and $m_\mu = 106$ MeV. The individual cases (a) through (e) indicate the form factors chosen with the others set equal to zero. Case (a): $a_1^A = 1.2b^2$; case (b): $a_1^V = b^2$; case (c): $a_2^A = 1.2b^2$; case (d): $a_2^V = b^2$; case (e): $a_1^V = -2.32a_2^V = 5.6b^2$.

Multiply each of the following σ entries by 10^{-40} cm^2. Read 4.9E2 as 4.9×10^2.

b	E_ν(lab)	(a)		(b)		(c)		(d)		(e)	
(m_π^2)	(BeV)	σ	F/B	σ	F/B	σ	F/B	σ	F/B	σ	F/B
1	0.75	0.064	4.9E2	0.0011	1.6E2	0.0076	4.1E2	0.0058	1.7E2	0.0077	3.8E1
	2.0	0.241	2.0E5	0.0036	1.6E4	0.028	2.0E4	0.020	1.2E4	0.015	1.6E3
	5.0	0.337	7.2E6	0.0050	2.9E5	0.039	1.5E5	0.027	1.1E5	0.019	1.5E4
	10.0	0.373	5.9E7	0.0055	1.8E6	0.042	4.3E5	0.029	3.7E5	0.020	4.8E4
20	0.75	4.05	3.6	0.131	1.7	0.573	3.9	0.674	1.7	2.99	1.1
	2.0	6.69	8.1E1	0.258	1.6E1	1.93	1.9E1	1.50	1.2E1	6.72	9.6
	5.0	7.74	1.4E3	0.306	1.5E2	2.57	7.9E1	1.82	6.2E1	8.38	5.4E1
	10.0	8.13	9.3E3	0.325	7.7E2	2.79	2.0E2	1.95	1.8E2	9.10	1.5E2
37.4	0.75	6.52	2.1	0.243	1.0	0.922	2.4	1.24	1.1	5.95	0.73
	2.0	12.0	2.4E1	0.654	6.1	4.70	7.5	3.97	4.7	20.0	4.3
	5.0	14.0	3.0E2	0.818	4.7E1	7.18	2.6E1	5.26	2.0E1	27.5	2.0E1
	10.0	14.8	1.8E3	0.891	2.2E2	8.17	6.2E1	5.82	5.4E1	31.0	5.4E1
60	0.75	9.06	1.5	0.359	0.77	1.27	1.8	1.83	0.79	9.14	0.55
	2.0	19.9	1.0E1	1.40	3.1	9.92	3.8	8.90	2.4	47.0	2.3
	5.0	24.0	9.2E1	1.92	1.9E1	18.2	1.1E1	13.5	8.7	74.7	9.4
	10.0	25.7	4.8E2	2.16	8.2E1	22.2	2.5E1	15.7	2.2E1	89.1	2.3E1
120	0.75	12.1	1.2	0.512	0.59	1.68	1.4	2.60	0.61	13.4	0.44
	2.0	36.7	4.2	3.48	1.5	23.7	1.9	23.5	1.2	127.0	1.2
	5.0	48.7	2.3E1	5.95	6.8	63.0	4.1	48.8	3.3	276.0	3.7
	10.0	54.3	9.2E1	7.27	2.5E1	89.1	8.2	64.3	7.3	376.0	8.2

[a]See reference 8.

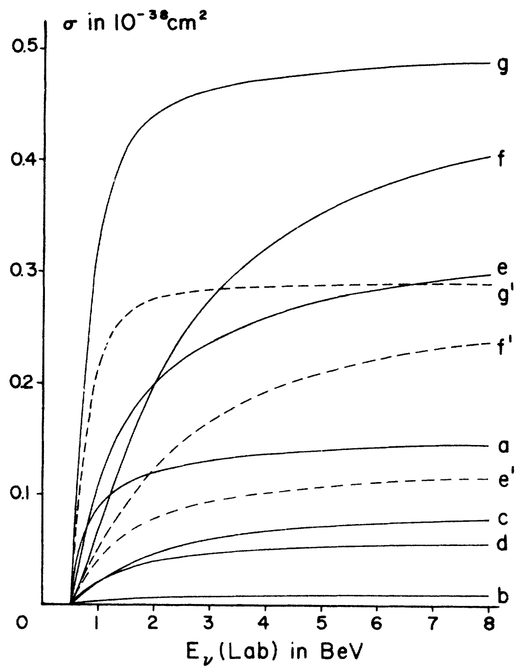

FIG. 2. Total cross section for N^{*+} production[8] with $b = 37.4m_\pi^2$. The letter attached to each curve refers to a particular case in Table I. Additional cases are case (f): $a_1^V = -2.32a_2^V = 5.6b^2$, $a_1^A = 1.2b$; case (g): $a_1^V = -2.32a_2^V = 5.6b^2$, $a_1^A = -1.2b^2$; case (e′): $a_1^V = -2.32a_2^V = 3.5b^2$; case (f′): $a_1^V = -2.32a_2^V = 3.5b^2$, $a_1^A = 1.2b^2$; case (g′): $a_1^V = -2.32a_2^V = 3.5b^2$, $a_1^A = -1.2b^2$.

(3) The V-A interference effect can make a significant contribution to the N^* production cross section.[12]

(4) For all choices of the form factors used, no maximum is obtained in either the neutrino or antineutrino cross sections in contrast to the "elastic" case.

(5) The upper limit found for the asymptotic cross section ranges from $\sim 10^{-40}$ cm^2 for $b = m_\pi^2$ up to $\sim 5 \times 10^{-38}$ cm^2 for $b = 120m_\pi^2$. Recall that in the "elastic" case,[13] this limit is predicted to be $\sim 0.75 \times 10^{-38}$ cm^2.

It is apparent from Fig. 2 that F_1^V [case (b)] is rather ineffective for the production process. On the basis of the CVC hypothesis and the N^* photoproduction analysis of Gourdin and Salin,[10] however, F_1^V and F_2^V are weighted heavily. In fact, for a reasonable cutoff parameter b and the coupling constant chosen, the predicted cross sections in cases (e), (f), and (g) are larger than the experimental result.[1] On the other hand, somewhat better agreement is obtained in cases (e′), (f′), and (g′) where $F_1^V(0)$ and $F_2^V(0)$ are deduced from the CVC hypothesis and the work of Bég, Lee, and Pais[11] on SU(6).

Direct form factors $F_1^{V,A}$ receive contributions only from Fig. 1(b) with vector or axial-vector meson exchange—aside from a possible fundamental four-fermion interaction: $(N^*N)(l\nu)$. Of these two, only F_1^V benefits from ρ-meson exchange. A substantial contribution by F_1^A indicates the existence of at least one of the following: the intermediate boson,[14] a fundamental $(N^*N)(l\nu)$ interaction,[15] or some axial-vector meson such as that proposed by several authors.[16]

One test of the strength of $F_1^{V,A}$ recognizes the following fact: Since $p_{1\mu}\psi_\mu^{(\lambda)}$ vanishes in the lab system for $\lambda=\pm\frac{3}{2}$, the reaction as measured in the lab system proceeds only via $F_1^{V,A}$ for these polarization states.

A detailed account of this work as well as of polarized N^* production will be published elsewhere.

It is a pleasure to thank Professor L. M. Brown for his continued interest and encouragement throughout the course of this work. We also wish to thank Professor M. M. Block for valuable comments concerning this work and for informing us of the latest experimental results obtained by the CERN heavy-liquid bubble chamber group.

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[1]G. Bernardini, Report of Argonne National Laboratory Accelerator Users Group Meeting, 5 December 1963 (unpublished). M. M. Block, H. Burmeister, D. C. Cundy, B. Eiben, C. Franzinetti, J. Keren, R. Møllerud, G. Myatt, A. Orkin-Lecourtois, M. Paty, D. Perkins, C. A. Ramm, K. Schultze, H. Sletten, K. Soop, R. Stump, W. Venns, and H. Yoshiki, to be published.

[2]Process (1) has been investigated theoretically by several authors. In these treatments the effect of the N^* is inserted to enhance the appropriate intermediate state in a dispersion-theoretical or static-model approach. See, e.g., J. S. Bell and S. M. Berman, Nuovo Cimento 25, 404 (1962); N. Cabibbo and G. Da Prato, Nuovo Cimento 25, 611 (1962); N. Dombey, Phys. Rev. 127, 653 (1962); P. Dennery, Phys. Rev. 127, 664 (1962); Nguyen Van-Hieu, Zh. Eksperim. i Teor. Fiz. 43, 1296 (1962) [translation: Soviet Phys.—JETP 16, 920 (1963)]; S. L. Adler, private communication.

[3]Process (2a) has been discussed recently by I. M. Zheleznykh [Phys. Letters 11, 251 (1964)] in the simplified case where the two direct form factors are taken equal and all others are set equal to zero. After submission of our note, the following preprints in which the authors have studied the same process from a somewhat different point of view have been brought to our attention: S. M. Berman and M. Veltman, to be published; C. W. Kim, to be published; D. L. Weaver, H. S. Song, C. L. Hammer, and R. H. Good, Jr., to be published.

[4]W. Rarita and J. Schwinger, Phys. Rev. 60, 61 (1940); S. Kusaka, Phys. Rev. 60, 61 (1940); L. M. Brown and V. L. Telegdi, Nuovo Cimento 7, 698 (1958).

[5]S. Gerschtein and J. Zeldovich, Zh. Eksperim. i Teor. Fiz. 29, 698 (1955) [translation: Soviet Phys.—JETP 2, 576 (1956)]; R. P. Feynman and M. Gell-Mann, Phys. Rev. 109, 193 (1958).

[6]See, e.g., R. Hofstadter, F. Bumiller, and M. R. Yearian, Rev. Mod. Phys. 30, 482 (1958).

[7]The results presented at the 1964 Dubna Conference suggest that the N^* production cross section saturates above several BeV; cf. reference 1. Thus the above t dependence of the form factors appears very reasonable.

[8]The cross section for $\nu_\mu+p\rightarrow N^{*++}+\mu^-$ is obtained by multiplying the tabulated values for N^{*+} production by 3; hence the weak N^* production cross section per nucleon is twice the listed numbers.

[9]At low momentum transfer, $t\approx 0$, the invariant differential cross section per nucleon is given approximately by $[2/(3\pi M_1^4)](GM_1^2)^2|F_1^A|^2$, where the lepton mass and the N^*-N mass difference are neglected. The experimental result quoted by the CERN group in the 1964 Dubna Conference Report is $(0.5\pm0.2)\times10^{-38}\ \mathrm{cm}^2/(\mathrm{GeV}/c)^2$ averaged over the ranges $0\leq -t\leq 0.2(\mathrm{GeV}/c)^2$ and $1.0\leq E_\nu\leq 3.0$ GeV. This suggests that the weak coupling constant for N^* production is comparable to that appearing in the elastic reaction.

[10]M. Gourdin and Ph. Salin, Nuovo Cimento 27, 193, 309 (1963).

[11]M. A. B. Bég, B. W. Lee, and A. Pais, Phys. Rev. Letters 13, 514 (1964).

[12]This feature was also observed by Berman and Veltman,[3] who included the effects of the N^* decay. For the specific cases considered by these two authors, our numerical results are in substantial agreement.

[13]T. D. Lee and C. N. Yang, Phys. Rev. Letters 4, 307 (1960); N. Cabibbo and R. Gatto, Nuovo Cimento 15, 304 (1960); Y. Yamaguchi, Progr. Theoret. Phys. (Kyoto) 23, 1117 (1960).

[14]T. D. Lee and C. N. Yang, Phys. Rev. 119, 1410 (1960).

[15]See, e.g., M. M. Block, Phys. Rev. Letters 12, 262 (1962).

[16]See, e.g., P. Dennery and H. Primakoff, Phys. Rev. Letters 8, 350, 466(E) (1962); P. G. O. Freund and Y. Nambu, to be published.