LARGE-SPIN ROTATIONAL STATES OF DEFORMED NUCLEI*

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In a recent Letter,[1] Stephens, Lark, and Diamond have strikingly demonstrated the increase of moment of inertia, $\mathcal{I}$, as one progresses to higher angular-momentum states for a given nucleus. They have investigated the ground-state bands of several rare-earth even-even nuclei, in some cases observing all levels up to $I=18$. The effect of increasing I can result in a two- to three-fold increase in $\mathcal{I}$ over this range. Since the effective moment of inertia determined from the $I=2\rightarrow I=0$ level spacing is normally about one-third to one-half of the rigid-body value, one sees that by the time the $I=18$ level is reached, the rigid-body $\mathcal{I}$ is almost attained.

There are many alternative explanations of this phenomenon. Mottelson and Valatin[2] have outlined one such possibility within the framework of nuclear pairing theory. One effect of the rotational motion is to reduce the effective pairing-force strength, G, from the no-rotation value. Since it has been shown that the pairing force can account for the reduction of $\mathcal{I}$ from the naively expected rigid value,[3] one would expect that an increase in the angular velocity ω (corresponding to larger I values) would result in less pairing (smaller G_{eff}) and consequently lead to an increase in $\mathcal{I}$ toward the rigid-body value. In addition, a critical value of G generally exists such that pairing solutions do not occur for $G_{\text{eff}}<G_{\text{crit}}$. This indicates that the energy gap should be completely destroyed for values of I corresponding to $\omega>\omega_{\text{crit}}$.

The connection between $\mathcal{I}$ and the energy gap is illustrated in the case of the $K^{\pi}=0^{-}$ band which occurs at low excitations in the deformed regions. This band may be based on a particular two-quasiparticle state or on a more collective linear superposition of such states.[4,5] In either case, the energy gap should be smaller (reduced by blocking) than it is in the low-lying levels of the ground-state band. Such an effect is observed; typically one finds $\mathcal{I}(K^{\pi}=0^{-})\sim 1.7\,\mathcal{I}_{\text{gnd}}$.[5]

Another explanation of the dependence of $\mathcal{I}$ on the degree of rotation comes from the theory of Davydov and Chaban.[6] They consider the coupling between the β vibrations and rotational motion which leads to a centrifugal stretching of the nucleus and a corresponding increase in $\mathcal{I}$ as the rotational frequency is increased. The authors of reference 1 use this theory to obtain a good two-parameter fit to their data.

The main purpose of this paper is to point out still another and perhaps simpler explanation for this effect. The moment of inertia is usually calculated from the cranking model.[7] One begins with a rotating, deformed, self-consistent potential well. The Hamiltonian is then transformed to a reference frame rotating with the nucleus. In this system, a new term appears, $H'=-\omega J_x$. If this term is treated as a perturbation, then upon transforming back to the lab coordinates one obtains, to lowest order,

$$\Delta E=\omega^2\sum_m \frac{|\langle m|J_x|0\rangle|^2}{E_m-E_0}. \tag{1}$$

Since ΔE must correspond to the rotational energy $(\mathcal{I}_0\omega^2/2)$, one can solve for $\mathcal{I}_0$,

$$\mathcal{I}_0=2\sum_m \frac{|\langle m|J_x|0\rangle|^2}{E_m-E_0}. \tag{2}$$

This is the usual cranking-model result. It is proposed here, however, that the next higher order term in this perturbation expansion is important. Retaining terms of order ω^4, one has

$$E=\tfrac{1}{2}(\mathcal{I}_0+3C\omega^2)\omega^2, \tag{3}$$

where C can be written as

$$C=2\sum_{mnp}\frac{\langle 0|J_x|n\rangle\langle n|J_x|m\rangle\langle m|J_x|p\rangle\langle p|J_x|0\rangle}{(E_n-E_0)(E_m-E_0)(E_p-E_0)} - \mathcal{I}_0\sum_m\frac{|\langle 0|J_x|m\rangle|^2}{(E_m-E_0)^2}. \tag{4}$$

This calculation also leads to the result

$$\langle J_x\rangle=\omega(\mathcal{I}_0+2C\omega^2). \tag{5}$$

Upon comparing Eqs. (3) and (5), one sees that

Table I. Parameters used in solving Eq. (6).

Nucleus	$\mathcal{I}_0$ (10^{-2} keV^{-1})	C (10^{-8} keV^{-3})
W^{174}	2.596	9.470
W^{176}	2.689	7.768
Hf^{166}	1.734	9.909
Hf^{168}	2.328	9.554
Hf^{170}	2.894	11.647
Hf^{172}	3.116	7.050
Yb^{164}	2.369	8.385
Yb^{166}	2.897	6.541

different effective values of $\mathcal{I}$ enter in calculations of the rotational energy and the angular momentum. Thus it seems as though a unique moment of inertia does not exist in the usual sense when one considers terms of higher order in ω.

The present model is based on the two equations

$$E = \tfrac{1}{2}\omega^2(\mathcal{I}_0 + 3C\omega^2), \tag{3}$$

and

$$[I(I+1)]^{1/2} = \omega(\mathcal{I}_0 + 2C\omega^2). \tag{5'}$$

The parameter ω may be eliminated from these, leading to a cubic equation in E involving only two parameters, $\mathcal{I}_0$ and C:

$$\frac{128C^2}{\mathcal{I}_0^3}\frac{E^3}{I(I+1)} - \frac{32C}{\mathcal{I}_0}\frac{E^2}{I(I+1)} + 2\mathcal{I}_0\frac{E}{I(I+1)} + \frac{72C}{\mathcal{I}_0^2}E - 1 - \frac{27C}{\mathcal{I}_0^3}I(I+1) = 0. \tag{6}$$

The third and fifth terms (those not containing C) lead directly to the simple rotational spectrum, $E = I(I+1)/2\mathcal{I}_0$.

A better idea of the effect of C on the spectrum may be obtained by solving Eq. (6) by iteration:

$$E = [I(I+1)/2\mathcal{I}_0]\{1 - x + 4x^2 - 24x^3 + \cdots\}, \tag{7}$$

where $x = CI(I+1)/\mathcal{I}_0^3$. All powers of $I(I+1)$ enter into the spectrum, although their coefficients are uniquely determined in terms of only two parameters, C and $\mathcal{I}_0$.

To see how well Eq. (6) fits the observed spectra, C and $\mathcal{I}_0$ are determined by a least-squares fit to the experimental energy levels.[1,8] Equation (6) is then solved for each value of I, and the resulting energy eigenvalues are compared with experiment. Results of this program for a few representative nuclei are shown in Tables I and II.

It can be seen that Eq. (6) represents the experimental spectra satisfactorily. The rms deviations given in the last column of Table II compare favorably with the accuracy of the experimental energies (±0.3%).[9]

Of course, perturbation theory is not expected to hold when the correction terms are as large as in the present situation (~50%). It may be argued on the grounds of self-consistency, however, that Eqs. (3) and (5) (and therefore all following equations) be valid to order ω^4 even when perturbation theory does not apply. In this latter case, $\mathcal{I}_0$ and C are no longer given by Eqs. (2) and (4). Since these parameters are determined empirically here, the results

Table II. Rotational energies (keV).

Nucleus		$J=2^+$	4^+	6^+	8^+	10^+	12^+	14^+	16^+	rms deviation (%)
W^{174}	Expt.	111.9	355.0	704.2	1137	1635	2186			0.30
	Theory	112.2	354.9	701.8	1133	1634	2196			
W^{176}	Expt.	108.7	348.5	699.4	1140	1648	2206			0.46
	Theory	109.1	348.7	696.4	1133	1645	2223			
Hf^{166}	Expt.	158.7	470.7	897.6	1407	1971	2565			0.59
	Theory	158.9	472.7	893.5	1396	1965	2591			
Hf^{168}	Expt.	123.9	385.0	756.1	1212	1734	2304			0.26
	Theory	123.8	386.2	755.0	1208	1731	2313			
Hf^{170}	Expt.	100.0	320.6	641.1	1041	1503	2013	2564	3147	0.76
	Theory	101.0	320.7	636.6	1031	1491	2008	2575	3186	
Hf^{172}	Expt.	94.5	307.9	627.0	1036	1519	2063	2651		0.46
	Theory	95.0	308.1	624.7	1030	1513	2062	2672		
Yb^{164}	Expt.	122.5	384.0	758.0	1219	1748				0.14
	Theory	122.4	384.8	757.0	1217	1750				
Yb^{166}	Expt.	101.8	329.7	667.1	1097	1604	2172			0.20
	Theory	102.0	329.7	665.9	1094	1602	2179			

of this paper are in no way dependent upon the validity of perturbation theory. A more complete discussion of the present work is now in preparation.

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[9]The measured quantities are the transition energies which also have experimental uncertainties of ±0.3%. The theoretical and experimental transition energies which may be obtained from Table II do not agree as well as the absolute energies. This, of course, is because the parameters g_0 and C were chosen to give the best fit to these absolute energies. If we attempted to fit the energy differences, the deviations given in the last column of Table II would be somewhat larger.

NEW ISOTOPE OF NITROGEN: N^{18}

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Evidence for the production of nitrogen-18 by the reaction $O^{18}(n,p)N^{18}$ has been obtained. The half-life of N^{18} has been measured, and information about its decay scheme has also been determined.

Preliminary estimates of the mass of N^{18} from the systematics of light nuclei in the vicinity of mass 18 indicated that N^{18} should be stable against breakup into $N^{17}+n$; namely, the mass excess of N^{18} should be less than 16.1 MeV on a scale where the mass excess of C^{12} is zero. If breakup does not occur by the emission of other heavy particles, N^{18} would decay by high-energy beta emission to states in O^{18}. Furthermore, from the nuclear systematics in this mass region, the ground state of N^{18} will almost certainly have odd parity and a spin ≤ 3. Of the 10 levels below 6 MeV in O^{18}, only the 4.45- and 5.09-MeV levels have odd parity (viz., 1^-, and 2^- or 3^-, respectively[1-5]). Therefore, an allowed β decay from N^{18} to one or both of these levels would be expected. In any case, a characteristic 1.98-MeV γ ray from γ-ray transitions through the first excited state of O^{18} should be seen in coincidence with β rays in the decay of N^{18}. In summary, N^{18} should exhibit a decay similar to that of N^{16}; namely, it should emit high-energy β rays, emit γ rays characteristic of transitions in the daughter nucleus, and possess a half-life of the order of a second.

Based on these predictions, an attempt was made to produce N^{18} by the reaction $O^{18}(n,p)N^{18}$ and to observe its decay. Neutrons with an energy of about 19 MeV were produced by the reaction $T(d,n)He^4$ using the Lockheed 3.5-MeV Van de Graaff accelerator. These neutrons irradiated an O^{18} sample which consisted of 6.33 g of water whose oxygen content was enriched to 97% in O^{18}. This sample was contained in a thin-walled stainless-steel cylinder. An identical but empty cylinder was used to determine background contributions. Bombardments of this second container filled with ordinary water were carried out to produce N^{16} by the reaction $O^{16}(n,p)N^{16}$; the known end-point energies of the β rays from the 7.4-sec N^{16} activity were used for energy calibration of the β counter.