Table II. Cross sections for ρ^0 production.

Photon-energy range (BeV)	ρ^0 in 3C events (%)	95 % confidence limits (%)	Experimental ρ^0 production σ (μb)
1.1-1.4	20	0-60	12.1
1.4-1.8	40	0-90	17.7
1.8-4.8	85	55-100	19.6

efficient reduction of the data, and our engineers and technicians for their help with the operation of the bubble chamber.

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†On leave from the Weizmann Institute, Rehovoth, Israel (1963-1964).

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[2]The 95 % confidence limits obtained from the χ^2 fits are, respectively, 55-100 %, 0-75 %, and 10-50 %. The N^* was taken to have a Breit-Wigner form with M_{N^*} = 1225 MeV and Γ = 100 MeV.

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[8]This is the best fit for the combination of ρ^0 and phase space. The ρ^0 was taken to have a Breit-Wigner form with M_{ρ^0} = 725 MeV and Γ = 150 MeV. The least-squares analysis gives 95 % probability that the fraction of ρ^0 in this region is between 20 % and 100 %.

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ELECTROMAGNETIC PROPERTIES OF BARYONS IN THE SUPERMULTIPLET SCHEME OF ELEMENTARY PARTICLES*

Bunji Sakita
Argonne National Laboratory, Argonne, Illinois
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Recently an extension of Wigner's supermultiplet theory of the nucleus to the elementary particles has been developed.[1-4] The group used in the theory is a group SU(6) which has a subgroup SU(2)⊗SU(3) identified with the direct product of the ordinary spin group [SU(2)] and the SU(3) internal-symmetry group. As mentioned in reference 3, the group SU(6) also has a subgroup SU(4) which can be identified with Wigner's SU(4).[5] It has been suggested that the pseudoscalar mesons and the vector mesons are members of a regular representation 35 of SU(6). In reference 1, the 20 representation was derived from the quark model[6] for the supermultiplet to which the baryons belong, while in references 3 and 4, the 56 representation has been chosen. Although the nucleon in the 56 representation does not have Wigner's representation[5] of SU(4), in contrast to the nucleon in the 20 representation, the 56 representation has the following interesting features, as noted by Pais[3]: (a) It contains a decuplet of spin-$\frac{3}{2}$ particles together with an octet of spin-$\frac{1}{2}$ particles; (b) it gives a relation between the mass difference among octet baryons and the mass difference among members of the decouplet.

The electromagnetic properties of baryons, especially the magnetic moment, have been discussed in reference 1 under the following assumptions: (i) The electric[7] (and magnetic) current is a tensor of the regular representation of SU(6), and also a tensor of the (1, 8) [and (3, 8)] representation of SU(2)⊗SU(3). (ii) The representation for baryons is 20. These assumptions are based on the quark model of elementary particles which is assumed to have the minimal electromagnetic current of quarks. In this note we shall discuss the same problem by replacing 20 by 56 for the baryon representation in assumption (ii). Although we might avoid the quark model in choosing a 56 representation for the baryons, the assumption (i) for the transformation property of the electromagnetic current is a natural one since it is the simplest generalization of the electromagnetic current from the SU(3) symmetry scheme to the SU(6) scheme. We shall also discuss the mass differences among the states in an isomultiplet under the assumption that they are due to a second-order electromagnetic perturbation.

A basis of the irreducible representation 56 of the group SU(6) can be described by a totally symmetric third-rank tensor in a complex six-dimensional vector space, which we denote by $\Psi_{ABC} = \Psi_{BAC} = \Psi_{ACB}$. We may interpret this tensor as a wave function for an octet of baryons and a decuplet of spin-$\frac{3}{2}$ particles, and may decompose Ψ_{ABC} into wave functions of the spin-$\frac{1}{2}$ octet $(N_{\alpha, i}{}^{\beta})$ and the spin-$\frac{3}{2}$ decuplet $(D_{\alpha\beta\gamma;\, ijk})$ as follows:

$$\Psi_{ABC} \equiv \Psi_{i\alpha, j\beta, k\gamma} = D_{\alpha\beta\gamma;\, ijk} + (1/3\sqrt{2})[\epsilon_{\alpha\beta\delta}\epsilon_{ij}N_{\gamma, k}{}^{\delta} + \epsilon_{\beta\gamma\delta}\epsilon_{jk}N_{\alpha, i}{}^{\delta} + \epsilon_{\gamma\alpha\delta}\epsilon_{ki}N_{\beta, j}{}^{\delta}], \tag{1}$$

where $D_{\alpha\beta\gamma;\, ijk}$ is totally symmetric with respect to the Greek indices and the Latin indices, respectively, so that it represents a decuplet of spin-$\frac{3}{2}$ particles.[8] $\epsilon_{\alpha\beta\gamma}$ and ϵ_{ij} are the totally antisymmetric tensors in a three- and two-dimensional vector space, respectively.

We first construct the effective current in a bilinear form in Ψ in such a way as to retain the transformation property of the 35 representation. Using the tensor notation, it is given by

$$J_A{}^{A'} = \bar{\Psi}^{A'BC}\Psi_{ABC} - \tfrac{1}{6}\delta_A{}^{A'}\langle\bar{\Psi}\Psi\rangle. \tag{2}$$

Inserting the expression Ψ into (2) we obtain

$$J_A{}^{A'} \equiv J_{i\alpha}{}^{i'\alpha'} = \bar{D}^{\alpha'\beta\gamma;\, i'jk}D_{\alpha\beta\gamma;\, ijk} - \tfrac{1}{6}\delta_\alpha{}^{\alpha'}\delta_i{}^{i'}\langle\bar{D}D\rangle$$
$$+ \tfrac{1}{3}\sqrt{2}[\epsilon^{\alpha'\beta\delta}\epsilon^{i'j}\bar{N}^{\gamma, k}D_{\alpha\beta\gamma;\, ijk} + \epsilon_{\alpha\beta\delta}\epsilon_{ij}\bar{D}^{\alpha'\beta\gamma;\, i'jk}N_{\gamma, k}{}^{\delta}]$$
$$+ \tfrac{1}{6}\{\delta_i{}^{i'}(\bar{N}N_F)_\alpha{}^{\alpha'} + \tfrac{2}{3}(\vec{\sigma})_i{}^{i'}\cdot[3(\bar{N}\vec{\sigma}N_D)_\alpha{}^{\alpha'} + 2(\bar{N}\vec{\sigma}N_F)_\alpha{}^{\alpha'} + \delta_\alpha{}^{\alpha'}\langle\bar{N}\vec{\sigma}N\rangle]\}, \tag{3}$$

where

$$\langle\bar{D}D\rangle = \bar{D}^{\alpha\beta\gamma;\, ijk}D_{\alpha\beta\gamma;\, ijk}, \qquad (\bar{N}N_F)_\alpha{}^{\alpha'} = \bar{N}_\beta{}^{\alpha'}N_\alpha{}^{\beta} - \bar{N}_\alpha{}^{\beta}N_\beta{}^{\alpha'},$$

$$(\bar{N}N_D)_\alpha{}^{\alpha'} = \bar{N}_\beta{}^{\alpha'}N_\alpha{}^{\beta} + \bar{N}_\alpha{}^{\beta}N_\beta{}^{\alpha'} - \tfrac{2}{3}\delta_\alpha{}^{\alpha'}\langle\bar{N}N\rangle, \qquad \langle\bar{N}N\rangle = \bar{N}_\beta{}^{\alpha}N_\alpha{}^{\beta}.$$

The last line in Eq. (3) is the octet baryonic current and the terms in this line correspond to the (1, 8), the (3, 8), and the (3, 1) representation of the SU(2)⊗SU(3) group, respectively. We may see that we have an F-type current for the (1, 8) current, while we have $(3D+2F)$-type coupling for the (3, 8) current.[9] From assumption (i), therefore, the type of coupling of the magnetic current is $3D+2F$. Since the SU(6) symmetry is to be applied only in the static limit,[10] the magnetic current is proportional to the total magnetic moment of the particle. From this consideration and the result of Eq. (3), we immediately derive the ratio of the total magnetic moment of the proton and the neutron:

$$\mu_p/\mu_n = -\tfrac{3}{2}. \tag{4}$$

This exceedingly good agreement of the magnetic moment with the empirical one seems to encourage us to choose the 56 representation for the baryons as well as the SU(6) symmetry scheme itself. Of course, we obtain all relations among the magnetic moments of baryons derived from the SU(3) symmetry.[11]

The second line in Eq. (3) gives the magnetic dipole transition between the spin-$\frac{3}{2}$ excited state and the spin-$\frac{1}{2}$ baryonic state.

Next let us consider the mass differences among the states in an isomultiplet due to the electromagnetic interaction. Since the electromagnetic mass splitting is of second order in the electromagnetic interaction and the electromagnetic current is a tensor in the $\underline{35}$ representation, the mass-splitting Hamiltonian should be a product of two $\underline{35}$ tensors, which is reduced as

$$\underline{35}\otimes\underline{35}=\underline{1}+\underline{35}+\underline{35}+\underline{189}+\underline{280}+\underline{280}^*+\underline{405}.$$

The mass term of the baryons formed from $\bar{\Psi}\Psi$ is reduced as

$$\underline{56}^*\otimes\underline{56}=\underline{1}+\underline{35}+\underline{405}+\underline{2695}.$$

The number of representations in common in these two products is the number of parameters (number of independent couplings) in the effective electromagnetic mass-splitting term of baryons. One of the $\underline{35}$'s in the first product is antisymmetric with respect to the interchange of two tensors in the product, so that it does not contribute to the electromagnetic mass term because these two tensors in the product in the electromagnetic mass-splitting Hamiltonian are symmetric. Therefore, we have three independent couplings: $\underline{1}$, $\underline{35}$, and $\underline{405}$. Since the terms from $\underline{1}$ do not contribute to the mass difference, however, we have essentially two parameters in an effective mass term. Using tensor notations, we may write the mass term of baryons as follows:

$$\alpha\bar{\Psi}^{i1,A,B}\Psi_{i1,A,B}+\beta\bar{\Psi}^{i1,j1,A}\Psi_{i1,j1,A}. \tag{4}$$

The first term is the contribution from $\underline{35}$ and $\underline{405}$, while the second term is from $\underline{405}$. If we insert the expression for Ψ [Eq. (1)], we obtain

$$\alpha\{\bar{D}^{1\beta\gamma}D_{1\beta\gamma}+\tfrac{1}{3}(\bar{N}N_F)_1{}^1+\tfrac{1}{3}\langle\bar{N}N\rangle\}$$
$$+\beta\{\bar{D}^{11\gamma}D_{11\gamma}$$
$$+\tfrac{1}{3}(\bar{N}_\alpha{}^1N_1{}^\alpha-\bar{N}_1{}^1N_1{}^1)\}. \tag{5}$$

From this we obtain the following mass relations:

$$N^{*+}-N^{*0}=Y_1{}^{*+}-Y_1{}^{*0}=p-n=\Sigma^+-\Sigma^0,$$
$$\Xi^{*-}-\Xi^{*0}=N^{*-}-N^{*0}=Y_1{}^{*-}-Y_1{}^{*0}$$
$$=\Xi^--\Xi^0=\Sigma^--\Sigma^0,$$
$$N^{*++}=3(N^{*+}-N^{*0})+N^{*-},$$

where we have denoted the mass of the particle by its symbol. The relations among the decuplet states are the consequences of SU(3) symmetry alone.[12] Experimental values of the mass differences are[13]

$$p-n=-1.3\text{ MeV},\quad \Sigma^+-\Sigma^0=-2.85\pm0.30\text{ MeV};$$
$$\Xi^--\Xi^0=6.1\pm1.6\text{ MeV},\quad \Sigma^--\Sigma^0=4.75\pm1.0\text{ MeV}.$$

Agreements with the theoretical predictions are not so impressive as for the magnetic moment. We note that our results, of course, satisfy the Coleman-Glashow mass relation for baryons,[11] but are not compatible with some of the predictions of the tadpole model.[14]

We would like to thank Professor L. Michel and Dr. H. J. Lipkin for their illuminating discussions, and Professor R. G. Sachs for his careful reading of this note.

Note added in proof.—After we had written this note, Professor L. Michel showed us a preprint by M. A. B. Bég, B. W. Lee, and A. Pais, in which they also obtained the ratio of magnetic moments.

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[7]The separation of "electric" and "magnetic" currents is based on the behavior of the current in the static limit. In the static limit, the electric current of a spin-$\frac{1}{2}$ particle has the form $\Psi^*\nabla\Psi-(\nabla\Psi^*)\Psi$ so that it transforms as a scalar under the SU(2) spin transformation, while the magnetic current has the form $\Psi^*\vec{\sigma}\Psi\times\vec{k}$ so that it transforms as a vector. Combining it with the SU(3) tensor property of the electromagnetic current, i.e., $T_1{}^1$, therefore, the electric current should be a tensor of the (1, 8) representation of SU(2)⊗SU(3), while the magnetic current is a tensor of the (3, 8) representation. The lowest representation which has (1, 8) and (3, 8) as its components is the regular representation $\underline{35}$.

[8]$D_{\alpha\beta\gamma}$ has 10 components, identified as follows:

$D_{333}=\Omega^-$, $D_{133}=\Xi^{*0}/\sqrt{3}$, $D_{233}=\Xi^{*-}/\sqrt{3}$,
$D_{113}=Y_1{}^{*+}/\sqrt{3}$, $D_{123}=Y_1{}^{*0}/\sqrt{6}$, $D_{223}=Y_1{}^{*-}/\sqrt{3}$,
$D_{111}=N^{*++}$, $D_{112}=N^{*+}/\sqrt{3}$, $D_{122}=N^{*0}/\sqrt{3}$,
$D_{222}=N^{*-}$

[9]The type of coupling obtained here is the same as

the type of coupling obtained in reference 4 for the pion-nucleon interactions. In our point of view, however, the Yukawa-type pion-nucleon interactions are symmetry-violating interactions of SU(6). If we assume that the Yukawa interaction transforms as the 35 representation [simplest SU(6)-violating interaction], we have two independent couplings so that we do not necessarily have a definite type of coupling for the pion-nucleon interactions.

[10]Since we are not convinced of the relativistic extension of the group SU(6), contrary to the statement in reference 2, we apply this group in the static limit by assuming that it is an approximate symmetry valid only in this limit. We thank Professor L. Michel for his informative discussion on the extension of the Poincaré group.

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ISOTOPIC STRUCTURE OF THE WEAK INTERACTIONS*

V. De Santis†
Northwestern University, Evanston, Illinois
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Recently,[1] striking evidence for the process

$$K_2^0 \to \pi^+ + \pi^- \qquad (1)$$

has been reported, with a branching ratio of about 2×10^{-3} to all the other decay modes of K_2^0. Since this process is forbidden by CP invariance, its existence means that CP invariance is violated in the weak interactions. In the light of the CPT theorem, T invariance, too, is violated. This development adds new fog to an already confused situation. So far, tentative partial explanations of the experimental facts have been made possible by assuming empirical nonconservation regularities, namely $\Delta I=\frac{1}{2}$ and $\Delta S=\Delta Q$ rules,[2,3] and, with the help of the spurion[4] trick, by pushing into the weak interactions, which are I- and S-nonconserving, the isotopic technique,[5] built into the strong ones on the assumption of I conservation. Even so, it is difficult to understand, from one side, the regularity in the violations, more than the violations in themselves. From another side there is much discussion yet about the validity of such rules and about the extent of their violation ($\Delta I=\frac{3}{2}$, $\Delta S=-\Delta Q$).[6-9] We are almost led to talk in terms, so to speak, of "regularities in the violation of the regularities of the violated conservation of isotopic spin." Still, the regularities in the weak interactions really exist, suggesting the existence of some underlying symmetry related to an isospinlike conservation. This idea is not new and several attempts in this sense are known,[10-16] but so far they have been limited mostly to the leptons. Summarizing, the trend has been to look at the weak interactions through the strong-interaction scheme, with its well conserved I and I_3, and to realize that they are no longer conserved: a curious, special way of being nonconserved, however.

The recent discovery of CP nonconservation, furthermore, robs the $\Delta I=\frac{1}{2}$ rule of most of its power. In fact, CP conservation together with the Pauli principle allowed in many cases the selection of one value for I out of the two consistent with $\Delta I=\frac{1}{2}$, thus allowing Clebsch-Gordan-type calculations to predict several ratios. If CP is not conserved, this is no longer possible: Two independent isotopic amplitudes are involved (for instance in K_2^0 decay), and even a more or less ad hoc statement about the "extent" of CP violation cannot give definite predictions without further assumptions on the relative "strength" of the two amplitudes.

It is the purpose of this note to suggest a way of reversing the trend by looking directly at the weak interactions to see if the present rules can be unified and understood on a more general basis. Namely, we propose to interpret the regularities shown by the weak interactions as due to the existence and to the conservation of a weak isotopic spin I' and of its third component I_3', related to the electric charge by

$$Q=\tfrac{1}{2}Y'+I_3', \qquad (2)$$

which is the weak equivalent of the well-known

$$Q=\tfrac{1}{2}Y+I_3, \qquad (2\ )$$