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†James Clerk Maxwell Fellow at Cornell University.

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PHONON-PUMPED SPIN-WAVE INSTABILITIES

H. Matthews
Bell Telephone Laboratories, Murray Hill, New Jersey

and

F. R. Morgenthaler*
Department of Electrical Engineering and Center for Materials Science and Engineering, Massachusetts Institute of Technology, Cambridge, Massachusetts

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We report here some theoretical results on the parametric excitation of spin waves by an elastic wave, together with experimental observations consistent with these results.

A phonon traveling in a magnetically saturated ferromagnet generates an effective magnetic field that can parametrically couple to spin waves. This field has the phonon momentum and, hence, spin-wave excitation by a traveling-wave pump is possible. The effective field may, in general, have components that are both transverse and parallel to the dc magnetic field. The former lead to modified Suhl instabilities,[1] the latter to modified "parallel pump" instabilities.[2,3]

We restrict ourselves here to a discussion of the instabilities excited by a longitudinal phonon of frequency ω_p, and elastic displacement

$$\vec{\rho} = \vec{i}_z \rho_0 \cos(\omega_p t - \vec{k}_p \cdot \vec{r}), \tag{1}$$

whose wave vector, $\vec{k}_p = \vec{i}_z k_p$, is parallel to the dc magnetic field.

We consider the first-order excitation of spin-wave pairs when the pump phonon travels in a cubic crystal of ellipsoidal shape magnetically saturated along a principal axis taken, for simplicity, parallel to a $\langle 100 \rangle$ crystal axis. The classical equations of motion governing spin-wave propagation follow from

$$\partial\vec{M}/\partial t = \gamma \vec{M} \times \vec{H}, \tag{2}$$

in which γ denotes the gyromagnetic ratio (negative). The magnetization is assumed to be composed of a spin-wave pair, $\vec{m}_1$ and $\vec{m}_2$, so that $\vec{M} = \vec{m}_1 + \vec{m}_2 + \vec{i}_z M_s$. The ith spin wave, whose wave vector is

$$\vec{k}_i = k_i \sin\Psi_i (\vec{i}_x \cos\xi_i + \vec{i}_y \sin\xi_i) + k_i \cos\Psi_i \vec{i}_z, \tag{3}$$

is represented by

$$\vec{m}_i = \vec{i}_x m_{ix} \cos(\omega_i t - \vec{k}_i \cdot \vec{r} + \varphi_{ix}) + \vec{i}_y m_{iy} \sin(\omega_i t - \vec{k}_i \cdot \vec{r} + \varphi_{iy}), \tag{4}$$

and M_s denotes the saturation magnetization. To account for the Zeeman, demagnetizing, exchange, dipolar, and magnetic anisotropy energies and to include loss, $\vec{H} = \vec{h}_1 + \vec{h}_2 + \vec{i}_z H_z$, where

$$\vec{h}_i = -\lambda k_i^2 \vec{m}_i - 4\pi \frac{\vec{k}_i(\vec{k}_i \cdot \vec{m}_i)}{k_i^2} + \frac{\Delta H_i}{2M_s}(\vec{i}_x m_{iy} - \vec{i}_y m_{ix}), \tag{5}$$

and

$$H_z = H_0 - 4\pi N_z M_s + 2K_1/M_s. \tag{6}$$

In these expressions H_0 denotes the external applied field, N_z the corresponding demagnetizing factor, λ the exchange constant, K_1 the magnetic anisotropy constant, ΔH_i the full linewidth of the ith spin wave.

The magnetoelastic energy is given by

$$U = (b_1/M_s^2) M_z^2 \partial\rho_z/\partial z. \tag{7}$$

Together with Eq. (1) this leads to the effective field

$$h_z = -(\partial U/\partial M_z)$$
$$= -(2b_1/M_s)k_p\rho_0 \sin(\omega_p t - \vec{k}_p\cdot\vec{r}), \quad (8)$$

when $M_z \approx M_s$. The effective field h_z is added to H_z to account for the magnetoelastic energy.

The spin-wave pair can be parametrically excited by the traveling-wave pump if

$$\omega_1 + \omega_2 = \omega_p, \quad \vec{k}_1 + \vec{k}_2 = \vec{k}_p. \quad (9)$$

The instability threshold may be obtained with the aid of a perturbation technique[4] by which we find the critical phonon amplitude to be given by

$$\frac{2b_1}{M_s}k_p\rho_0^{\text{crit}} = \frac{2(\Delta H_1\Delta H_2\omega_1\omega_2)^{1/2}}{(A_1A_2)^{1/2}-(B_1B_2)^{1/2}}, \quad (10)$$

where

$$\omega_i^2 = A_iB_i,$$

and

$$A_i = -\gamma(H_0 - 4\pi N_zM_s + 2K_1/M_s + \lambda k_i^2M_s + 4\pi M_s\sin^2\Psi_i),$$

$$B_i = -\gamma(H_0 - 4\pi N_zM_s + 2K_1/M_s + \lambda k_i^2M_s), \quad (11)$$

and Eq. (10) is subject to the energy and momentum constraints given by Eq. (9). If $k_1 = k_2$, then $\sin\Psi_1 = \sin\Psi_2$, $\Delta H_1 = \Delta H_2$, and the spin-wave frequencies are degenerate, $\omega_1 = \omega_2 = \frac{1}{2}\omega_p$. Eq. (10) reduces to the form

$$\frac{2b_1}{M_s}k_p\rho_0^{\text{crit}} = \frac{\omega_p\Delta H}{-\gamma 4\pi M_s\sin^2\Psi}. \quad (12)$$

The position in ω-$\vec{k}$ space of the spin-wave pair with the minimum threshold will depend critically on the value of H_0. We will refer to Fig. 1, for which we have set $\omega_z = -\gamma H_z$, $\omega_M = -\gamma 4\pi M_s$, and $\omega_T = [\omega_z(\omega_z + \omega_M)]^{1/2}$. For values of H_0 such that $\frac{1}{2}\omega_p$ lies appreciably above the "top" of the spin-wave band [Fig. 1(a)], energy and momentum can be conserved only is both partners have large k. By comparison, $k_p \approx 0$ and $k_1 \approx k_2$. The modes are degenerate and Eq. (12) is applicable. The minimum threshold occurs for $\Psi \approx \frac{1}{2}\pi$. For values of H_0 such that $\frac{1}{2}\omega_p$ lies within the spin-wave band [Fig. 1(b)], nondegenerate pairs with small k (a-a', c-c', d-d') are possible, as well as de-

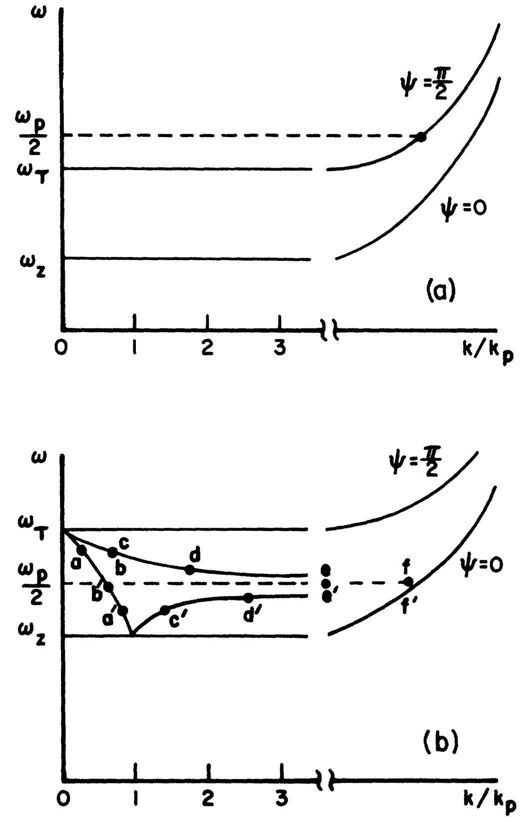

FIG. 1. Sketch shows the effect of the position of the spin-wave band on the loci of the spin-wave pairs, a-a' b-b', etc., that can be parametrically excited by a longitudinal phonon. Symbols are defined in the text.

generate pairs of both small k (b-b') and large k (e-e' to f-f'). The threshold for any pair is given by Eq. (10), but note that the threshold depends on the $\vec{k}$-dependent nature of ΔH_i as well as the parametric efficiency, which is determined by the ellipticity of the polarization of the spin-wave modes. The threshold for any pair is finite unless both partners are circularly polarized.[5]

It appears likely that minimum thresholds will occur for values of H_0 such that $\frac{1}{2}\omega_p$ is near the "top" of the spin-wave band, since then both partners will have high ellipticity and low loss.

A breakdown effect has been observed which is consistent with the theory discussed in this note. A pulse-echo technique employing a piezoelectric transducer was used to observe the propagation, at room temperature, of 1000-Mc/sec longitudinal phonons, typically of 2-μsec duration, traveling along the axis of a single-crystal rod ($L = 1.01$ cm, $D = 0.432$

cm) of gallium-substituted yttrium iron garnet ($4\pi M_S = 294$ G).

The rod axis was parallel to a ⟨100⟩ crystal axis. The phonon propagation was studied as a function of the peak input power P to the transducer circuit and the field H_0 applied parallel to the rod axis.

With this configuration, longitudinal phonons are not linearly coupled to spin waves, and with P sufficiently small, the observed propagation is independent of H_0. However, with sufficiently large P, the breakdown effect shown in Fig. 2 is observed for a range δH of applied field. δH converged to a specific value and the breakdown of the trailing edge diminished as P was reduced to the minimum value, P_c, for which the effect would be detected. P_c decreased with increasing pulse length. With a 2-μsec pulse, $P_c = 70$ mW and $H_0 = 230$ Oe.

As noted above, we may expect minimum thresholds for the excitation of spin-wave pairs when $\frac{1}{2}\omega_p$ lies near the top of the spin-wave band. For our rod $2K_1/M_S = -135$ Oe, and on the axis at the center $4\pi N_z M_S = 21$ Oe, so that with $H_0 = 230$ Oe the top of the spin-wave band is at 462 Mc/sec, whereas $\frac{1}{2}f_p$ is 500 Mc/sec. If we assume the observed breakdown to result from the excitation of a degenerate spin-wave pair with $\sin\Psi \approx 1$, then, after converting Eq. (12) into an expression for the threshold phonon power density on the assumption of plane waves,

$$p_c = \tfrac{1}{2}c_{11}v_l\left(\frac{\omega_p \Delta H_1}{-8\pi\gamma b_1}\right)^2, \tag{13}$$

we obtain an expected threshold power density of 100 mW/cm^2 using the material parameters for unsubstituted yttrium iron garnet; namely, $\gamma = -2.8$ Mc/Oe, $v_l = 7.21 \times 10^5$ cm/sec,[6] $c_{11} = 26.9 \times 10^{11}$ dyn/cm^2,[6] and $b_1 = 3.48 \times 10^6$ erg/cm^3.[7] We take $\Delta H = 0.25$ Oe.[8] From measured insertion loss our transducer had a conversion efficiency of 4.5% and an area of approximately 0.015 cm^2, so that the observed threshold power density was 210 mW/cm^2.

The experimental conditions impose limitations, i.e., short interaction time and bias-field nonuniformity, which make precise quantitative comparisons difficult. The results are consistent with the theory, however, and strongly imply that the observed breakdown is due to parametric excitation of spin waves.

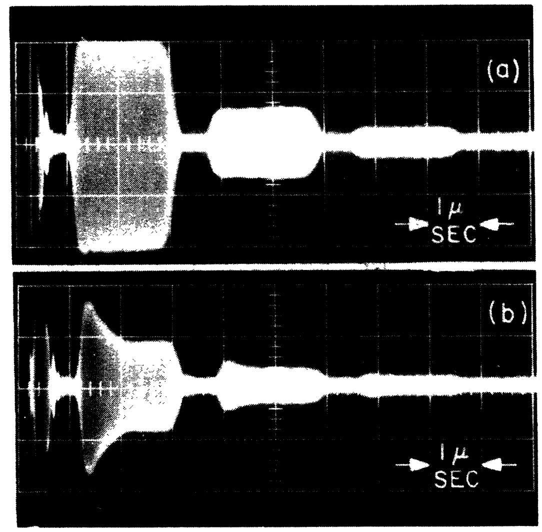

FIG. 2. Detected echoes of 1000-Mc/sec longitudinal phonon pulse traveling in magnetically biased Ga-substituted yttrium iron garnet. (a) Normal propagation, independent of the biasing field. (b) Breakdown that occurs for a range of bias field when phonon power density is sufficiently large.

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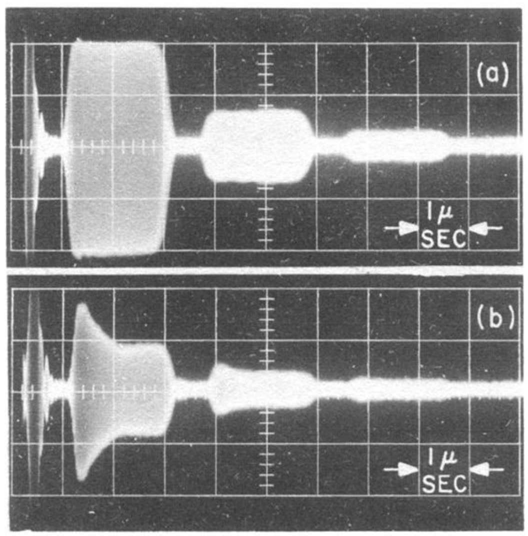

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