The meson-baryon squared coupling constant is given by

$$\gamma_1 = 3\left(\frac{1}{\pi}+\beta_0\right)\frac{1-s_1}{1+s_1}\left(1+\frac{1}{\kappa}\right)^4. \tag{13}$$

The threshold condition reads

$$s_1^{-1} + 2\,\mathrm{Im}(q_0^{-1} - q_1^{-1}) = 1 + 2\kappa^{-1}, \tag{14}$$

which, for $\kappa \sim 1$, approximately reduces to

$$\beta_1 + \beta_0 \approx \tfrac{3}{2}(1+\kappa^{-1})^{-3}. \tag{15}$$

There is resonance in the $J=\frac{3}{2}$ channel represented by a pole of $S_{3/2}$ at $\omega = y$ on the second Riemann sheet. For $\kappa \sim 1$ the pole is near threshold, with position and width given, respectively, by the approximate expressions

$$\mathrm{Re}y \approx 1 + \tfrac{1}{4}(1+\kappa^{-1})^{-2},$$
$$\mathrm{Im}y \approx \frac{\sqrt{3}}{4}(1+\kappa^{-1})^{-2}. \tag{16}$$

(5) Applying the usual bootstrap philosophy to these models, we would expect the positions ω_i and squared coupling constants λ_i of bound states (or virtual states, or resonances) to be determined by a set of equations, which for the case of two bound states (one of them being the target baryon located at $\omega_1 = 0$) should be of the form

$$\begin{aligned} 0 &= f_1(\lambda_1, \lambda_2, \omega_2, g, \kappa),\\ \omega_2 &= f_2(\lambda_1, \lambda_2, \omega_2, g, \kappa),\\ \lambda_1 &= F_1(\lambda_1, \lambda_2, \omega_2, g, \kappa),\\ \lambda_2 &= F_2(\lambda_1, \lambda_2, \omega_2, g, \kappa), \end{aligned} \tag{17}$$

where g is the subtraction constant and κ the cutoff momentum. (The mass of the scattered meson acts as a scaling parameter, which is set equal to unity.) From (17) we should expect ω_2, λ_1, and λ_2 to be determined up to one arbitrary parameter. The results in all the models studied disagree with this counting. We may perhaps understand the nature of this disagreement by considering the present theory as the limit of a relativistic theory in which the mass of the target baryon (M) is to be inserted into Eq. (17) as follows:

$$\begin{aligned} M &= M + f_1(\lambda_1, \lambda_2, \omega_2, g, \kappa, M),\\ M + \omega_2 &= M + f_2(\lambda_1, \lambda_2, \omega_2, g, \kappa, M),\\ \lambda_1 &= F_1(\lambda_1, \lambda_2, \omega_2, g, \kappa, M),\\ \lambda_2 &= F_2(\lambda_1, \lambda_2, \omega_2, g, \kappa, M). \end{aligned} \tag{18}$$

We see that Eqs. (18) have the correct number of variables to allow two free parameters. If now a static limit is to make sense, the functions f_i and F_i must be insensitive to M, and therefore one of Eqs. (17) must be an identity.

Details of this work will be published elsewhere.

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SPIN AND UNITARY-SPIN INDEPENDENCE IN A PARAQUARK MODEL OF BARYONS AND MESONS

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Wigner's supermultiplet theory,[1] transplanted independently by Gürsey, Pais, and Radicati,[2] and by Sakita,[2] from nuclear-structure physics to particle-structure physics, has aroused a good deal of interest recently. In the nuclear supermultiplet theory, the approximate independence of both spin and isospin of those forces relevant to the energies of certain low-lying bound states (nuclei) makes it useful to classify the states according to irreducible representations of SU(4). Parallel to this, in the particle supermultiplet theory, the possible independence of both spin and unitary spin of those forces relevant to the masses of certain low-lying bound states (particles) makes it interesting to classify the states according to irreducible representations of SU(6). Three results associated with this SU(6) classification indicate its usefulness: (1) The best known baryons (in particular, the spin-$\frac{1}{2}^+$ baryon octet and the spin-$\frac{3}{2}^+$ baryon decuplet) are grouped into a supermultiplet containing 56 particles.

The pseudoscalar 0^- octet, vector 1^- octet, and vector 1^- singlet of mesons are grouped into a supermultiplet containing 35 particles. (2) Rather accurate mass formulas have been written down (partly on heuristic grounds) for the supermultiplets.[2,3] (3) In the approximation where the spin and unitary-spin independence of the forces relevant to the $\underline{56}$ is broken only by electromagnetic coupling, the magnetic moments of all the baryons in the $\underline{56}$ have been calculated up to a single common factor.[4] This calculation predicts the ratio $\beta = \mu_n/\mu_p = -\frac{2}{3}$, which agrees with the experimental value to within 3%.

Analogy with the Wigner supermultiplet theory leads us to adopt an atomic model in which all of the baryons and mesons are composite objects made up of basic particles. We assume that the basic forces are independent of unitary spin, but are in general spin dependent, and that the approximate symmetry group of the theory is $\mathcal{P}'\otimes SU(3)$, where $\mathcal{P}'$ is the covering group of the Poincaré group. However, for low-lying bound states, in particular when all orbital angular momenta are zero, all orbit-orbit and spin-orbit forces vanish so that there will be an additional degeneracy which will allow SU(6) invariance for these particular states, provided the spin-spin forces are not too large.[5] The desired supermultiplets, the $\underline{56}$ for the baryons and the $\underline{35}$ for the mesons, contain, respectively, the symmetric direct product of three SU(6) multiplets, and the antisymmetric direct product of an SU(6) and an SU(6)* multiplet. These assignments suggest that the basic particles are quarks.[6-9] For the mesons the $\underline{35}$ can be achieved by a state containing a fermion and an antifermion in s states; however, for the baryons the assignment of three fermions to the $\underline{56}$ is not possible if all three fermions are in s states.[10] We will return to this problem later, but for the moment assume that we can arrange to construct both the $\underline{35}$ and the $\underline{56}$ with all particles in s states.

The notion that an atomic model underlies the SU(6) invariance gives some hints for the derivation of mass formulas. In analogy with atomic and nuclear physics we suggest that the terms violating the exact SU(6) symmetry, for those states where the symmetry is relevant, arise either from one-body or two-body forces. A particularly simple assumption is that all the SU(6)-violating terms come from the $J=I=Y=0$ member of the (35-dimensional) adjoint representation of SU(6). With this assumption, the one-body force gives for the mass operator

$$\mathfrak{M}^{(1)} = \sum_i M_0 V_0(r_i) + \sum_i [T_3{}^3(i) + T_6{}^6(i)] V_1(r_i),$$

where $T_i{}^j$ is a tensor operator which transforms in the same way as the adjoint representation. The two-body contribution gives

$$\mathfrak{M}^{(2)} = \tfrac{1}{2} \sum_{i\neq j} \sum_{\lambda} [(T_3{}^3 + T_6{}^6)_i \times (T_3{}^3 + T_6{}^6)_j]^{(\lambda)} V_2{}^{(\lambda)}(|r_i - r_j|),$$

where λ distinguishes the different contributions which can occur from the two-body operator in the brackets. These contributions are $M_{(1)}{}^{(1)}$, $M_{(35)}{}^{(8)}$, $M_{(405)}{}^{(1)}$, $M_{(405)}{}^{(8)}$, and $M_{(405)}{}^{(27)}$, using the notation of Bég and Singh.[3] Note that with this assumption, only the representations $\underline{1}$, $\underline{35}$, and $\underline{405}$ can contribute to $M^{(2)}$, because these are the only symmetric real representations contained in $\underline{35\otimes 35}$. For the $\underline{56}$, the contributions from $M_{(1)}{}^{(1)}$ and $M_{(35)}{}^{(8)}$ can be absorbed in $\mathfrak{M}^{(1)}$. Contributions from $M_{(405)}{}^{(1)}$ and $M_{(405)}{}^{(8)}$, together with the one-body operator, already suffice to give the mass formula which has been suggested for the $\underline{56}$:

$$M = M_0 + \alpha Y + \beta[I(I+1) - \tfrac{1}{4}Y^2] + \gamma J(J+1).$$

The magnetic-moment calculation,[4] as already pointed out by the authors, can be obtained by assuming that the entire magnetic moments of the baryons in the $\underline{56}$ are produced by the intrinsic magnetic moments of the quarks,[11] and that the quark moments are proportional to their Dirac moments, $Q\hbar/2Mc$, Q = quark charge, M = quark mass. If the free quark moments equal their Dirac moments, then one can resolve the incompatibility of assumptions (I)-(IV) discussed by Bég, Lee, and Pais[4] by saying that quarks, rather than nucleons, have minimal electromagnetic coupling, and that the radiative corrections (in empty space) to the quark moments are small (as is the case for the electron and muon), or proportional to their Dirac moments. However, if the quarks bound in nucleons have Dirac moments, then $M = m_p/2.79 = 336$ MeV,[12] so that one must look for a mechanism (perhaps acting only in strongly bound states) which enhances quark magnetic moments in proportion to their Dirac moments.

Now we return to the question of placing three spin-$\frac{1}{2}$ quarks in s states in the baryon $\underline{56}$.

This can be done if the quarks are parafermions of order $p=3$. This suggestion is the main new idea of this article.[13] For parafermions of order 3, one has the Green Ansatz for the creation and annihilation operators,

$$a_\lambda^\dagger = \sum_{\alpha=1}^{3} a_\lambda^{(\alpha)\dagger}, \quad a_\lambda = \sum_{\alpha=1}^{3} a_\lambda^{(\alpha)},$$

where the $a^{(\alpha)}$ and $a^{(\beta)\dagger}$ satisfy the anticommutation rules for the same α,

$$[a_\lambda^{(\alpha)}, a_\mu^{(\alpha)\dagger}]_+ = \delta_{\lambda,\mu}, \quad [a_\lambda^{(\alpha)\dagger}, a_\mu^{(\alpha)\dagger}]_+ = 0,$$

and commute for different α and β,

$$[a_\lambda^{(\alpha)}, a_\mu^{(\beta)\dagger}]_- = 0, \quad \alpha \neq \beta;$$

$$[a_\lambda^{(\alpha)\dagger}, a_\mu^{(\beta)\dagger}]_- = 0, \quad \alpha \neq \beta.$$

Here λ stands for the single-particle quantum numbers; for example, momentum, spin, I_z, and Y. Let

$$f_{\lambda\mu\nu}^\dagger \equiv [[a_\lambda^\dagger, a_\mu^\dagger]_+, a_\nu^\dagger]_+$$

$$= 4 \sum_{\substack{\alpha,\beta,\gamma=1 \\ \alpha\neq\beta\neq\gamma\neq\alpha}}^{3} a_\lambda^{(\alpha)\dagger} a_\mu^{(\beta)\dagger} a_\nu^{(\gamma)\dagger}$$

Then the state $f_{\lambda\mu\nu}^\dagger\Phi_0$ is symmetric under all permutations of λ, μ, and ν.[14] This composite state is a fermion,[15] since $[f_{\lambda\mu\nu}^\dagger, a_\sigma^\dagger]_+ = 0$, which implies $[f_{\lambda\mu\nu}^\dagger, f_{\sigma\tau\eta}^\dagger]_+ = 0$.

The comparison of superselection sectors for paraquarks due to their para nature with the charge and baryon-number superselection sectors is given in Table I.

The suggestion that quarks are parafermions (and, in a field theory, the quanta of para-Fermi fields) is allowed by the selection rules which follow from locality.[13] Relevant[16] local interactions (in the sense of spacelike commutativity of the interaction Hamiltonian density) can be constructed with these fields; for example, the Yukawa interaction

$$H_I = g[[\bar{\Psi}, \Psi]_+ - \langle[\bar{\Psi}, \Psi]_+\rangle_0, \varphi]_+,$$

or the Fermi interaction

$$H_I = G\{[[\bar{\Psi}, \Psi]_-, [\bar{\Psi}, \Psi]_-]_+ - \langle\text{same}\rangle_0\},$$

where Ψ and φ are, respectively, para-Fermi and para-Bose of order 3. The paraquark suggestion does not fall under the quantum-mechanical theorem[17] that particles with anomalous permutation properties cannot be produced from initial states ($\mathfrak{F}^\times$) with at most one such particle because, in this theorem, it was assumed that the only superselection rules were those generated by charge, baryon number, and lepton number, while in parafield theories there are additional superselection rules.[13,18] The question of the compatibility of parafield theory with intuitive notions concerning the behavior of quantum systems when separated into subsystems has been raised.[19] This question deserves serious attention; however, we do not consider it here.

Table I. Comparison of para, charge, and baryon-number superselection sectors for $p=3$ para-Fermi quarks.

State[a]	Para superselection sector	Q	B
$a^\dagger\Phi_0$	para-Fermi	$\frac{2}{3}, -\frac{1}{3}$	$\frac{1}{3}$
$[a^\dagger, a^\dagger]_+\Phi_0$	para-Bose	$\frac{4}{3}, \frac{1}{3}, -\frac{2}{3}$	$\frac{2}{3}$
$[a^\dagger, b^\dagger]_+\Phi_0$	para-Bose	$1, 0, -1$	0
$[a^\dagger, a^\dagger]_-\Phi_0$	Bose	$\frac{4}{3}, \frac{1}{3}, -\frac{2}{3}$	$\frac{2}{3}$
$[a^\dagger, b^\dagger]_-\Phi_0$	Bose	$1, 0, -1$	0
$[[a^\dagger, a^\dagger]_+, a^\dagger]_+\Phi_0$	Fermi	$2, 1, 0, -1$	1
$[[a^\dagger, a^\dagger]_+, b^\dagger]_+\Phi_0$	Fermi	$5/3, \frac{2}{3}, -\frac{1}{3}, -\frac{4}{3}$	$\frac{1}{3}$

[a] $a^\dagger$ is the creation operator for a quark, $b^\dagger$ for an antiquark.

After this brief defense of the possibility that quarks are para-Fermi, we proceed to the classification of baryon states in the paraquark model. The baryon wave functions must be symmetric under permutations. We present in Table II states in which the wave functions[5] are sums of products of space wave functions and spin–unitary-spin wave functions. All the quarks are in the lowest radial state for a given l. The Young diagrams for both the space and spin–unitary-spin wave functions are the same; the numbers listed give the number of boxes in successive rows. Even if the SU(6) classification is useful only for the lowest $\underline{56}$ with configuration s^3, the (SU(3), J) classification given in column 5 should be useful for higher states. We have continued the table up to the p^3 configuration in order to obtain states with $J=\frac{9}{2}$, since there is some experimental evidence for such states.[20] However, we postpone assigning the known baryons to multiplets. The fact that only the $\underline{1}$, $\underline{8}$, and $\underline{10}$ SU(3) mul-

Table II. Low-lying states in paraquark model of baryons.[a]

Orbitals configuration	L	Parity	Young diagram	(SU(3), J) decomposition	Total multiplicity	Total no. of I multiplets
s^3 (pure)	0	+	(3)	($\underline{8}$, $J = 1/2$) ($\underline{10}$, $J = 3/2$)	56	8
s^2p^1 (spurious)	1	−	(3)	($\underline{8}$, $J = 1/2, 3/2$) ($\underline{10}$, $J = 1/2, 3/2, 5/2$)	168	20
s^2p^1 (pure)	1	−	(2, 1)	($\underline{1}$, $J = 1/2, 3/2$) ($\underline{8}$, $J = 1/2, 3/2, 5/2$) ($\underline{10}$, $J = 1/2, 3/2$) ($\underline{8}$, $J = 1/2, 3/2$)	210	30
s^1p^2, s^2d^1 (mixed)	2	+	(3)	($\underline{8}$, $J = 3/2, 5/2$) ($\underline{10}$, $J = 1/2, 3/2, 5/2, 7/2$)	280	24
s^1p^2, s^2d^1 (mixed)	2	+	(2, 1)	($\underline{1}$, $J = 3/2, 5/2$) ($\underline{8}$, $J = 1/2, 3/2, 5/2, 7/2$) ($\underline{10}$, $J = 3/2, 5/2$) ($\underline{8}$, $J = 3/2, 5/2$)	350	34
s^1p^2, s^2d^1 (mixed)	1	+	(2, 1)	($\underline{1}$, $J = 1/2, 3/2$) ($\underline{8}$, $J = 1/2, 3/2, 5/2$) ($\underline{10}$, $J = 1/2, 3/2$) ($\underline{8}$, $J = 1/2, 3/2$)	210	30
s^1p^2 (pure)	1	+	(1, 1, 1)	($\underline{8}$, $J = 1/2, 3/2$) ($\underline{1}$, $J = 1/2, 3/2, 5/2$)	60	11
s^1p^2, s^2d^1 (mixed)	0	+	(3)	($\underline{8}$, $J = 1/2$) ($\underline{10}$, $J = 3/2$)	56	8
s^1p^2, s^2d^1 (mixed)	0	+	(2, 1)	($\underline{1}$, $J = 1/2$) ($\underline{8}$, $J = 3/2$) ($\underline{10}$, $J = 1/2$) ($\underline{8}$, $J = 1/2$)	70	13
$p^3, s^1p^1d^1, s^2f^1$ (mixed)	3	−	(3)	($\underline{8}$, $J = 5/2, 7/2$) ($\underline{10}$, $J = 3/2, 5/2, 7/2, 9/2$)	392	24
$p^3, s^1p^1d^1$ (mixed)	2	−	(2, 1)	($\underline{1}$, $J = 3/2, 5/2$) ($\underline{8}$, $J = 1/2, 3/2, 5/2, 7/2$) ($\underline{10}$, $J = 3/2, 5/2$) ($\underline{8}$, $J = 3/2, 5/2$)	350	34
$p^3, s^1p^1d^1, s^2f^1$ (mixed)	1	−	(3)	($\underline{8}$, $J = 1/2, 3/2$) ($\underline{10}$, $J = 1/2, 3/2, 5/2$)	168	20
$p^3, s^1p^1d^1$ (mixed)	1	−	(2, 1)	($\underline{1}$, $J = 1/2, 3/2$) ($\underline{8}$, $J = 1/2, 3/2, 5/2$) ($\underline{10}$, $J = 1/2, 3/2$) ($\underline{8}$, $J = 1/2, 3/2$)	210	30
$p^3, s^1p^1d^1$ (mixed)	0	−	(1, 1, 1)	($\underline{8}$, $J = 1/2$) ($\underline{1}$, $J = 3/2$)	20	5
				Totals:	2600	291

[a]See reference 23.

tiplets occur is common to any three-quark model. The table can be constructed using only elementary facts[21] known to nuclear physicists. A useful check[22] on the construction of the table is the observation that if the s- and p-orbital states are considered equivalent, then these four states transform as the fundamental representation of an "orbital" SU(4) group. When this last group is combined with SU(6), one reaches SU(24), whose three-particle symmetric representation has dimension 2600.[23]

If this model is correct, then it should be possible to produce real paraquarks in high-energy interactions. The superselection rules for production of paraquarks from normal matter are the same as for Fermi quarks (see Table I); for example, $b + m \rightarrow 3q$, but $b + m \not\rightarrow 2q$

or q, where b, m, q stand for baryon, meson, quark. However, the threshold behavior of $b+m \to 3q$ for paraquarks would reflect the s-state wave functions and would differ from the threshold behavior for Fermi quarks.

In Coulomb scattering from normal matter, the lowest order cross section for para particles is the same as for normal particles of the same charge, so para particles should be detected as easily as normal particles.

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[5]We have no theoretical justification for the use of a nonrelativistic classification of forces and states.

[6]M. Gell-Mann, Phys. Letters 8, 214 (1964); G. Zweig, to be published.

[7]B. Sakita, reference 2.

[8]P. G. O. Freund and B. W. Lee (private communication) have constructed models of baryons using Fermi quarks.

[9]It is amusing that one reaches the concrete notion of atomic structure after a lengthy detour through the special unitary groups.

[10]This was already pointed out by Sakita,[2] who suggested for this reason the use of the antisymmetric 20-dimensional representation for the baryons. However, this suggestion has two defects: First, the $\underline{20}$ no longer accommodates the decuplet; and secondly, the SU(6)-invariant electromagnetic coupling to the $\underline{20}$ leads to the wrong value of β.[4]

[11]This will be the case if the quarks are all in s states.

[12]Quarks having this mass would already have been detected. See W. Blum et al., Phys. Rev. Letters 13, 353a (1964); V. Hagopian et al., Phys. Rev. Letters 13, 280 (1964); and references in these articles.

[13]All statements made here concerning para particles are straightforward consequences of results contained in O. W. Greenberg and A. M. L. Messiah, to be published. The "absence of para particles" in the title of this article refers to the presently known particles.

[14]Permutations of these indices are related to permutations of the particles in the state for one-dimensional representations of the permutation group, but not in general.[13]

[15]We can also construct the meson $\underline{35}$ using a particle-antiparticle pair of order 3 para-Fermi quarks in s states. Let

$$b_{\lambda\mu}^{\dagger} = [a_{\lambda}^{\dagger}, a_{\mu}^{\dagger}]_{-} = 2 \sum_{\alpha \neq \beta = 1}^{3} a_{\lambda}^{(\alpha)\dagger} a_{\mu}^{(\beta)\dagger}.$$

Then the state $b_{\lambda\mu}^{\dagger}\Phi_0$ is antisymmetric under permutation of λ and μ, and this composite state is a boson since $[b_{\lambda\mu}^{\dagger}, a_{\nu}^{\dagger}]_{-} = 0$, which implies $[b_{\lambda\mu}^{\dagger}, b_{\nu\sigma}^{\dagger}]_{-} = 0$. This construction works for all para orders, including $p=1$, the Fermi case. So the mesons constructed with parafermions are the same as those constructed with fermions; for this reason we will not consider the mesons further in this article.

[16]By "relevant," we mean that these interactions lead to connected Feynman graphs with only the desired $\underline{56}$ (or $\underline{35}$) in the initial and final states.

[17]A. M. L. Messiah and O. W. Greenberg, Phys. Rev. 136, B248 (1964).

[18]We expect that when this theorem[17] is amended to include the parafield superselection rules, the absolute selection rule for production of paraquarks from $\mathfrak{F}^{\times}$ will allow production, from ordinary states, of both symmetric and antisymmetric paraquark states, provided these states occur in different superselection sectors.

[19]C. N. Yang, private communication.

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[22]We thank P. G. O. Freund and B. W. Lee for pointing this out.

[23]Because Table II lists shell-model states, some spurious states are present due to incorrect treatment of the center-of-mass motion. Only the configuration s^2p^1, with Young diagram (3), is purely spurious; for the other configurations we have indicated the higher shell-model states which must be mixed in to produce proper internal states. (Harmonic oscillator wave functions were used.) We plan to do more detailed calculations with more specific assumptions about the forces to try to estimate where the various SU(3) multiplets should lie. Perhaps other orbitals, such as the second radial s state, should be considered. Empirical information will be helpful in guiding such calculations. See J. P. Elliott and T. H. R. Skyrme, Proc. Roy. Soc. (London) A232, 561 (1955), and E. Baranger and C. W. Lee, Nucl. Phys. 22, 157 (1961), for discussions of spurious states.