The proposed symmetry breaking[6] by use of a subgroup of SU(6) denoted by $W(Y)\otimes SU(4)(T)\otimes SU(2)(X)$, and the properties thereby obtained for the $\omega$ and $\varphi$, are simply described by quark-spin conservation. Breaking SU(6) while preserving invariance under the above subgroup means conservation of $S_{p'}$, $S_{n'}$, $S_{\lambda'}$, isospin, and hypercharge while breaking the conventional SU(3). Thus the selection rule inhibiting strange-particle production still holds even after the inclusion of symmetry-breaking effects producing $\omega$-$\varphi$ mixing and the observed mass splitting in the SU(3) multiplets. The physical $\omega$ and $\varphi$ states proposed are those mixtures of the unitary singlet and octet that are eigenstates of $S_{\lambda'}$, namely $\bar{p}'p'$ and $\bar{n}'n'$ having $S_{\lambda'}=0$, and $\lambda\bar{\lambda}'$ having $S_{\lambda'}=1$. The physical $\varphi$ has $S_{\lambda'}=1$. Its decay into pions is forbidden by quark-spin conservation, and its production without associated strange-particle production is forbidden in $\pi p$, $pp$, and $\bar{p}p$ reactions.

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[1]F. Gürsey and L. A. Radicati, Phys. Rev. Letters 13, 173 (1964); A. Pais, Phys. Rev. Letters 13, 175 (1964).

[2]B. Sakita, Phys. Rev. (to be published).

[3]H. J. Lipkin, Phys. Letters 9, 203 (1964).

[4]M. Gell-Mann, Physics 1, 63 (1964).

[5]A detailed analysis of this reaction may give some reduction in the cross section resulting from quark-spin conservation. However, such a reduction can be expected to be of the order of a factor of 2 for Reaction (2) rather than 16 as for Reaction (1). The analysis can be made by coupling quark spins by traditional angular momentum techniques.

[6]F. Gürsey, L. A. Radicati, and A. Pais, Phys. Rev. Letters 13, 299 (1964).

# SHELL MODEL OF BARYONS*

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It has recently been suggested that by coupling spin and unitary spin, new insights can be attained into the mass spectrum of hadrons.[1] Such a point of view is strongly reminiscent of the shell theory of nuclei and one is tempted to ask whether a similar substructure of baryons is responsible for the "marriage" of spin and unitary-spin symmetries.

Specifically, we shall investigate some implications of the hypotheses that (1) baryons are built up of spin-$\frac{1}{2}$ fermions belonging to a unitary triplet (which we shall call baryonettes); (2) within a baryon, these baryonettes move in a self-consistent central field, exhibiting a shell structure; (3) coupled spin–unitary-spin symmetry is not an intrinsic symmetry of the "fundamental" Lagrangian, but a symmetry of the baryon wave function (in terms of baryonette coordinates) in the central-field approximation. This symmetry is broken by the residual interaction between baryonettes which is not accounted for by the central field. The basic triplet will be assumed to have the charge structure $(\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3})$, just as quarks,[2] but for the moment we shall leave the baryon number $b$ of baryonettes unspecified.

We shall discuss both $LS$- and $jj$-coupling shell models of baryons.

$\underline{LS\text{ coupling}}$.—We assume that the central field is spin–unitary-spin independent. This is the generalization of Wigner's supermultiplet theory[3] in nuclear physics. The spin–unitary-spin part of the baryon wave function transforms according to some representation of SU(6).

If a baryon is made up of three baryonettes ($b=\frac{1}{3}$; in this case our baryonettes are identical to quarks) in the ground $s$ state, the spin–unitary-spin wave function must be completely antisymmetric. The baryons transform according to the 20-dimensional representation of SU(6); physical consequences of this assignment have been amply discussed by Sakita.[1]

Alternatively, we assume that $b=\frac{1}{9}$,[4] and a baryon is made up of nine baryonettes. The first six will fill the $s$ shell and form a unitary-singlet, spinless boson of baryon number $\frac{2}{3}$ (analog of the $\alpha$ particle). If the following shell to be filled is a $p$ shell, then the baryon number = 1 state is reached in the configuration $(1s)^6(2p)^3$. If the interaction among baryonettes can be described in terms of Wigner forces of range much longer than the baryon size, all 18 components of a $p$-shell baryonette are degenerate and the baryons will have the symmetry of SU(18).[5] Because of the Pauli principle, the baryon states will belong to the totally antisymmetric 816-dimensional representation of SU(18).

The short-range Wigner and Majorana forces which are independent of spin and unitary spin will considerably reduce this symmetry. They will break SU(18) down to SU(6)⊗O(3). Let $N_6$ and $2L+1$ be the dimensionalities of representations of SU(6) and O(3), respectively, and $(N_6, L)$ the corresponding representation of SU(6)⊗O(3). The 816-dimensional representation of SU(18) breaks down to $(56,S)+(70,P)+(70,D)+(20,P)+(20,F)$. The mass formula describing this breakdown is a straightforward generalization of the mass formula of Hund[6] and Wigner,[7]

$$m(N_6, L)=m_0+m_1W(N_6)+m_2L(L+1), \qquad (1)$$

where $m_0,\cdots, m_3$ are constant parameters and $W$ is essentially the quadratic Casimir operator of SU(6) and takes the following values: $W(56)=+3$, $W(70)=0$, $W(20)=-3$. At this stage $\vec{L}\cdot\vec{S}$ and $F^2$ [=quadratic Casimir operator of SU(3)] terms will split the supermultiplets (1) into multiplets of SU(3)⊗SU(2) and a Gell-Mann–Okubo mechanism will further break this down to SU(2)⊗U(1)⊗SU(2). The mass $m(N_6, N_3, J, L, S, I, Y)$ of a baryon of spin $J$, isospin $I$, and hypercharge $Y$ belonging to the $(N_3, 2S+1; L)$ multiplet of SU(3)⊗SU(2)⊗O(3) and to the $(N_6, L)$ supermultiplet of SU(6)⊗O(3) [e.g., the nucleon mass $m_N$ in this notation is $m_N=m(56, 8, \frac{1}{2}, 0, \frac{1}{2}, \frac{1}{2}, 1)$ as we naturally[1] assign the well-known $J^P=\frac{1}{2}^+$ baryon octet and $J^P=\frac{3}{2}^+$ baryon decuplet to the representation $(56,S)$ of SU(6)⊗O(3)] is given by the formula[8]

$$\begin{aligned} m(N_6,N_3,J,L,S,I,Y) &= m_0+m_1W(N_6)+m_2L(L+1)+2m_3\vec{L}\cdot\vec{S} \\ &\quad +m_4F^2(N_3)+m_5Y+m_6[I(I+1)-\tfrac{1}{4}Y^2], \end{aligned} \qquad (2)$$

with

$$2\vec{L}\cdot\vec{S}=J(J+1)-L(L+1)-S(S+1),$$

and $F^2(1)=0$, $F^2(8)=3$, $F^2(10)=6$. All 95 baryon states (we do not count states differing only by the values of their $I_3$ and/or $J_3$ separately) being built up of nine baryonettes in the $(1s)^6(2p)^3$ configuration will have the same parity. Of course this represents an extremely rich baryon spectrum. One can show that the experimentally known baryons are insufficient for the determination of the seven parameters appearing in Eq. (2). However, numerous sum rules for masses can be derived from (2). Without going into details let us point out that, whichever of the $J^P=\frac{5}{2}^+$ octets, contained in 816, $N^*(1685)$ should belong to, the $I=Y=0$ member of this octet has to be a $J^P=\frac{5}{2}^+$ $Y_0^*$ of mass $1685+m_\Lambda-m_N=1860$ MeV. Experimentally there exists a $J^P=\frac{5}{2}^+$ $Y_0^*$ at $1815\pm35$ MeV.[9] Furthermore, provided $Y_1^*(1660)$ has $J^P=\frac{3}{2}^+$, independently of its $N_3$ (8 or 10) or $N_6$ (70 or 20), the accompanying $J^P=\frac{3}{2}^+$ $\Xi^*$ will have a mass $1660+m_\Xi-m_\Sigma=1790$ MeV. It is again gratifying that an $I=\frac{1}{2}, J^P=?$ $\Xi^*$ has been observed[9] at 1770 MeV. In a less obvious way a $P_{11}$ $\pi N$ resonance at $\simeq 1485$ MeV can be predicted by (2) again in agreement with experiment.[10] Even though these predictions are in agreement with the existing baryon spectrum one should be worried by the very large number of baryons predicted by the $LS$ shell model. We shall see that this difficulty can be much reduced in the framework of $jj$ coupling.

First, however, let us remark that all the experimental predictions[11] made on the basis of SU(6) for the 56-dimensional supermultiplet can be reproduced in the $LS$-coupling shell model.

*jj coupling.*—This is presumably a better approximation, since the baryonette mass appears to be large and the binding energy of the baryon must also be large. If the baryon is made up of three quarks ($b=\frac{1}{3}$) in the $j=\frac{1}{2}$ shell, the symmetry of the wave function is SU(6)⊃SU(2)⊗SU(3). This case therefore reduces to that considered by Sakita.[1]

If $b=\frac{1}{9}$ and the $p_{1/2}$ shell has lower energy than the $p_{3/2}$ shell, we again obtain Sakita's model.[1]

We shall assume that the strong spin-orbit coupling decreases the energy of the $p_{3/2}$ shell below that of the $p_{1/2}$ shell. There will then be three baryonettes in the (outer) $p_{3/2}$ shell. In the unitary-spin-independent central field, the 12 components of a $j=\frac{3}{2}$ baryonette are degenerate and the baryons transform according to the totally antisymmetric, 220-dimensional representation of SU(12). A straightforward generalization of the corresponding problem in nuclear physics[12] suggests that we reduce the symmetry of the wave function according

Table I. Symmetry chain in $jj$ coupling.

| Group | SU(12) | SU(4)⊗SU(3) | Sp(4)⊗SU(3)[a] | SU(2)⊗SU(3)[b] |
|---|---|---|---|---|
| Representations and values of $Q$ and $F^2$ | 220 | (20, 1) | (20, 1) | $(\frac{3}{2}, 1)$ $Q=21$ $F^2=0$<br>$(\frac{5}{2}, 1)$<br>$(\frac{9}{2}, 1)$ |
| | | (20, 8) | (16, 8) | $(\frac{1}{2}, 8)$ $Q=15$ $F^2=3$<br>$(\frac{5}{2}, 8)$<br>$(\frac{7}{2}, 8)$ |
| | | | (4, 8) | $(\frac{3}{2}, 8)$ $Q=5$ $F^2=3$ |
| | | (4, 10) | (4, 10) | $(\frac{3}{2}, 10)$ $Q=5$ $F^2=6$ |

[a] In this column, the first entry is the dimensionality $N_{S4}$ of the irreducible representation of Sp(4).
[b] In this column the spins, rather than the spin multiplicities, are used in the representation symbol.

to the chain SU(12) → SU(4)⊗SU(3) where SU(4) is the symmetry of the spin-orbit part of the wave function, followed by SU(4) → Sp(4) → SU(2), and SU(3) → broken SU(3). The appearance of Sp(4) in the last part of the chain is intimately related to the seniority scheme.[13] The successive reduction of 220 is shown in Table I. The linear mass formula corresponding to the above chain of symmetry breaking is

$$m(N_3, N_{S4}, J, I, Y) = m_0 + m_1 F^2 + m_2 Q(N_{S4}) + m_3 J(J+1) + m_4 Y + m_5[I(I+1) - \tfrac{1}{4}Y^2], \qquad (3)$$

where $Q(N_{S4})$ is the eigenvalue of the quadratic Casimir operator of Sp(4) corresponding to the $N_{S4}$-dimensional representation, and can be read in Table I.[14] Now, all six parameters of Eq. (3) can be determined and in addition to the well-known $J^P = \frac{1}{2}^+$ octet and $J^P = \frac{3}{2}^+$ decuplet, three more octets and three more singlets are predicted and the masses of the corresponding 15 particles are listed in Table II. As can be seen from this table, the $\Lambda_{5/2}(1815)$ and $\Xi_{3/2}(1770)$ are present also in this scheme, whereas the 1485 $P_{11}$ $\pi N$ resonance predicted in $LS$ coupling does not appear for $jj$ coupling. The important feature is, however, the relatively small number (23) of baryons as opposed to the 95 baryons predicted in $LS$ coupling. Since now SU(6) symmetry is not realized in any intermediate

Table II. Baryon mass spectrum in $jj$ coupling.[a]

| | | Mass (MeV) | | | | | |
|---|---|---|---|---|---|---|---|
| | | $Y=0$ | | $Y=-\frac{1}{2}$ | $Y=1$ | | $Y=-2$ |
| $J^P$[b] | SU(3) multiplicity | $I=0$ | $I=1$ | $I=\frac{1}{2}$ | $I=\frac{1}{2}$ | $I=\frac{3}{2}$ | $I=0$ |
| $(\frac{1}{2})^+$ | 8 | 1115 | 1190 | 1320 | 940 | | |
| $(\frac{3}{2})^+$ | 10 | | 1385 | 1535 | | 1238 | 1685 |
| $(\frac{3}{2})^+$ | 8 | 1585 | 1660[c] | 1790[d] | 1410 | | |
| $(\frac{5}{2})^+$ | 8 | 1860[e] | 1935 | 2065 | 1685 | | |
| $(\frac{7}{2})^+$ | 8 | 2510 | 2585 | 2715 | 2335 | | |
| $(\frac{3}{2})^+$ | 1 | 1555 | | | | | |
| $(\frac{5}{2})^+$ | 1 | 2020 | | | | | |
| $(\frac{9}{2})^+$ | 1 | 3510 | | | | | |

[a] Underlined numbers are input; all others are predictions.
[b] All parities are defined relative to the nucleon.
[c] The spin-parity of $Y_1^*(1660)$ is assumed to be $\frac{3}{2}^+$.
[d] This could be identified with $\Xi_{1/2}^*(1770)$, $J^P(\text{expt}) = ?$: M. Roos, reference 9.
[e] This could be identified with $\Lambda^*(1815 \pm 35)$, $J^P = (\frac{5}{2})^+$: M. Roos, reference 9.

step of the breakdown of SU(12), the predictions of coupling constants, magnetic moments, etc., made on the basis of SU(6) have to be revised. In particular we find[15]

$$\mu(n)/\mu(p) = -\tfrac{4}{5},$$

to be compared with $\mu(n)/\mu(p)|_{\rm expt} \cong -0.68$. For the $D/F$ ratio of the axial vector current in weak interactions, $jj$ coupling gives $(D/F)_A = 2$ in good agreement with $(D/F)_A \cong 1.7$ (experimental).[16,17] Finally, a naive application of the vector addition of angular momenta leads to the following generalized Landé formula:

$$\frac{m_b}{m_N} = \frac{2(2+\lambda)}{3\mu_n},$$

where $\mu_n = -1.91$ is the magnetic moment of the neutron, $\lambda$ the anomalous magnetic moment of the baryonettes, and $m_b$ the baryonette mass. A large baryonette mass is therefore expected to be accompanied by a large baryonette anomalous magnetic moment.

The mesons ($0^-$, $1^-$) may be considered as composite systems of a baryonette and an antibaryonette bound in the $s$ state. Independent of the value of $b$, we obtain a 35-plet of bosons.[1,18]

We emphasize that, in the present consideration, the couplet spin–unitary-spin symmetry is characteristic of the shell structure of baryons, rather than of the baryon-meson interaction.

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[1]F. Gürsey and L. A. Radicati, Phys. Rev. Letters 13, 173 (1964); A. Pais, Phys. Rev. Letters 13, 175 (1964); B. Sakita, to be published.

[2]M. Gell-Mann, Phys. Letters 8, 216 (1964); G. Zweig, to be published.

[3]E. P. Wigner, Phys. Rev. 51, 105 (1937).

[4]Another possibility is to consider quarks obeying parastatistics as first suggested by O. W. Greenberg, private communication. Of course even within the framework of conventional statistics there are many more possibilities than just introducing baryonettes of $b = \frac{1}{9}$.

[5]The correspondent of our group SU(8) in nuclear physics is SU[4(2$l$ +1)] ; see, e.g., G. Racah, "Group Theory and Spectroscopy," unpublished lecture notes, The Institute for Advanced Study, Princeton, 1951.

[6]F. Hund, Z. Physik 105, 202 (1937).

[7]E. P. Wigner, Phys. Rev. 51, 947 (1937).

[8]Our mass formulas (2) and (3) are simple generalizations of well-known mass formulas of nuclear physics. A discussion of baryon states in the SU(6) scheme without considering orbital angular momenta has been given by A. Pais, reference 1. Mass formulas for this case have been discussed by T. K. Kuo and T. Yao, Phys. Rev. Letters 13, 415 (1964); and M. A. B. Bég and V. Singh, Phys. Rev. Letters 13, 418 (1964).

[9]M. Roos, Nucl. Phys. 52, 1 (1964).

[10]L. D. Roper, Phys. Rev. Letters 12, 340 (1964).

[11]F. Gürsey, A. Pais, and L. A. Radicati, Phys. Rev. Letters 13, 299 (1964); M. A. B. Bég, B. W. Lee, and A. Pais, Phys. Rev. Letters 13, 517 (1964); H. J. Lipkin, to be published.

[12]See, for instance, A. de Shalit and I. Talmi, Nuclear Shell Theory (Academic Press, New York, 1963), P.III.

[13]B. H. Flowers, Proc. Roy. Soc. (London) A212, 248 (1952).

[14]For the eigenvalue of the quadratic Casimir operator of Sp($2n$) in a representation characterized by the partition $(\sigma_1, \sigma_2, \cdots, \sigma_n)$, we find $Q[(\sigma_1, \sigma_2, \cdots, \sigma_n)] = \sigma_1(\sigma_1 + 2n) + \sigma_2(\sigma_2 + 2n-2) + \cdots \sigma_n(\sigma_n + 2)$.

[15]This result follows from the vector addition of (spin and orbital) magnetic moments.

[16]W. Willis et al., Phys. Rev. Letters 13, 291 (1964); N. Brene et al., Phys. Letters 11, 344, (1964).

[17]These predictions follow not only from SU(12), but also from the first stage in broken symmetry, SU(4) ⊗ SU(3). The SU(12) symmetry is badly broken and predictions valid only at the SU(12) stage are presumably not reliable. Note that at the stage SU(4) ⊗ SU(3) the baryon octet and decuplet do not belong to the same supermultiplet. [This is to be contrasted with the $LS$ case where the baryon octet and decuplet belong to the same supermultiplet at the corresponding stage SU(3)⊗SU(6) where SU(3) is the symmetry of the $p$-shell wave function.] Hence, at this stage octet-decuplet transition matrix elements (for processes, e.g., $N^* \to N + \pi$, $N^* \to N + \gamma$) are not related to the corresponding octet-octet matrix elements (e.g., $N \to N + \pi$, $N \to N + \gamma$).

[18]The mass formula for bosons, derived from the shell-model point of view, is

$$m = m_0 + m_1 J(J+1) + m_2 F^2 + m_3 [I(I+1) - \tfrac{1}{4} Y^2].$$

Physical implications of this mass formula are discussed in reference 1 and Kuo and Yao, reference 8. The shell-model picture we adopt here does not relate the charge-type coupling of the vector mesons to the pseudovector or magnetic-moment–type coupling of the 35-plet bosons. We do obtain an equality between the pseudovector coupling of the pseudoscalar mesons and the Pauli-type coupling of the vector mesons to the baryons, as noted in reference 1 and Gürsey, Pais, and Radicati, reference 11.