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[1]R. Armenteros et al., Proceedings of the International Conference on High-Energy Nuclear Physics, Geneva, 1962, edited by J. Prentki (CERN Scientific Information Service, Geneva, Switzerland, 1962), p. 351; G. B. Chadwick et al., ibid., p. 69.

[2]A paper by K. Tanaka [Phys. Rev. 135, B1186 (1964)] makes a similar assumption, but neglects the singlet vector-meson contribution to the intermediate state—a dubious approximation.

[3]P. Tarjanne and V. Teplitz, Phys. Rev. Letters 11, 447 (1963); Y. Hara, Phys. Rev. 134, B701 (1964); D. Amati et al., Phys. Letters 11, 190 (1964); B. J. Bjørken and S. L. Glashow, Phys. Letters 11, 255 (1964); I. S. Gerstein and M. L. Whippman, Phys. Rev. 136, B829 (1964).

[4]We denote the unmixed isosinglet vector mesons as ω_8 and ω_1, belonging to the octet and singlet, respectively. We will take the mixing angle as 35.3° for simplicity ($\cos 35.3° = \sqrt{\frac{2}{3}}$).

[5]It should be noted that in an SU(4)-invariant theory based on a fundamental set of four particles and their antiparticles, the smallest multiplet in which the baryons can be placed is the icosaplet.

[6]For the baryon icosaplet, we replace d by $\frac{1}{2}(d+f)$ in the last term.

[7]Units are $\hbar = m_\pi = c = 1$.

[8]See, for example, C. Bouchiat and G. Flammand, Nuovo Cimento 23, 13 (1962).

[9]R. Armenteros et al., reference 1. Y. Dothan et al., Phys. Letters 1, 308 (1962), also derived theoretical branching relations for these final states, assuming $d = 3f$.

HIGHER SYMMETRIES AND STRANGE-PARTICLE PRODUCTION*

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The consistently low experimental cross sections observed for strange-particle production in nucleon-nucleon, pion-nucleon, and antinucleon-nucleon reactions provide an important test for any higher symmetry theory which assumes a universal coupling for strange and nonstrange particles. The rapid decrease in cross section with increasing strangeness of the particles (N, Λ, Ξ, Ω) produced seems to indicate the existence of a selection rule. Unitary symmetry does not provide such a selection rule and fits the experimental data only by use of the large number of free parameters arising in the coupling of two unitary octets. Any proposed higher symmetry should have fewer free parameters and may be restricted so much that it must either exhibit a selection rule or disagree with experiment.

The purpose of this Letter is to point out that a selection rule may be obtained from higher symmetry schemes[1-3] which are extensions of the Wigner supermultiplet classification. In the following discussion, a quark[4] model is used as the simplest method of presentation, but is not necessary to obtain the result. The elementary quarks are a unitary triplet of spin $\frac{1}{2}$ having third-integral values for electric charge, hypercharge, and baryon number, but conventional values for isospin and strangeness. By analogy with the Sakata model, the members of the triplet are designated as follows: p' and n' form an isodoublet of strangeness zero, and λ' is an isosinglet with strangeness -1. The physical baryons are states of three quarks, while the physical pseudoscalar mesons are states of a quark-antiquark pair.

In the Wigner supermultiplet theory, systems of nucleons are treated assuming invariance of the interactions under the group SU(4); i.e., the four spin and charge states of the nucleon are assumed to be equivalent. By use of SU(2) subgroups of SU(4), many conserved quantities having the properties of an angular momentum can be defined; e.g., in addition to the total spin of the system, the total spin of all the neutrons and the total spin of all the protons are conserved separately. Similar conservation laws are found in a quark model under the assumption of SU(6) invariance. In any system of quarks, the total spins $S_{p'}$, $S_{n'}$, and $S_{\lambda'}$ of the p', n', and λ' components are separately conserved.

This "quark-spin" conservation leads to selection rules inhibiting the production of strange particles. In any nucleon-nucleon, pion-nucleon, or antinucleon-nucleon reaction, the initial state contains only the nonstrange quarks p' and n' and the corresponding antiquarks, but contains no strange quarks λ'. The total λ' and $S_{\lambda'}$ is therefore zero. In associated production of strange particles, the final state contains a strange-quark pair, λ' and $\bar{\lambda}'$. If $S_{\lambda'}$ is conserved, the $\lambda'\bar{\lambda}'$ pair must be produced in a state having $S_{\lambda'}=0$, and its production in a state having $S_{\lambda'}=1$ is forbidden. This introduces a statistical factor of $\frac{1}{4}$ in the strange-particle-production cross section. With increasing strangeness, more $\lambda'\bar{\lambda}'$ particles are produced and the restriction to $S_{\lambda'}=0$ reduces the cross section by a greater statistical factor.

Detailed analyses of reactions are complicated because the individual quark spins do not commute with the total spin. The observed particles are usually not eigenstates of $S_{p'}$, $S_{n'}$, and $S_{\lambda'}$. We shall examine one particularly simple example.

Consider the reaction

$$\underset{(p'\bar{n}')}{\pi^+} + \underset{(p'p'n')}{p} \to \underset{(p'\bar{\lambda}')}{K^+} + \underset{(p'p'\lambda')}{\Sigma^+}, \qquad (1)$$

where the quark constituents are written underneath each particle. The final state in this reaction is not an eigenstate of $S_{\lambda'}$. However, since the spin of the $\bar{\lambda}'$ in the K^+ is coupled with a p' to a total spin of zero, the orientation of the $\bar{\lambda}'$ spin is completely uncorrelated with the λ' spin in the Σ^+. Thus, the final state is a linear combination of states having $S_{\lambda'}=0$ and 1 with a probability $\frac{1}{4}$ for $S_{\lambda'}=0$. This leads to a reduction in the cross section by a factor of 4. A similar argument can be given for conservation of $S_{n'}$ since no n' is present in the final state. Only the component of the initial state with $S_{n'}=0$ contributes to the reaction cross section and reduces the cross section by another factor of 4. Thus, individual quark-spin conservation reduces the cross section for Reaction (1) by a factor 16.

The reaction

$$\underset{(\bar{p}'\lambda')}{K^-} + \underset{(p'p'n')}{p} \to \underset{(\bar{p}'n')}{\pi^-} + \underset{(p'p'\lambda')}{\Sigma^+} \qquad (2)$$

appears superficially to be very similar to Reaction (1), but its experimental cross section is very much higher. A difference between these reactions can be seen from quark-spin conservation. Both the initial and the final states of Reaction (2) are eigenstates of $S_{\lambda'}$ and $S_{n'}$ with the eigenvalue $\frac{1}{2}$, and there is no effect analogous to the factor 16 for Reaction (1).[5]

The exact meaning of individual quark-spin conservation in a relativistic many-quark system is not obvious because of the role of orbital angular momentum. For this reason Reaction (1) provides a particularly simple case. In systems containing no λ' nor n', the corresponding total quark spins must vanish no matter how the orbital angular momentum is included.

The application to nucleon-antinucleon annihilation processes is more complicated because all particles have finite spin, and spin correlations between different quarks must be calculated by use of Racah algebra. The extreme case of $\bar{p}+p\to\bar{\Omega}+\Omega$ is simplest since the Ω consists of three λ' quarks coupled to $S_{\lambda'}=S=\frac{3}{2}$, and $S_{\lambda'}$ for the $\bar{\Omega}\Omega$ system is equal to the total spin. The only final-state spin allowed by quark-spin conservation is zero; transitions to the states of total spin 1, 2, and 3 are forbidden. No similar selection rule exists for the corresponding reaction $\bar{p}+p\to\bar{N}^*+N^*$. Thus one can say qualitatively that quark-spin conservation in the final state reduces the $\Omega\Omega$ cross section by a factor of roughly 16. An additional reduction is found in the requirement for $S_{p'}=S_{n'}=0$ in the initial state. Exact calculation of this factor involves angular-momentum couplings and some assumptions regarding the roles of the orbital angular momenta.

The arguments given above can be rewritten in an abstract form independent of the quark model by defining the quark-spin operators as generators of SU(2) subgroups of SU(6) which act on the states of the fundamental six-dimensional representation of SU(6) in the same way as the quark-spin operators act on the quark states. If the mesons and baryons are classified in the 35- and 56-dimensional representations of SU(6),[1] the SU(2) generators will act on these states in exactly the same way as the quark-spin operators in the quark model. These results hold as well in any higher symmetry scheme that includes SU(6); e.g., an SU(9) classification including the baryon number.[3]

Quark-spin conservation can be used to obtain relations between cross sections for various processes without requiring the full SU(6) algebra. This is similar to the use of U spin for obtaining SU(3) results. The relations obtained are not always easily interpreted because of the presence of orbital angular momentum. For example, conservation of total quark spin with neglect of orbital angular momentum would forbid the decay of the decuplet ($S=\frac{3}{2}$) into a baryon ($S=\frac{1}{2}$) and a pseudoscalar meson ($S=0$), and also the decay of a vector meson into two pseudoscalar mesons.

The proposed symmetry breaking[6] by use of a subgroup of SU(6) denoted by $W(Y)\otimes SU(4)(T)\otimes SU(2)(X)$, and the properties thereby obtained for the ω and φ, are simply described by quark-spin conservation. Breaking SU(6) while preserving invariance under the above subgroup means conservation of $S_{p'}$, $S_{n'}$, $S_{\lambda'}$, isospin, and hypercharge while breaking the conventional SU(3). Thus the selection rule inhibiting strange-particle production still holds even after the inclusion of symmetry-breaking effects producing ω-φ mixing and the observed mass splitting in the SU(3) multiplets. The physical ω and φ states proposed are those mixtures of the unitary singlet and octet that are eigenstates of $S_{\lambda'}$, namely $\bar{p}'p'$ and $\bar{n}'n'$ having $S_{\lambda'}=0$, and $\lambda\bar{\lambda}'$ having $S_{\lambda'}=1$. The physical φ has $S_{\lambda'}=1$. Its decay into pions is forbidden by quark-spin conservation, and its production without associated strange-particle production is forbidden in πp, pp, and $\bar{p}p$ reactions.

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[1]F. Gürsey and L. A. Radicati, Phys. Rev. Letters 13, 173 (1964); A. Pais, Phys. Rev. Letters 13, 175 (1964).

[2]B. Sakita, Phys. Rev. (to be published).

[3]H. J. Lipkin, Phys. Letters 9, 203 (1964).

[4]M. Gell-Mann, Physics 1, 63 (1964).

[5]A detailed analysis of this reaction may give some reduction in the cross section resulting from quark-spin conservation. However, such a reduction can be expected to be of the order of a factor of 2 for Reaction (2) rather than 16 as for Reaction (1). The analysis can be made by coupling quark spins by traditional angular momentum techniques.

[6]F. Gürsey, L. A. Radicati, and A. Pais, Phys. Rev. Letters 13, 299 (1964).

SHELL MODEL OF BARYONS*

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It has recently been suggested that by coupling spin and unitary spin, new insights can be attained into the mass spectrum of hadrons.[1] Such a point of view is strongly reminiscent of the shell theory of nuclei and one is tempted to ask whether a similar substructure of baryons is responsible for the "marriage" of spin and unitary-spin symmetries.

Specifically, we shall investigate some implications of the hypotheses that (1) baryons are built up of spin-$\frac{1}{2}$ fermions belonging to a unitary triplet (which we shall call baryonettes); (2) within a baryon, these baryonettes move in a self-consistent central field, exhibiting a shell structure; (3) coupled spin–unitary-spin symmetry is not an intrinsic symmetry of the "fundamental" Lagrangian, but a symmetry of the baryon wave function (in terms of baryonette coordinates) in the central-field approximation. This symmetry is broken by the residual interaction between baryonettes which is not accounted for by the central field. The basic triplet will be assumed to have the charge structure $(\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3})$, just as quarks,[2] but for the moment we shall leave the baryon number b of baryonettes unspecified.

We shall discuss both LS- and jj-coupling shell models of baryons.

$\underline{LS\text{ coupling}}$.—We assume that the central field is spin–unitary-spin independent. This is the generalization of Wigner's supermultiplet theory[3] in nuclear physics. The spin–unitary-spin part of the baryon wave function transforms according to some representation of SU(6).

If a baryon is made up of three baryonettes ($b=\frac{1}{3}$; in this case our baryonettes are identical to quarks) in the ground s state, the spin–unitary-spin wave function must be completely antisymmetric. The baryons transform according to the 20-dimensional representation of SU(6); physical consequences of this assignment have been amply discussed by Sakita.[1]

Alternatively, we assume that $b=\frac{1}{9}$,[4] and a baryon is made up of nine baryonettes. The first six will fill the s shell and form a unitary-singlet, spinless boson of baryon number $\frac{2}{3}$ (analog of the α particle). If the following shell to be filled is a p shell, then the baryon number $=1$ state is reached in the configura-