SU(3) AND SU(4) RELATIONS IN ANTIPROTON-NUCLEON ANNIHILATION*

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Experiment seems to indicate that $\bar{p}$-p annihilation at rest proceeds from a $^3S_1$ state.[1] This state has quantum numbers 1^-, $C=-1$ which are just those of the ρ, ω, and φ. It is natural to assume, then, that the annihilation proceeds via these states or, equivalently, via the vector-meson octet and singlet of the eightfold way.[2] In terms of SU(4),[3] we assume that the vector-meson pentadecuplet (15 members) dominates. Isospin symmetry alone and, of course, these larger symmetries imply that $\bar{p}$-n annihilation should behave similarly. So, we will consider the general case of $\bar{p}$-nucleon annihilation into vector plus pseudoscalar mesons.

The annihilation of $\bar{p}$-n is simple for both SU(3) and SU(4) since only the ρ^- intermediate state is allowed. However, $\bar{p}$-p annihilation is more complex, since there are three intermediate states, as already mentioned. With SU(3), we essentially get three independent couplings to the ρ, ω_8, and ω_1,[4] since both f and d couplings contribute to the $\bar{p}p$ (octet) interaction. With SU(4), the $\bar{p}p\omega_1$ coupling is related to the octet coupling, so that we get only two independent parameters. If we put the baryons into a pentadecuplet (which would predict another strange baryon with the quantum numbers of the Λ), the relation between the singlet and octet couplings is a very nice one, and enables us to determine the f to d ratio two nucleon masses off the mass shell for the vector-meson-nucleon-nucleon interaction merely by examining the K^*K final states. When the baryons are placed in an icosaplet (20 members),[5] one neither obtains quite so simple a situation nor predicts a ninth baryon. It is, of course, not known at the present time whether SU(4) holds at all.

For an example of the kind of arithmetic involved here, suppose we consider the process $\bar{p}+p\to\omega+\eta$. The interactions involved, assuming SU(4) symmetry and the baryon pentadecuplet, are as follows:

$$\bar{p}p\{6^{-1/2}(3f-d)\omega_8+(d/\sqrt{3})\omega_1\},^6 \tag{1}$$

$$\omega_8\{3^{-1/2}\omega_1\eta-6^{-1/2}\omega_8\eta\}. \tag{2}$$

Thus neglecting the (small) propagator differences, we find the following amplitudes:

State	Amplitude
$\omega_8\eta$	$\frac{2}{3}d-f$
$\omega_1\eta$	$18^{-1/2}(3f-d)$

Decomposing these by means of $\omega_8=\varphi\cos\theta+\omega\sin\theta$ and $\omega_1=-\varphi\sin\theta+\omega\cos\theta$, we obtain $d/3\sqrt{3}$ for the $\omega\eta$ amplitude; the $K^{*+}K^-$ amplitude is simply d. So, in this case, $27(p_{K^{*+}}/p_\omega)^3=27(89/105)$ gives the branching ratio.[7] (We have used a p^3 weighting factor to account for phase space and the momentum dependence of the matrix element.) Other branching ratios are computed in the same way.

One may note above the lack of an f/d ambiguity in the vector-vector-pseudoscalar interaction; this is because d type only is allowed, f type yielding a $\rho\rho\pi$ coupling. We also call attention to the fact that we have treated the particles as though stable. Thus, for example, we ignore the effects of overlapping resonances in the final state.[8]

The results of the calculation are shown in Tables I, II, III, and IV, where all relations obtained by charge conjugation have not been shown explicitly. For the case of SU(3), we have taken the couplings as follows:

$$\bar{p}p(a\omega_1+b\omega_8+c\rho^0),\quad d\omega_1\mathrm{Tr}(VP),$$
$$\text{and } e\,\mathrm{Tr}\{P[V,V]_+\},$$

where P and V are the pseudoscalar and vector octets, respectively. The notation for the "charmed" particles is that of Bjørken and

Table I. Relative amplitudes for $\bar{p}$-n annihilation.

State	Symmetry used SU(3)	SU(4)
$K^{*-}K^0$	$(\frac{3}{2})^{1/2}c$	1
K^-K^{*0}	$(\frac{3}{2})^{1/2}c$	1
$\rho^-\eta$	c	$\sqrt{\frac{2}{3}}$
$\omega_1\pi^-$	a	$\sqrt{\frac{1}{3}}$
$\omega_8\pi^-$	c	$\sqrt{\frac{2}{3}}$
$D_p{}^-D_v{}^0$	...	1
$D_p{}^0D_v{}^-$	...	1
$\rho^-\chi$	...	$\sqrt{\frac{1}{3}}$

Table II. Relative amplitudes for $\bar{p}$-p annihilation.

	Symmetry used		
		SU(4)	
State	SU(3)	Baryon pentadecuplet	Baryon icosaplet
$\pi^+\rho^-$	$ad+2be$	f	$\frac{1}{6}(7f-d)$
$\pi^0\rho^0$	$ad+2be$	f	$\frac{1}{6}(7f-d)$
$K^{*0}\bar{K}^0$	$ad-be-\sqrt{3}ce$	$-f$	$-\frac{1}{6}(5f+d)$
$K^{*+}K^-$	$ad-be+\sqrt{3}ce$	d	$+\frac{1}{6}(f+5d)$
$\omega_8\eta$	$ad-2be$	$\frac{2}{3}d-f$	$\frac{1}{6}(3d-5f)$
$\omega_1\eta$	bd	$18^{-1/2}(3f-d)$	$18^{-1/2}(3f-d)$
$\omega_8\pi^0$	$2ce$	$3^{-1/2}(f+d)$	$3^{-1/2}(f+d)$
$\omega_1\pi^0$	cd	$6^{-1/2}(f+d)$	$6^{-1/2}(f+d)$
$\rho^0\eta$	$2ce$	$3^{-1/2}(f+d)$	$3^{-1/2}(f+d)$
$D_p{}^+D_v{}^-$	$\cdots$	$-d$	$-\frac{1}{6}(5d+f)$
$D_p{}^0D_v{}^0$	$\cdots$	f	$+\frac{1}{6}(5f+d)$
$\rho^0\chi$	$\cdots$	$6^{-1/2}(f+d)$	$6^{-1/2}(f+d)$
$S_p{}^+S_v{}^-$	$\cdots$	$-f$	$-\frac{1}{6}(7f-d)$
$\omega_8\chi$	$\cdots$	$18^{-1/2}(3f-d)$	$18^{-1/2}(3f-d)$
$\omega_1\chi$	$\cdots$	$-2d/3$	$-\frac{1}{3}(d+f)$

Glashow.[3] ($S_p{}^+$ and $S_V{}^+$ are the $C=+1$ pseudoscalar and vector isosinglets, respectively. $D_p{}^+$ and $D_p{}^0$ make a pseudoscalar $C=+1$ doublet; similarly for $D_v{}^+$ and $D_v{}^0$.) We have also used the masses they derived with the exception of that for χ, the ninth pseudoscalar meson, whose mass we took as that of the $\eta\pi\pi$ resonance, since that is now definitely 0^{-+}.

Table III. Branching ratios for $\bar{p}n$ annihilation.

SU(3) symmetry

(1) $\Gamma(K^{*-}K^0)=\Gamma(K^{*0}K^-)=\Gamma(\rho^-\eta)$

(2) $[\Gamma(\rho^-\eta)]^{1/2}=|[\Gamma(\varphi\pi^-)]^{1/2}\pm[\Gamma(\omega\pi^-)/5]^{1/2}|$

SU(4) symmetry

(3) $3\Gamma(K^{*0}K^-)=3\Gamma(K^{*-}K^0)=30\Gamma(\varphi\pi^-)=2\Gamma(\omega\pi^-)$
$=4\Gamma(\rho^-\eta)=40\Gamma(\rho^-\chi)=6\Gamma(D_p{}^-D_v{}^0)$
$=6(D_p{}^0D_v{}^-)$

It can be seen that the "charmed" doublets are expected to be produced rather copiously, if their masses are as assumed. All the predictions in Tables III and IV are to one significant figure in the coefficients, since one only expects order-of-magnitude accuracy (or perhaps better). This is borne out by comparing the experimental results of $\bar{p}$-p annihilation into two pseudoscalars against the theoretical prediction obtained using the methods of this paper. This particular process is fairly simple to analyze because $\omega_1 \not\to P+P$. One finds the prediction

$$[\Gamma(\pi^+\pi^-)]^{1/2}=2|[\Gamma(K_1K_2)]^{1/2}\pm[\Gamma(K^+K^-)]^{1/2}|.$$

The experiment[9] gave $\Gamma(\pi^+\pi^-):\Gamma(K^+K^-):\Gamma(K_1K_2)=395:131:56$, which satisfies the above relation without the factor of 2.

Table IV. Branching ratios for $\bar{p}$-p annihilation.

SU(3) symmetry

(1) $[\Gamma(\rho^0\eta)]^{1/2}=|[\Gamma(\varphi\pi^0)]^{1/2}\pm[\Gamma(\omega\pi^0)/5]^{1/2}|$

(2) $[2\Gamma(\rho^0\eta)]^{1/2}=|[\Gamma(K^{*+}K^-)]^{1/2}\pm[\Gamma(K^{*0}\bar{K}^0)]^{1/2}|$

(3) $[\Gamma(\rho^+\pi^-)/8]^{1/2}=[\Gamma(\rho^0\pi^0)/8]^{1/2}$
$=\big||3\Gamma(\varphi\eta)]^{1/2}\mp[\Gamma(\omega\eta)/2]^{1/2}|$
$\pm|[\Gamma(K^{*+}K^-)]^{1/2}\mp[\Gamma(K^{*0}\bar{K}^0)]^{1/2}|\big|$

Choose either the upper or lower sign between the K^*K terms in (2) and (3); do the same for the $\varphi\eta$, $\omega\eta$ terms in (1) and (3). The other sign is arbitrary.

SU(4) symmetry

(a) Assuming baryon pentadecuplet.

(1) $\Gamma(\omega\pi^0)=10\Gamma(\varphi\pi^0)=2\Gamma(\rho^0\eta)=20\Gamma(\rho^0\chi)$

(2) $\Gamma(\rho^+\pi^-)=\Gamma(\rho^0\pi^0)=80\Gamma(S_p{}^+S_v{}^-)=2\Gamma(K^{*0}\bar{K}^0)$

(3) $\Gamma(K^{*0}\bar{K}^0)=2\Gamma(D_p{}^0\bar{D}_v{}^0)$

(4) $\Gamma(K^{*+}K^-)=2\Gamma(D_p{}^+D_v{}^-)=20\Gamma(\omega\eta)$

(5) $\Gamma(K^{*+}K^-)/\Gamma(K^{*0}K^0)=(d/f)^2$

(6) $[\Gamma(\varphi\eta)]^{1/2}=|[\Gamma(K^{*+}K^-)/4]^{1/2}\mp[\Gamma(K^{*0}\bar{K}^0)]^{1/2}|$

(7) $[2\Gamma(\rho^0\eta)]^{1/2}=|[\Gamma(K^{*+}K^-)]^{1/2}\mp[\Gamma(K^{*0}\bar{K}^0)]^{1/2}|$

(8) $[\Gamma(\omega\chi)]^{1/2}=|[\Gamma(K^{*+}K^-)/30]^{1/2}\mp[\Gamma(K^{*0}\bar{K}^0/10]^{1/2}|$

In relations (6) through (8), choose either upper or lower sign for all.

(b) Assuming baryon icosaplet

(1) $\Gamma(\omega\pi^0)=10\Gamma(\varphi\pi^0)=2\Gamma(\rho^0\eta)=20\Gamma(\rho^0\chi)=80\Gamma(\omega\eta)$

(2) $\Gamma(\rho^+\pi^-)=\Gamma(\rho^0\pi^0)=80\Gamma(S_p{}^+S_v{}^-)$

(3) $\Gamma(K^{*0}\bar{K}^0)=2\Gamma(D_p{}^0\bar{D}_v{}^0)$

(4) $\Gamma(K^{*+}K^-)=2\Gamma(\bar{D}_p{}^+D_v{}^-)$

(5) $[10\Gamma(\varphi\eta)]^{1/2}=|[\Gamma(K^{*+}K^-)]^{1/2}\pm[9\Gamma(K^{*0}K^0)]^{1/2}|$

(6) $[80\Gamma(\rho^0\eta)]^{1/2}=|[\Gamma(K^{*+}K^-)]^{1/2}\pm[\Gamma(K^{*0}\bar{K}^0)]^{1/2}|$

(7) $[5\Gamma(\omega\chi)]^{1/2}=|[4\Gamma(K^{*+}K^-)]^{1/2}\mp[\Gamma(K^{*0}\bar{K}^0)]^{1/2}|$

(8) $[2\Gamma(\rho^0\pi^0)]^{1/2}=|[\Gamma(K^{*+}K^-)]^{1/2}\mp[9\Gamma(K^{*0}K^0)]^{1/2}|$

In relations (5) through (8), choose either upper or lower sign for all.

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[1]R. Armenteros et al., Proceedings of the International Conference on High-Energy Nuclear Physics, Geneva, 1962, edited by J. Prentki (CERN Scientific Information Service, Geneva, Switzerland, 1962), p. 351; G. B. Chadwick et al., ibid., p. 69.

[2]A paper by K. Tanaka [Phys. Rev. 135, B1186 (1964)] makes a similar assumption, but neglects the singlet vector-meson contribution to the intermediate state—a dubious approximation.

[3]P. Tarjanne and V. Teplitz, Phys. Rev. Letters 11, 447 (1963); Y. Hara, Phys. Rev. 134, B701 (1964); D. Amati et al., Phys. Letters 11, 190 (1964); B. J. Bjørken and S. L. Glashow, Phys. Letters 11, 255 (1964); I. S. Gerstein and M. L. Whippman, Phys. Rev. 136, B829 (1964).

[4]We denote the unmixed isosinglet vector mesons as ω_8 and ω_1, belonging to the octet and singlet, respectively. We will take the mixing angle as 35.3° for simplicity ($\cos 35.3° = \sqrt{\frac{2}{3}}$).

[5]It should be noted that in an SU(4)-invariant theory based on a fundamental set of four particles and their antiparticles, the smallest multiplet in which the baryons can be placed is the icosaplet.

[6]For the baryon icosaplet, we replace d by $\frac{1}{2}(d+f)$ in the last term.

[7]Units are $\hbar = m_\pi = c = 1$.

[8]See, for example, C. Bouchiat and G. Flammand, Nuovo Cimento 23, 13 (1962).

[9]R. Armenteros et al., reference 1. Y. Dothan et al., Phys. Letters 1, 308 (1962), also derived theoretical branching relations for these final states, assuming $d = 3f$.

HIGHER SYMMETRIES AND STRANGE-PARTICLE PRODUCTION*

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The consistently low experimental cross sections observed for strange-particle production in nucleon-nucleon, pion-nucleon, and antinucleon-nucleon reactions provide an important test for any higher symmetry theory which assumes a universal coupling for strange and nonstrange particles. The rapid decrease in cross section with increasing strangeness of the particles (N, Λ, Ξ, Ω) produced seems to indicate the existence of a selection rule. Unitary symmetry does not provide such a selection rule and fits the experimental data only by use of the large number of free parameters arising in the coupling of two unitary octets. Any proposed higher symmetry should have fewer free parameters and may be restricted so much that it must either exhibit a selection rule or disagree with experiment.

The purpose of this Letter is to point out that a selection rule may be obtained from higher symmetry schemes[1-3] which are extensions of the Wigner supermultiplet classification. In the following discussion, a quark[4] model is used as the simplest method of presentation, but is not necessary to obtain the result. The elementary quarks are a unitary triplet of spin $\frac{1}{2}$ having third-integral values for electric charge, hypercharge, and baryon number, but conventional values for isospin and strangeness. By analogy with the Sakata model, the members of the triplet are designated as follows: p' and n' form an isodoublet of strangeness zero, and λ' is an isosinglet with strangeness −1. The physical baryons are states of three quarks, while the physical pseudoscalar mesons are states of a quark-antiquark pair.

In the Wigner supermultiplet theory, systems of nucleons are treated assuming invariance of the interactions under the group SU(4); i.e., the four spin and charge states of the nucleon are assumed to be equivalent. By use of SU(2) subgroups of SU(4), many conserved quantities having the properties of an angular momentum can be defined; e.g., in addition to the total spin of the system, the total spin of all the neutrons and the total spin of all the protons are conserved separately. Similar conservation laws are found in a quark model under the assumption of SU(6) invariance. In any system of quarks, the total spins $S_{p'}$, $S_{n'}$, and $S_{\lambda'}$ of the p', n', and λ' components are separately conserved.

This "quark-spin" conservation leads to selection rules inhibiting the production of strange particles. In any nucleon-nucleon, pion-nucleon, or antinucleon-nucleon reaction, the initial state contains only the nonstrange quarks p' and n' and the corresponding antiquarks, but