from one or more compound states, probably in the ^3P and ^1S configurations.[1,2]

The position of the hydrogen resonance on the energy scale is in very good agreement with theoretical predictions, which range from 9.6 to 9.8 eV.

Because of the difficulty of the present experiment the author had to seek advice on many aspects of the experiment. He is indebted to A. O. McCoubrey, R. F. C. Vessot, and F. Kaufman for advice on handling of atomic hydrogen; to B. R. McAvoy, J. L. Pack, and J. L. Moruzzi for advice on and loan of high-power microwave equipment; to A. V. Phelps and P. J. Chantry for frequent discussions; and to W. J. Uhlig, J. Kearney, and H. T. Garstka for technical assistance.

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[11]In addition to the usual problems encountered in calibrating energy scales, the charging of the glass and the existence of a residual plasma in the region in which the electron beam traverses the gas stream may play a role in establishing the potential in that region.

[12]The elastic cross section in both molecular and atomic hydrogen decreases with electron energy; thus the transmitted current vs electron energy under our operating conditions is a steeply rising function. On such a curve it would be very difficult to observe a resonance. Fortunately, the elastic cross section of H_2O increases with energy in the 9- to 10-eV range and thus it is possible to alter the slope of the transmitted current vs electron energy by admixing various amounts of H_2O to H_2.

[13]In a mixture of H_2 and H_2O it is difficult to establish the proper energy scale. In a mixture of H_2 and Ne, the rare gas serves both as a buffer gas for enhanced dissociation and as a calibrating gas.

GLOBAL CONSERVATION LAWS AND MASSLESS PARTICLES*

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In all of the fairly numerous attempts to date to formulate a consistent field theory possessing a broken symmetry, Goldstone's remarkable theorem[1] has played an important role. This theorem, briefly stated, asserts that if there exists a conserved operator Q_i such that

$$[Q_i, A_j(x)] = \sum_k t_{ijk} A_k(x),$$

and if it is possible consistently to take $\sum_k t_{ijk} \times \langle 0|A_k|0\rangle \neq 0$, then $A_j(x)$ has a zero-mass particle in its spectrum. It has more recently been observed that the assumed Lorentz invariance essential to the proof[2] may allow one the hope of avoiding such massless particles through the introduction of vector gauge fields and the consequent breakdown of manifest covariance.[3] This, of course, represents a departure from the assumptions of the theorem, and a limitation on its applicability which in no way reflects on the general validity of the proof.

In this note we shall show, within the framework of a simple soluble field theory, that it is possible consistently to break a symmetry (in the sense that $\sum_k t_{ijk}\langle 0|A_k|0\rangle \neq 0$) without requiring that $A(x)$ excite a zero-mass particle. While this result might suggest a general procedure for the elimination of unwanted massless bosons, it will be seen that this has been accomplished by giving up the global conservation law usually implied by invariance under a local gauge group. The consequent time dependence of the generators Q_i destroys the usual global operator rules of quantum field theory (while leaving the local algebra unchanged), in such a way as to preclude the possibility of applying the Goldstone theorem. It is clear that such a modification of the basic operator relations is a far more drastic step than that taken in the usual broken-symmetry theories in which a degenerate vacuum is the sole symmetry-breaking agent, and the operator algebra possesses the full symmetry. However, since superconductivity appears to display a similar behavior, the possibility of breaking such global conservation laws must not be lightly discarded.

Normally, the time independence of

$$Q_i = \int d^3x\, j_i^{\,0}(\vec{x}, t)$$

is asserted to be a consequence of the local conservation law $\partial_\mu j^\mu = 0$. However, the relation

$$\partial_\mu \langle 0 | [j_i^{\,\mu}(x), A_j(x')] | 0 \rangle = 0$$

implies that

$$\int d^3x \langle 0 | [j_i^{\,0}(x), A_j(x')] | 0 \rangle = \text{const}$$

only if the contributions from spatial infinity vanish. This, of course, is always the case in a fully causal theory whose commutators vanish outside the light cone. If, however, the theory is not manifestly covariant (e.g., radiation-gauge electrodynamics), causality is a requirement which must be imposed with caution. Since Q_i consequently may not be time independent, it will not necessarily generate local gauge transformations upon $A_j(x')$ for $x^0 \neq x'^0$ despite the existence of the differential conservation laws $\partial_\mu j^\mu = 0$.

The phenomenon described here has previously been observed by Zumino[4] in the radiation-gauge formulation of two-dimensional electrodynamics where the usual electric charge cannot be conserved. The same effect is not present in the Lorentz gauge where zero-mass excitations which preserve charge conservation are found to occur. (These correspond to gauge parts rather than physical particles.) We shall, however, allow the possibility of the breakdown of such global conservation laws, and seek solutions of our model consistent only with the differential conservation laws.

We consider, as our example, a theory which was partially solved by Englert and Brout,[5] and bears some resemblance to the classical theory of Higgs.[6] Our starting point is the ordinary electrodynamics of massless spin-zero particles, characterized by the Lagrangian

$$\mathcal{L} = -\tfrac{1}{2}F^{\mu\nu}(\partial_\mu A_\nu - \partial_\nu A_\mu) + \tfrac{1}{4}F^{\mu\nu}F_{\mu\nu} + \varphi^\mu \partial_\mu \varphi + \tfrac{1}{2}\varphi^\mu \varphi_\mu + ie_0 \varphi^\mu q \varphi A_\mu,$$

where φ is a two-component Hermitian field, and q is the Pauli matrix σ_2. The broken-symmetry condition

$$ie_0 q \langle 0 | \varphi | 0 \rangle = \eta \equiv \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix}$$

will be imposed by approximating $ie_0 \varphi^\mu q \varphi A_\mu$ in the Lagrangian by $\varphi^\mu \eta A_\mu$. The resulting equations of motion,

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu,$$

$$\partial_\nu F^{\mu\nu} = \varphi^\mu \eta,$$

$$\varphi^\mu = -\partial^\mu \varphi - \eta A^\mu,$$

$$\partial_\mu \varphi^\mu = 0,$$

are essentially those of the Brout-Englert model, and can be solved in either the radiation[7] or Lorentz gauge. The Lorentz-gauge formulation, however, suffers from the fact that the usual canonical quantization is inconsistent with the field equations. (The quantization of A_μ leads to an indefinite metric for one component of φ.) Since we choose to view the theory as being imbedded as a linear approximation in the full theory of electrodynamics, these equations will have significance only in the radiation gauge.

With no loss of generality, we can take $\eta_2 = 0$, and find

$$(-\partial^2 + \eta_1^{\,2})\varphi_1 = 0,$$

$$-\partial^2 \varphi_2 = 0,$$

$$(-\partial^2 + \eta_1^{\,2})A_k^{\,T} = 0,$$

where the superscript T denotes the transverse part. The two degrees of freedom of $A_k^{\,T}$ combine with φ_1 to form the three components of a massive vector field. While one sees by inspection that there is a massless particle in the theory, it is easily seen that it is completely decoupled from the other (massive) excitations, and has nothing to do with the Goldstone theorem.

It is now straightforward to demonstrate the failure of the conservation law of electric charge. If there exists a conserved charge Q, then the relation expressing Q as the generator of rotations in charge space is

$$[Q, \varphi(x)] = e_0 q \varphi(x).$$

Our broken symmetry requirement is then

$$\langle 0 | [Q, \varphi_1(x)] | 0 \rangle = -i\eta$$

or, in terms of the soluble model considered here,

$$\int d^3x' \, \eta_1 \langle 0 | [\varphi_1^0(x'), \varphi_1(x)] | 0 \rangle = -i\eta_1.$$

From the result

$$\langle 0 | \varphi_1^0(x') \varphi_1(x) | 0 \rangle = \partial_0 \Delta^{(+)}(x'-x; \eta_1^2),$$

one is led to the consistency condition

$$\eta_1 \exp[-i\eta_1(x_0'-x_0)] = \eta_1,$$

which is clearly incompatible with a nontrivial η_1. Thus we have a direct demonstration of the failure of Q to perform its usual function as a conserved generator of rotations in charge space. It is well to mention here that this result not only does not contradict, but is actually required by, the field equations, which imply

$$(\partial_0^2 + \eta_1^2) Q = 0.$$

It is also remarkable that if A_μ is given any bare mass, the entire theory becomes manifestly covariant, and Q is consequently conserved. Goldstone's theorem can therefore assert the existence of a massless particle. One indeed finds that in that case φ_1 has only zero-mass excitations.

In summary then, we have established that it may be possible consistently to break a symmetry by requiring that the vacuum expectation value of a field operator be nonvanishing without generating zero-mass particles. If the theory lacks manifest covariance it may happen that what should be the generators of the theory fail to be time-independent, despite the existence of a local conservation law. Thus the absence of massless bosons is a consequence of the inapplicability of Goldstone's theorem rather than a contradiction of it. Preliminary investigations indicate that superconductivity displays an analogous behavior.

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