CHARGE SYMMETRY, CHARGE INDEPENDENCE, AND THE NUCLEON-NUCLEON SCATTERING LENGTHS*

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(I) Charge symmetry. – Several years ago it was suggested[1] that measurement of the neutron-neutron scattering length a_{nn} would provide a very sensitive test of charge symmetry in a strong interaction. An approximate relation[2] between the p-p and n-n scattering lengths,

$$\frac{1}{a_{nn}}=\frac{1}{a_{pp}}+\frac{1}{R}\left(\ln\frac{R}{r_0}-0.33\right),$$

predicts $a_{nn}=-17$ F. Jackson and Blatt's[3] simple two-parameter potentials, obtained by fitting low-energy p-p data, lead to $a_{nn}=-16.5$ to -19 F by simply turning off the Coulomb potential; Wong and Noyes[1] later predicted -27 ± 1.4 F from a two-parameter dispersion equation model. Recent measurements of the reaction $\pi^-+d\to n+n+\gamma$ have yielded neutron-neutron scattering lengths of $-17.0_{+4.3}^{-7.7}$ F due to Ryan[4] and -17 ± 4 F due to Haddock et al.[5,6] Due to the discrepancy in the above numbers, and the availability of improved p-p potentials, it seemed of interest to further examine the predicted values. From a detailed comparison of potential model predictions, we find the value of a_{nn} on the basis of charge symmetry to be almost certainly in the interval -16.5 to -19 F, and probably to be in the range -16.6 to -16.9 F. Thus the recently reported measurements can be regarded as confirming charge symmetry.

Our compilation of potential model predictions for a_{nn} is shown in Table I. The method of calculation in all cases is to determine the parameters of the potential by fitting p-p data, then turn off the Coulomb potential and compute a_{nn}. The first four values were computed from Blatt and Jackson's interpolation formulas,[7] using the Jackson-Blatt[3] potentials. The one-pole-amplitude Bargmann potential[8] was used both alone and added to the one-pion-exchange potential (OPEP). The Hamada-Johnston[9] and Yale[10] predictions were computed from the respective potential parameters. The YS potential is merely an updated Hamada-Johnston type po-

Table I. Neutron-neutron scattering length predictions from various potentials.

Potential	Number of pieces of pp data below 9 MeV used for model	10- to 320-MeV pp data used for model	a_{nn}
Square well	24		−16.5 F
Gaussian	24		−17.1
Exponential	24		−17.5
Yukawa	24		−19.0
Bargmann	2[a]		−17.9
Bargmann + OPEP	2[a]		−19.0
Hamada-Johnston (HJ)		Yes	−16.7
HJ′	2[a]	Yes	−16.8
Yale	4[b]	Yes[b]	−16.8
YS		Yes	−16.6
YS′	2[a]	Yes	−16.9

[a]Exact fit to 1.397- and 2.425-MeV $^1S_0$ pp phases.
[b]Phases, not data.

tential. In the primed potentials, two of the parameters were readjusted so as to produce an exact fit to the two pp phases used by Wong and Noyes; this did not substantially alter the fit at higher energies.

The fact that the Bargmann and Bargmann-plus-OPEP potentials give a_{nn} values within the range quoted above is interesting because the Wong-Noyes one-pole and one-pole-plus-OPEP models should yield results almost identical to those given by these two potentials. Specifically, we find $a_{nn}=-18$ F and -19 F for the two potentials, respectively, whereas Wong and Noyes found -29 F and -27 F for their corresponding models. It appears, then, that the Wong-Noyes estimate of electrostatic effects is incorrect.[11] Our best estimates of a_{nn} are those of the last five potentials in Table I, since they are the more realistic models.

(II) Charge independence.—How is the difference between $a_{nn}=-16.7$ F and $a_{np}=-23.7$ F[12] to be explained? One contribution arises from the different signs, ranges, and strengths of the potentials associated with the exchange of charged and neutral pions. Part of this difference can be computed unambiguously, namely that due to OPEP.[13,14] Making this modification in the OPEP part of the YS potential, we find $a_{np}=-18.8$ F for $g_0^2=g_\pm^2$ and -13.4 F for $f_0^2=f_\pm^2$. Thus equality of pseudoscalar coupling constants produces an $a_{nn}-a_{np}$ splitting of the correct sign, while equality of the pseudovector coupling constants does not, but even in the former case the splitting is only $\sim\frac{1}{3}$ of that needed.

We have considered three ways to try to produce the experimental value of a_{np}, including the appropriate OPEP in each case. (i) Proceeding as in a previously reported calculation,[15] we can accomplish this by setting the effective pion mass which occurs in the TMO[16] two-pion-exchange ps-pv potential 1.3 MeV larger for nn than for np. This demonstrates that the scattering length is very sensitive to the effective two-pion mass; perhaps the correct n-p scattering length would emerge if the proper masses were used in all the two-pion-exchange diagrams. (ii) Choosing $g_0^2=g_\pm^2$ in OPEP, and using a static potential[17] due to the exchange of a single ρ meson, a mass splitting $m(\rho_0)-m(\rho_\pm)=2$ MeV will produce the desired effect. (iii) Another way to account for the remaining $a_{nn}-a_{np}$ difference is to put $g_\pm^2$ larger than g_0^2 by 3.5% in OPEP.

So far no account has been taken of the electromagnetic (other than point Coulomb) and vacuum polarization potentials, both of which tend to close the gap in the scattering lengths because they are repulsive for p-p (and n-n) and attractive for n-p. On the basis of Riazuddin's calculations,[18] which consider the electromagnetic structure of the nucleons with a hard core in the nuclear potential, it appears that $\simeq 0.8$ F of the $a_{nn}-a_{np}$ difference will be removed by electromagnetic effects. We estimate[19] that proper inclusion of vacuum polarization may remove an additional $\simeq 0.2$ F; the two effects together reduce the burden upon the charge-dependent part of the effective nuclear potential.

We wish to point out that it will be difficult to find a theoretical understanding of a_{np} because of its extreme sensitivity to quantum mass splittings. However, the matter of charge symmetry is much simpler, and now appears to be confirmed by experiment.

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SU(4) MASS FORMULA*

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There have been a number of speculations on the possibility of a symmetry scheme of fundamental particles based on the group SU(4).[1] These ideas extend the SU(3) symmetry scheme[2] in much the same way that the SU(3) scheme extends the isospin symmetry. It must be stressed, however, that unlike the search for SU(3) there is very little if any experimental motivation for this extension. In view of the success of the empirical SU(3) mass formula,[3] which says that the mass splittings transform like the $Y=0$ isospin-singlet component of the regular representation of SU(3), i.e., the 8, it is a natural first guess to presume that the mass splittings in the SU(4) scheme transform like a component of the regular representation of SU(4), i.e., the 15. In the following, we give the mass formula for an arbitrary representation of SU(4) based on the assumption that the mass splittings transform like a component of the 15. This general result is greatly simplified by using different SU(3) subgroups of SU(4). To emphasize the general usefulness of the various subgroups, the electromagnetic properties in SU(4) are briefly discussed also.

Since SU(4) is a group of rank 3, there are three independent conserved quantum numbers which we take to be the charge Q, the hypercharge Y,[4] and a new quantum number X. (Q is more convenient than I_3 for discussing electromagnetic interactions.) The 15 contains the following SU(3) submultiplets: a 3 with $X=1$, a 1 and an 8 with $X=0$, and a 3* with $X=-1$. The most general form of the mass splittings contained in the 15 compatible with Q, Y, X, and I conservation is a superposition of the SU(3) singlet and the isoscalar component of the SU(3) octet ($Q=Y=X=0$). [The SU(3) subgroup we refer to here is the one whose multiplets lie in planes orthogonal to the direction of the new quantum number X.]

Consider first the splittings which transform like the $Q=Y=X=0$ SU(3) singlet. These do not give splittings within a given SU(3) multiplet but do split apart different SU(3) multiplets. Since the $Q=Y=X=0$ SU(3) singlet is the component $\psi_4{}^4$ of the traceless tensor $\psi_\mu{}^\nu$ representing the 15, the general form of these mass splittings for any SU(4) representation[5] (L,M,N) is obtained by applying the Wigner-Eckart theorem for SU(4) to the matrix element of the component $T_4{}^4$ of a traceless tensor $T_\mu{}^\nu$. The general form of the matrix element of the component $T_4{}^4$ of an arbitrary traceless tensor $T_\mu{}^\nu$ in the SU(4) representation $(L,M,N,)$ turns out to be

$$T_4{}^4 = a + bX + c[C_2{}^{(x)} - X^2] + d[C_3{}^{(x)} - 3XC_2{}^{(x)} + X^3], \quad (1)$$