CALCULATION OF NUCLEON-DEUTERON SCATTERING AND THE TRITON BINDING ENERGY*

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We present a calculation of the binding energy, the low-energy scattering parameters, and angular distributions, for the three-nucleon system. This calculation extends the exact three-body theory introduced previously[1,2] to include spin-dependent two-body forces. These are forces separable in momentum space, but made to fit the low-energy two-nucleon data (deuteron binding energy, triplet and singlet scattering lengths, and singlet effective range). The theory then computes the three-nucleon system exactly (on a high-speed computer) using these forces. Thus we take three-body effects completely into account including inelastic processes and the complicated couplings between channels. The results clearly remove the famous scattering-length ambiguity for the three-nucleon system and also give a good value for the three-body binding energy, particularly if the effect of the tensor force and hard cores—so far not included in our calculations—is simulated by weakening the triplet force without changing the deuteron binding energy. The calculated angular distribution for nucleon-deuteron scattering is also in close agreement with experiment.

Although recently reduced to Fredholm form,[3] the general integral equations for the quantum mechanical three-body problem are still beyond the capabilities of even the largest computer available. This is not surprising since the central difficulty of the problem, the multiplicity of coordinates, is still present in these formulations. However, a few restricted three-body systems are now, for the first time, amenable to calculation. In all of these the essential "trick" is to reduce the number of coordinates needed to specify intermediate states in the integral equations to one. This reduction is exact if, for example, one introduces a potential between pairs separable in momentum space in the full equations; or if one introduces a particle or quasiparticle through which pairs interact. In fact, these two methods are exactly equivalent in the limit of zero wave-function renormalization for the particle introduced.[4] If one wishes to do better, one can use two two-body separable potentials or, equivalently, two quasiparticles. This gives a set of two coupled integral equations for the three-body system, each equation still having single-variable intermediate states. Since any reasonable local potential can be represented more and more accurately by more and more separable potentials, and hence more and more coupled equations, we see that our method is one that replaces a single many-variable integral equation by an infinite set of coupled one-variable equations. The obvious approximation scheme is then to truncate this set. In doing so one keeps certain parts of the two-particle force and treats their effect in the three-particle system exactly. For low-energy and bound-state systems it seems clearly more important to treat these three-particle effects than to sacrifice them for a better treatment of the high-energy behavior of the two-particle interaction.

As a first step toward implementing this scheme, we study the three-nucleon system. We generalize the procedure used previously to include spin and statistics and to take into account the singlet and triplet nucleon-nucleon interaction.[5] We get a set of coupled integral equations for the scattering of a nucleon on a triplet (T) or singlet (S) pair of the other two, which equations are represented diagrammatically in Fig. 1. Since we have no tensor forces or other coupling involving spin and orbit, it is convenient to study the equations in the L-S representation. The total S can of course be $\frac{1}{2}$ or $\frac{3}{2}$. In the quartet state only the triplet two-body state enters and the Born term is repulsive in states of even L. In the doublet spin state both the triplet and singlet two-body states enter and the Born terms are attractive for even L, as we expect. As before, we take a Hulthén form for the two-nucleon vertex in momentum space.[2] The parameters at this vertex are fitted to a

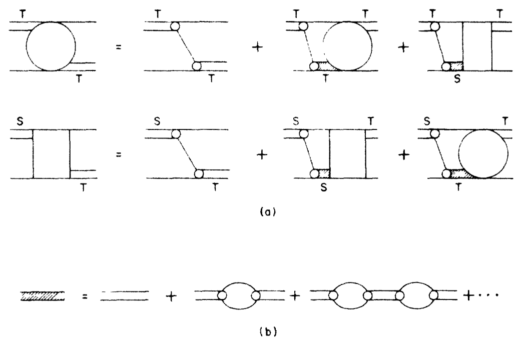

FIG. 1. The single line represents a nucleon, the double lines represent a two-nucleon pair—S for the singlet, and T for the triplet. The small circle is the nucleon-nucleon vertex; the large circle and rectangle are the three-body amplitudes. (a) Coupled integral equations for neutron-deuteron scattering. (b) The renormalized "propagator" for either the singlet or triplet pair.

scattering length $a_S = -23.7$ fermi and an effective range of $r_S = 2.7$ fermi for the singlet, and to a scattering length of 5.38 fermi and to the deuteron binding energy for the triplet.[6] This ensures that our two-body system is correct at low energies. This choice still leaves open the value of the deuteron wave-function renormalization, Z, which we shall vary away from zero—the value which corresponds to a separable potential and to a pure bound-state deuteron. With the parameters chosen, we can solve the coupled equations; the method of solution and analysis on the computer is similar to that used in B.

We present in Table I our results (AAY) for the the binding energy of the triton (BET), the quartet scattering length ($a_{3/2}$), and the doublet scattering length ($a_{1/2}$). Other calculations using essentially the $Z = 0$ limit of our model have been been performed by Mitra and Bhasin,[7] and Sitenko and Kharchenko.[8] Their results are shown also (MB and SK). Mitra and Bhasin's results suffer from an incorrect analytic approximation and the latter is presumably a cruder machine calculation than ours.

It is gratifying that we get too much binding with the potential model ($Z = 0$) since the major effects we have neglected, e.g., tensor force, hard cores, will reduce the binding energy. Some of these effects can be simulated by making Z slightly positive. This weakens the nucleon-nucleon interaction in the triplet state without changing the deuteron binding energy. This clearly cannot be done in a pure potential theory. Such weakening would be the effect of a hard core in the triplet state which would be more effective in the more compact triton than in the spread-out deuteron. It also represents the effect of the tensor force which is less attractive in the more symmetric triton than in the deuteron. The value $Z = 0.0488$ is chosen to give the correct doublet scattering length. The validity of the arguments presented above is shown by the fact that we also get a good fit to the triton binding energy with that single choice of adjustable parameters.[9] Furthermore, the size of Z is reasonable. Z measures in our theory the probability that the deuteron is not a triplet nucleon-nucleon pair interacting through the attractive central force. 5% is certainly a reasonable number for this. We are presently setting up a calculation which will include both the tensor force and the effect of hard cores.[10] From the results presented in the table we believe that one can say with confidence that the ambiguity previously existing in the two sets of experimental scattering lengths for the nucleon-deuteron system has been removed. Set I is the correct set.

That the theory does give a good account of the low-energy scattering is further shown in Fig. 2 where the angular distribution for 2.7-MeV neutron-deuteron scattering is compared with experiment.[11] (Here $Z = 0$ and $Z = 0.0488$ are very similar.) What data there are agree

Table I. Summary of theoretical and experimental results for the neutron-deuteron system. The values of the binding energy of the triton (BET) are given in MeV. The scattering lengths ($a_{3/2}$ and $a_{1/2}$) are in fermis.

			AAY		Experiment[a]	
	MB	SK	$Z = 0.0$	$Z = 0.0488$	Set I	Set II
$a_{3/2}$	5.91	6.28	6.28	6.14	6.38 ±0.06	2.6 ±0.2
$a_{1/2}$	9.25	−2.76	−1.01	0.7	0.7 ±0.3	8.26 ±0.12
BET		12.5	11.0	8.76	8.49	

[a]D. Hurst and N. Alcock, Can. J. Phys. 29, 36 (1951).

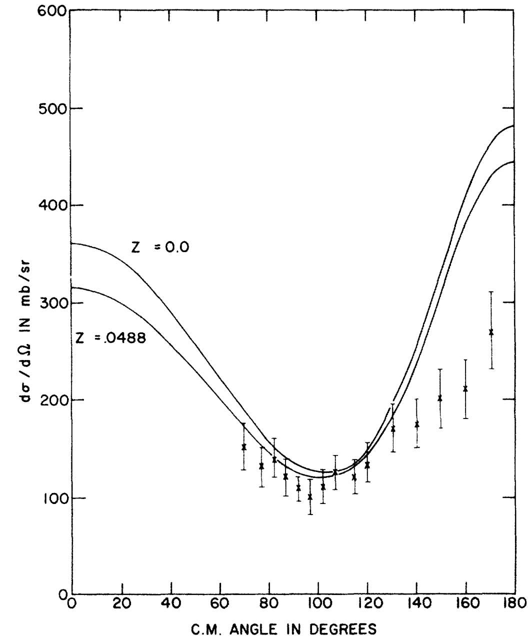

FIG. 2. Angular distributions for neutron-deuteron scattering. The theoretical curves are for 2.7 MeV. The experimental points are for 2.5 MeV.

quite well and indicate that nucleon exchange is the dominant mechanism for the interaction of a neutron with a loosely bound structure like the deuteron. The fits are not quite so good at lower energies. We also have spin-flip and spin-nonflip cross sections, but there seems to be little data. Results on the scattering above break-up threshold and on the break-up amplitude itself will be presented shortly, as will our calculation including tensor forces. We hope that this will stimulate more experimental work.

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[1]R. D. Amado, Phys. Rev. 132, 485 (1963). Hereafter referred to as A.

[2]R. Aaron, R. D. Amado, and Y. Y. Yam, to be published. Hereafter referred to as B.

[3]L. D. Faddeev, Zh. Eksperim. i Teor. Fiz. 39, 1459 (1960) [translation: Soviet Phys.–JETP 12, 1014 (1961)]; S. Weinberg, Phys. Rev. 133, B232 (1964); C. A. Lovelace, in Strong Interactions and High Enengy Physics, edited by R. G. Moorhouse (Plenum Press, New York, 1964).

[4]M. T. Vaughn, R. Aaron, and R. D. Amado, Phys. Rev. 124, 1258 (1961); S. Weinberg, Phys. Rev. 131, 440 (1963); L. Rosenberg, Phys. Rev. 135, B715 (1964).

[5]The triplet interaction may be taken to proceed via the deuteron state. The singlet interaction may be considered to proceed via a fictitious particle with a vanishing wave-function renormalization and positive bare rest-energy or via a separable potential.

[6]M. J. Moravcsik, The Two-Nucleon Interaction (Oxford University Press, New York, 1963).

[7]A. N. Mitra and V. S. Bhasin, Phys. Rev. 131, 1265 (1963).

[8]A. G. Sitenko and V. F. Kharchenko, Nucl. Phys. 49, 15 (1963).

[9]The triton binding energy is quite sensitive to the other input parameters as well as to Z, except for a_s, which is so very large that small changes in it do not really affect the force.

[10]For a crude estimate of the hard-core effect, see F. Tabakin, to be published.

[11]R. K. Adair, A. Okazaki, and M. Walt, Phys. Rev. 89, 1165 (1953). We have put in an experimental error of about 15% as suggested in the reference.