EXPERIMENTAL OBSERVATION OF ELECTRIC-QUADRUPOLE HYPERFINE EFFECTS IN MUONIC X-RAY SPECTRA*

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Theorists[1-3] pointed out a decade ago that muonic x-ray spectra should exhibit certain remarkable effects due to the hyperfine coupling between the muon charge cloud and the nuclear quadrupole moment. In addition to their intrinsic interest, these effects can provide a novel and, in some sense, unique tool to study nuclear properties. Experimental progress in the field has, however, been rather slow; quantitative measurements even of the fine-structure splitting are comparatively recent.[4] Magnetic focusing channels that increase the muon flux by an order of magnitude have given a great impetus to muonic spectroscopy. Using such a muon channel[5] and a particularly suitable NaI spectrometer,[6] we now have studied the muonic L and K x rays from a number of heavy elements and have obtained quantitative evidence for the predicted hyperfine effects. We report here the salient experimental results and a preliminary comparison with theory. Similar work (on K-muonic x rays of U^{238} and Th^{232}) has been performed independently at CERN[7] and our results and analysis for the K x-ray spectra are in reasonably good agreement with theirs.

Before discussing our results, we shall summarize the theoretical background. The ratio of the hyperfine splitting ΔE_{hfs} to the fine-structure splitting ΔE_{fs} in a muonic atom is expected to be much larger than the corresponding ratio for an ordinary atom, particularly if the nucleus possesses an electric quadrupole ($E2$) moment eQ. In such a case one has

$$\frac{\Delta E_{\text{hfs}}}{\Delta E_{\text{fs}}} \approx \frac{e^2Q}{Z\mu^2} = \left(\frac{m_\mu}{m_e}\right)^2 \frac{e^2Q}{Z\mu_0^{\,2}}, \qquad (1)$$

where m_μ and m_e are the masses of the muon and electron, Z the atomic number of the nucleus, and μ_0 the magnetic moment of the electron.

This fact was stressed by Wheeler[1] in his detailed consideration of nuclei having an observable eQ, i.e., a spin $I \geq 1$. Shortly thereafter, Jacobsohn[2] and, independently, Wilets[3] noted that the hfs of muonic atoms is even qualitatively different from the usual atomic case. These authors noted that the $E2$ hf interaction between muon and nucleus has off-diagonal matrix elements (say, $E_{\text{hfs}}' \approx \Delta E_{\text{hfs}}$) connecting the ground state I and an excited state I', and that these can become comparable in magnitude to the nuclear excitation energy $E_{\text{nucl}} \equiv E(I') - E(I)$. In that situation the electric quadrupole interaction will mix the two states I and I' (as well as fine-structure components j, j'). The muon no longer "sees" the nucleus in its ground state I, and can hence interact with the electric quadrupole moment $eQ(I')$ of the excited state. Thus one can have electric quadrupole splitting even for $I = 0$ nuclei! The necessary condition $E_{\text{hfs}}'/E_{\text{nucl}} \approx 1$ is particularly well met by the nuclei with strong deformations because these have both large $E2$ (transition) moments and low-lying first excited (rotational) states, $E_{\text{nucl}} = E_{\text{rot}}$. In fact, in the extreme Bohr-Mottelson[8] model, such nuclei are attributed an intrinsic $E2$ moment eQ_0 present even in an $I = 0$ ground state. With this model, Q_0 is the sole nuclear parameter governing the hfs effects in question. Conversely, observations on muonic x-ray spectra can be used, as emphasized by Jacobsohn[2] and Wilets,[3] to check this model and, in particular, to determine the sign of Q_0 for $I = 0$ nuclei.

The Jacobsohn-Wilets formalism needs to be extended in order to explain the observed muonic spectra. Previous theory considered only the effect of the interaction on the muonic $2p$ levels, whereas we found that $3d$-level mixing must also be considered in order to obtain the observed intensity distribution. Similar conclusions have been reached by the group working at CERN.[7]

In the present paper, experimental data taken during a recent run at the University of Chicago synchrocyclotron are compared with the relevant muonic spectra predicted by use of the model described above. The experimental details will appear in another paper.[9] In the notation of Wi-

Table I. Parameters used to calculate the muonic x-ray spectra from U^{238} and Th^{232}.

Parameter	U^{238}	Th^{232}
$E(1s_{1/2})$	-6192 keV[a]	-6029 keV[b]
$E(2p_{1/2})$	0 keV[c]	0 keV[c]
$E(2p_{3/2})$	228 keV[a]	207 keV[d]
$E(3d_{3/2})$	3300 keV[b]	3078 keV[b]
$E(3d_{5/2})$	3365 keV[b]	3139 keV[d]
E_{rot}	44.7 keV[e]	50.0 keV[e]
ϵ_{dd}	1.61 keV[f]	1.3 keV[f]
ϵ_{pp}	9.75 keV[f]	8.37 keV[f]
Q_0	10.5 b[g]	9.25 b[g]

[a] G. E. Pustovalov, Zh. Eksperim. i Teor. Fiz. 36, 1806 (1959) [translation: Soviet Phys.–JETP 9, 1288 (1959)].

[b] Empirically determined.

[c] Arbitrarily defined as 0. Other atomic energy values are given relative to this level.

[d] R. McKee, private communications. This value was calculated by use of the Dirac equation for a constant charge distribution, and $R_0 = 1.25$ F.

[e] F. S. Stephens, R. M. Diamond, and I. Perlman, Phys. Rev. Letters 3, 435 (1959).

[f] Calculated by use of the Wheeler formalism (reference 1).

[g] R. E. Bell, S. Bjørnholm, and J. C. Severiens, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Medd. 32, No. 12 (1960).

lets[3] the matrix elements for the quadrupole interaction are given by

$$\langle IjK;FM|H_q'|I'j'K;F'M'\rangle = -\tfrac{1}{2}e^2Q_0\langle j\|f(r)\|j'\rangle$$
$$\times(-1)^{I'-I+j+F+K}(2I+1)^{1/2}(2I'+1)^{1/2}$$
$$\times\begin{Bmatrix} F & j & I \\ 2 & I' & j' \end{Bmatrix}\begin{pmatrix} I & 2 & I' \\ -K & 0 & K \end{pmatrix}\langle j\|C_\mu^{(2)}\|j'\rangle\delta_{FF'}\delta_{MM'}. \qquad (2)$$

The curly brackets indicate a 6-j symbol and the large parentheses denote a 3-j symbol as described by Edmonds.[10]

In calculating the hf levels and muonic x-ray intensities, we use the parameters listed in Table I. The energy of the unperturbed $2p_{1/2}$ level was arbitrarily taken as zero. The parameters ϵ_{dd} and ϵ_{pp} represent the quantity $(e^2/10)Q_0\langle j\|f(r)\|j'\rangle$ evaluated for d and p states, respectively. The penetrability factor $f(r)$ depends on the distribution of charge in the nucleus.

The following approximations were employed: (a) The hyperfine interaction was neglected in levels for which $n > 3$. (b) $3s$-$3d$ mixing was ignored because of the low statistical weight of the $3s$ state. (c) Contributions to the hfs from the $I=4$ second excited state were ignored as small corrections which would not be discerned within our experimental resolution. (d) The quantities ϵ_{dd} and ϵ_{pp} were calculated by use of the Wheeler approximation of nonrelativistic hydrogenic wave functions and constant charge distribution.
(e) The atomic $E1$ transition probabilities include the standard energy dependence, but the radial dependence of the matrix elements was neglected.

Figure 1 shows the observed muonic L x-ray spectra for two so-called stiff nuclei (having no low-lying levels): natural lead (52% Pb^{208}) and Bi^{209}. The "classical" doublet without quadrupole interaction is observed. The intensity ratio of

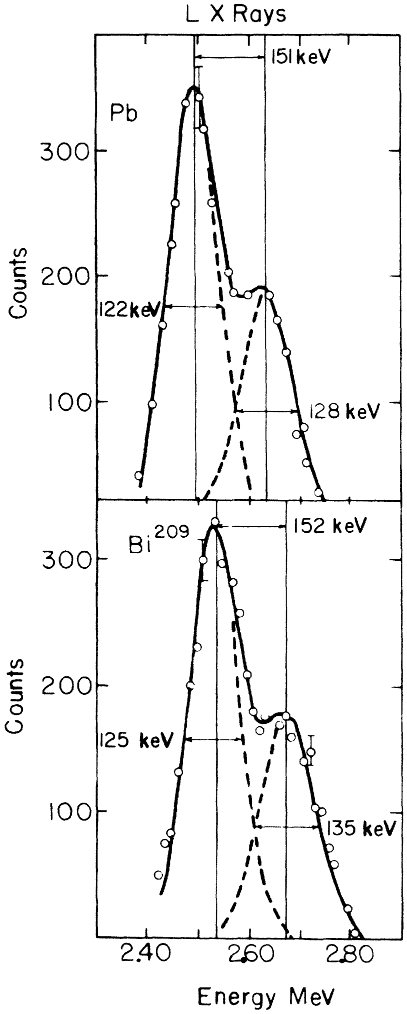

FIG. 1. Experimental muonic L x-ray spectra for natural lead (52% Pb^{208}) and Bi^{209} with the observed fine-structure splitting and resolution width (full width at half-maximum) indicated. The observed spectra are decomposed to show the two components.

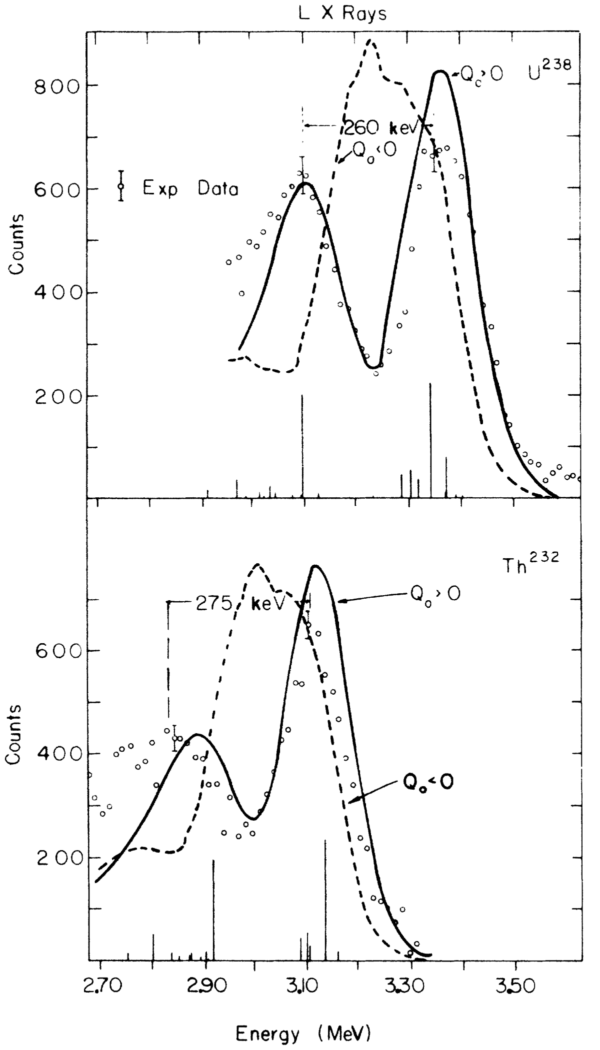

FIG. 2. Observed and theoretically predicted muonic L x-ray spectra for U^{238} and Th^{232}. The open circles are experimental data (corrected for capture gammas, accidentals, etc.). The vertical bars at the bottom of the graph show the theoretical spectra for $Q_0>0$. The solid curve, to be compared with experiment, was obtained by folding the spectrometer resolution into that spectrum. The broken curve is the analogous prediction for $Q_0<0$. The observed "apparent" fine-structure splittings are indicated.

the two components is 2:1, aside from small corrections. This is to be contrasted with Fig. 2, which shows (open circles) the muonic L x-ray spectra observed from the deformable even-even nuclei U^{238} and Th^{232}. These show an apparent "doublet," but the intensity ratios of the two "components" and their separations are drastically different from those shown in Fig. 1 which are due to the fine structure mentioned above.

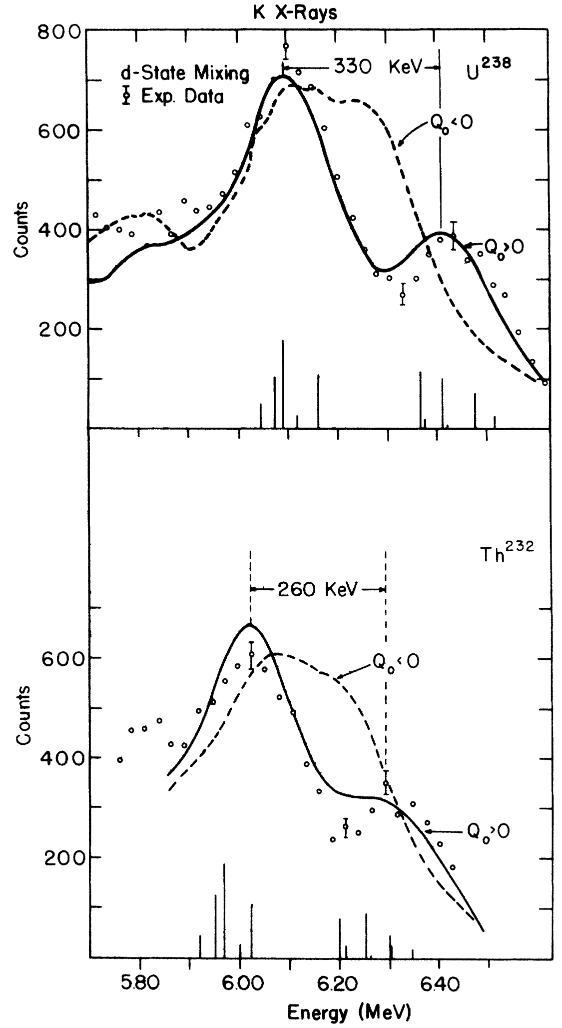

FIG. 3. Experimental and theoretical muonic K x-ray spectra for U^{238} and Th^{232}. The vertical bars at the bottoms of the graphs show the theoretical spectrum for $Q_0>0$. The solid line is the theoretical curve for $Q_0>0$ with the spectrometer resolution folded in; the broken line is that for $Q_0<0$. The open circles are experimental data. Observed fine-structure splittings are indicated. All notations are analogous to those in Fig. 2.

On the other hand, good agreement is obtained with predictions (solid curve) based on the dynamic electric quadrupole interaction with the parameters of Table I and $Q_0>0$. The observed spectra are not compatible with the prediction using $Q_0<0$ (broken line).

In Fig. 3 we present the muonic K x-ray spectra of U^{238} and Th^{232}. The results are in good agreement with the theoretical predictions based on the same parameters and $Q_0>0$; $Q_0<0$ is excluded.

The spectrometer response for the muonic L and K x rays is determined experimentally by study of the 2.75-MeV line from Na^{24} and the 6.14-MeV line following the β decay of N^{16}.

We have comparable experimental data for Ta^{181}, U^{235}, and Pu^{239}, but are still working on the theory for odd-A nuclei. It is worth noting that the observed spectra from the last two of these targets are very similar in appearance to those presented here.

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POSSIBLE EXISTENCE OF A p-He^3 RESONANCE WITH $Q = 11$ MeV PRODUCED IN $_\Lambda He^4$ DECAY*

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We are currently surveying the nuclear emulsion data on π^--mesic decays of hyperfragments in order to obtain statistically improved observations of the final-state interactions present in these events.[1,2] Previously other workers have looked for states of Li^4, primarily in reactions such as p-He^3 scattering[3-7] or $He^3(d,n)$.[8] A careful analysis of p-He^3 scattering data by Tombrello[9] shows only the possibility of some very broad states of Li^4, for proton energies of ≤ 11.5 MeV. The hyperfragment data for the decay mode (decay at rest)

$$_\Lambda He^4 \rightarrow \pi^- + p + He^3 \qquad (1)$$

indicate the possible existence of a very sharply defined state of Li^4 corresponding to a "two-body configuration" in ~20% of the events, namely:

$$_\Lambda He^4 \rightarrow \pi^- + Li^{4*} \rightarrow p + He^3 + (10.62 \pm 0.20)\text{ MeV} \qquad (2)$$

[using $Q_\Lambda = (37.58 \pm 0.15)$ MeV and $B_\Lambda(_\Lambda He^4) = 2.33 \pm 0.10)$ MeV[10]].

This phenomenon was first noticed in Fig. 1, where a scattergram of proton energy versus pion energy is given for all identified cases of Reaction (1). The solid curve in Fig. 1 is the boundary of the allowed region calculated using a binding energy $B_\Lambda(_\Lambda He^4)$ of 2.33 MeV.[10] The extraordinary feature is the spike in the pion energy which has its center at (23.68 ± 0.08) MeV

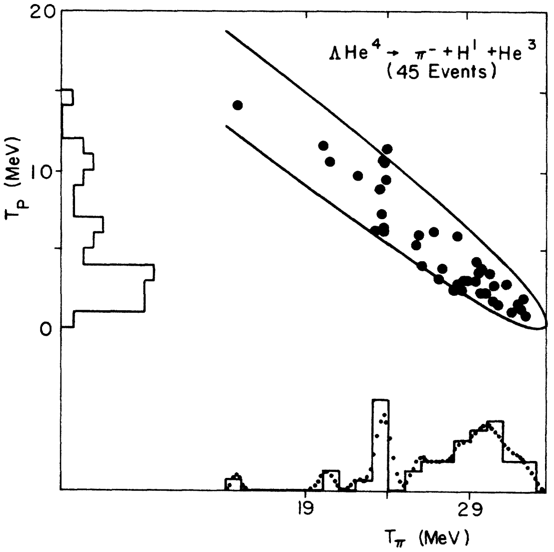

FIG. 1. A scattergram of proton energy versus pion energy for the decay $_\Lambda He^4 \rightarrow \pi^- + H^1 + He^3$. Solid circles: A Gaussian ideogram which provides an indication of the significance of the pion energy peak between 23 and 24 MeV. Solid line: The kinetic boundary based on $B_\Lambda(_\Lambda He^4) = 2.33 \pm 0.10$ MeV.