ENERGIES OF ISOBARIC MULTIPLETS IN $A = 16$ and 20*

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If the specifically nuclear part of nuclear interactions is charge independent then the energies of the various members of an isobaric multiplet will differ only because of the neutron-proton mass difference and the electromagnetic interactions among the protons. These electromagnetic contributions are calculable[1] and could be removed; the remaining masses of the multiplets thus could be examined with regard to charge independence in nuclear forces. These calculations, however, are somewhat model dependent, but to first order in the Coulomb energy it has been shown,[2] quite generally, that the masses within an isobaric multiplet are characterized by

$$M(A,T,T_z) = a(A,T) + b(A,T)T_z + c(A,T)T_z^2, \quad (1)$$

where M is the mass of a member of the multiplet, A is the number of nucleons, T is the isobaric spin, and $T_z = \frac{1}{2}(N-Z)$; $a(A,T)$, $b(A,T)$, and $c(A,T)$ are taken as constants within a given multiplet. The adequacy of this formula has never been tested empirically because at most three members of a multiplet are known so that no verifiable predictions can be made using Eq. (1).

Recent (p,t) experiments,[3] however, have been able to find a $T = 2$ level in certain $T_z = 0$ nuclei. The $T = 2$ level that is located is the isobaric analog of the ground state of the $T = 2$, $T_z = 2$ isobar. For example, a $T = 2$ level is found in Mg^{24} that is the analog to the ground state of Ne^{24}. In the mass-24 system the three lowest lying $T = 1$ levels with $T_z = 0, \pm 1$ are also known. In a recent paper[4] Wilkinson suggested that one assume the coefficients $b(A,T)$ and $c(A,T)$ within the same A be taken to be T independent. With this assumption, using the levels mentioned above he was able to show that the resulting prediction for the mass of the ground state of Al^{24} is in agreement with the observed mass,[5] though the experimental uncertainties are large.

We have just completed a study[6] of (p,t) and (p,He^3) reactions on O^{18} and Ne^{22}. These reactions on O^{18} allowed us to locate the analog to the ground state of C^{16} in N^{16} (9.91 ± 0.1 MeV) and in O^{16} (22.9 ± 0.1 MeV). Similarly, with the Ne^{22} target the analogs to the O^{20} ground state were located in F^{20} (6.43 ± 0.1 MeV) and in Ne^{20} (16.8 ± 0.1 MeV). Thus in each case we have three members of a $T = 2$ isobaric multiplet and in each case a set of $T = 1$ isobars is also known.[7,9] Therefore, the coefficients $b(A,1)$, $c(A,1)$, $b(A,2)$, and $c(A,2)$ can be determined from the data and compared for $A = 16$ and 20. Table I shows the values obtained. Two values are given for the $T = 1$, $A = 16$ multiplet because the spin of the ground state of F^{16} is not known[9] with certainty. The ground state of N^{16} is 2^-, but in O^{16} the lowest lying $T = 1$ state is 0^-. This inversion is probably due to the Thomas-Ehrman effect which would be most pronounced for the s-state proton. Thus it would seem that the ground state of F^{16} is also 0^-, but values for the coefficients assuming it to be 2^- are also included. From the values given in Table I it would seem that the assumption suggested by Wilkinson is not generally valid. However, it is instructive to take a more detailed look at the factors which bring about this disagreement between the coefficients. In each case it seems to be the position of the $T = 1$, $T_z = -1$ member of the multiplet. Using the assumption of reference 4 [that $b(A,1) = b(A,2)$ and

Table I. Comparison of the coefficients of Eq. (1) for $A = 16$ and $A = 20$ in the $T = 1$ and $T = 2$ multiplets. The two values shown for the $A = 16$, $T = 1$ case use the levels with the spin-parity listed before the set of coefficients.

	$T = 2$ (MeV)	$T = 1$ (MeV)
$A = 16$	$b = -2.93 \pm 0.25$	(2^-) $b = -2.61 \pm 0.03$
	$c = 0.35 \pm 0.12$	$c = 0.05 \pm 0.025$
		(0^-) $b = 2.45 \pm 0.03$
		$c = 0.19 \pm 0.025$
$A = 20$	$b = -3.69 \pm 0.25$	$b = -4.15 \pm 0.15$
	$c = 0.35 \pm 0.12$	$c = 0.91 \pm 0.15$

Table II. Results obtained using Eq. (2). The 2^- states are used in the $A=16$ case as they are not shifted by the Thomas-Ehrman effect as severely as the 0^- states.

	$E(A,2,1)$ (MeV)	$E(A,2,0)-E(A,1,0)$ (MeV)
$A=16$	9.91 ± 0.1	9.93 ± 0.1
$A=20$	6.43 ± 0.1	6.53 ± 0.1

$c(A,1)=c(A,2)]$, a prediction can be made which does not involve the mass of the $T=1$, $T_z=-1$ member:

$$E(A,2,1)=E(A,2,0)-E(A,1,0), \qquad (2)$$

where E stands for the excitation energy above the respective ground states and the terms in the parentheses have the same meaning as before. The results of this prediction are shown in Table II and are seen to show much better agreement than the coefficients in Table I. Further, to show that the $T=1$, $T_z=-1$ levels are the ones mainly responsible for the disagreement between the two sets of coefficients, one can use $b(A,2)$ and $c(A,2)$ to predict the mass differences in the $T=1$ multiplets. The resulting values obtained for N^{16}–O^{16} and F^{20}–Ne^{20} are in excellent agreement with experiment, whereas the value obtained for F^{16}–$O^{16}=16.07\pm0.27$ MeV (experimentally[9] 15.43 ± 0.05 MeV is obtained) and for Na^{20}–$Ne^{20}=14.33\pm0.27$ (reference 5 gives 15.3 ± 0.3).[10] The lower value observed for the fluorine-oxygen mass difference is most certainly due to the Thomas-Ehrman shift as the F^{16} ground state is unbound to proton decay.

In mass 20 a different situation holds: Using the lowest $T=1$ level in Ne^{20} and adding to it the Coulomb-energy difference obtained from Na^{21}–Ne^{21} corrected for radius to get a value of the Na^{20} ground state, one obtains Na^{20}–$Ne^{20}=13.86$, which is in very substantial disagreement with both the prediction in the paragraph above and the value currently accepted for this mass difference.[5] Considering the magnitude of the disagreement it seems that the current value for the mass of Na^{20} is too high by at least 1 MeV. It should be noted that it has only been reported once,[11] and therefore certainly should be remeasured.

Using Eq. (1), which is presumably good within a given multiplet, in the absence of Thomas-Ehrman shifts, we predict Mg^{20}–$Ne^{20}=25.6\pm0.7$ MeV and Ne^{16}–$O^{16}=30.2\pm0.7$ MeV.

It appears from this work that if the assumptions made in reference 4 are applied to levels that are not shifted because they are unbound, the agreement found is within experimental error. Further, from the fact that the energy of the $T=2$ analog state in O^{16} is shifted downward in energy less severely than the F^{16} ground state, one has further evidence that it has a very small width for charged-particle emission.

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[10]We believe the best number at present to be Na^{20}–$Ne^{20}=15.0\pm0.3$, based on a recent remeasurement of the mass of N^{12} [W. Whaling and T. R. Fisher, Bull. Am. Phys. Soc. 8, 598 (1963)], on which the Na^{20} mass determination [Luis Alvarez, Phys. Rev. 80, 519 (1950)] was based.

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