POSSIBLE CHAIN OF RESONANCES

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Recently, Dreitlein has investigated the $(\Xi, \bar{K})$ bound-state model of the Ω^- particle, and has pointed out that other bound states of the type $(\Xi, n\bar{K})$ may exist.[1] In this note we generalize this idea to SU(3), using static-model dispersion relations to study the dynamics. The result is a possible chain of resonances involving very many particles.

The baryon multiplets in the proposed chain are denoted by B^0, B^1, etc.; the SU(3) and spin assignments of these multiplets are

$$B^0 \sim D^8(1,1)d^2(\tfrac{1}{2}),$$

$$B^1 \sim D^{10}(3,0)d^4(\tfrac{3}{2}),$$

$$n>1, \quad B^n \sim D(n+2, n-1)d(n+\tfrac{1}{2}).$$

The symbol $D^i(p,q)$ is a standard notation for the SU(3) multiplet involved,[2] and $d^{2j+1}(j)$ denotes the spin-j representation of SU(2). We assume that the dynamics of the B^0 and B^1 multiplets are described by the well-known reciprocal bootstrap model.[3] The B^{n+1} are assumed to be resonances in two-particle, P-wave states of the type (n,P), where n is shorthand for B^n and P denotes the pseudoscalar meson octet. The force producing the B^{n+1} multiplet in this state (for $n>0$) is assumed to result from the exchange of the baryon multiplet B^{n-1}. The lightest undiscovered multiplets in the proposed chain are $B^2(35)$ and $B^3(81)$, where the number in parentheses is the SU(3) multiplicity.[4]

We first study the magnitudes of the forces. In the P-wave static model, each force corresponds to a simple pole. For the present, we neglect the mass splitting within each multiplet, so that there is only one force pole for each resonance. The symbol $F(A\to B,C)$ denotes the residue of the force pole in the elastic scattering amplitude in the partial wave corresponding to A of the two-particle state (B,C). Similarly, the coupling constant $\gamma^2(A\to B,C)$ is the negative of the residue of the corresponding resonance or bound-state pole. If B and C refer to multiplets, rather than single particles, these residues are "total" residues, i.e., sums over all the (B,C) states coupled to a particular particle of multiplet A. The bracketed symbol $[n]$ is used to denote the state of maximum I_z and maximum J_z in the multiplet B^n. The residue of the force that produces the state $[n+1]$ in the model is given by the crossing formula,

$$F([n+1]\to[n],\pi^{++}) = \gamma^2([n-1]\to[n],\pi^{--})$$
$$= \gamma^2([n]\to[n-1],\pi^{++}), \qquad (1)$$

where π^{++} denotes a π^+ meson of orbital angular momentum component $L_z=+1$. If $n>1$, the state $[n]$ is coupled only to the two-particle state $([n-1],\pi^{++})$ of the multiplet $(n-1,P)$, and Eq. (1) leads directly to the equation

$$F(n+1\to n,P) = \gamma^2(n\to n-1,P), \quad n>1. \qquad (2a)$$

Thus, the force producing the resonance B^{n+1} is attractive, and the residue is equal to the coupling constant of the previous multiplet in the chain. Equation (1) applies also to the case $n=1$, if the state $[0]$ is defined to be a proton of $J_z=\frac{1}{2}$. In this case the amplitude $[n]\to[n-1],\pi^{++}$ is the $N^{*++}\to p,\pi^+$ amplitude in the states of maximum spin. The (p,π^+) coupling is only half the N^{*++} coupling in SU(3), the other half being supplied by the (Σ^+,K^+) state. Therefore, the appropriate modification of Eq. (2a) for $n=1$ is

$$F(2\to 1,P) = \tfrac{1}{2}\gamma^2(1\to 0,P). \qquad (2b)$$

The significance of Eqs. (2a) and (2b) is particularly simple in the effective range approximation, in which F and γ^2 are equal for any resonance.[3] It is seen that in this approximation the residue of the force that produces each resonance above the decuplet in the chain is equal to $\frac{1}{2}$ the residue of the force that produces the decuplet.

We next show that the model is consistent in the sense that the B^{n-1} exchange force in the (n,P) state is less attractive in all other representations than it is in the (assumed resonating) representation $D(n+3,n)d(n+\frac{3}{2})$. For this purpose we rewrite Eq. (1) in terms of residues applying to entire multiplets, i.e.,

$$F(n+1\to n,P)$$
$$= \gamma^2(n-1\to n,P)C_{n+1,n-1}^{\;n}$$
$$= (W_n/W_{n-1})\gamma^2(n\to n-1,P)C_{n+1,n-1}^{\;n}, \qquad (3)$$

where $C_{n+1,n-1}{}^{n}$ is the element of the elastic crossing matrix in the state (n, P) referring to the force in the $n+1$ channel resulting from exchange of the multiplet B^{n-1}, and W_n is the number of particles in the multiplet B^n. One may compute the crossing-matrix elements of Eq. (3) by comparing this equation with Eqs. (2a) and (2b); the result is

$$C_{n+1,n-1}{}^{n} = W_{n-1}/W_n, \quad n > 1, \tag{4a}$$

$$C_{2,0}{}^{1} = \tfrac{1}{2}W_0/W_1. \tag{4b}$$

Each crossing-matrix element may be regarded as the product of an element of the spin crossing matrix and an element of the SU(3) crossing matrix. Equation (4a) is valid for each of the two crossing matrices separately. The factor $\frac{1}{2}$ in Eq. (4b) goes with the SU(3) crossing matrix.

We assert that the crossing-matrix elements of Eqs. (4a) and (4b) are larger than any other elements associated with the exchange of the B^{n-1} multiplet. This assertion may be proved for the SU(3) crossing matrices corresponding to $n > 2$ from the following three facts: (1) The crossing matrix C is related to a unitary matrix U by the equation $C_{ij} = U_{ij}(W_j/W_i)^{1/2}$, so that C satisfies the sum rule $\sum_i C_{ij}{}^2 W_i = W_j$.[5] (2) The elastic crossing matrix satisfies the sum rule $\sum_i C_{ij} W_i = W_j$.[6] (3) The representation of smallest dimension in the reduction of the direct product $D(1,1)\otimes D(n+2,n-1)$ is one of the two representations $D(n+1,n-2)$ or $D(n+3,n-3)$. [The multiplicity of $D(p,q)$ is given in reference 2.] One can prove the assertion by showing that if $C_{ij}{}^{n} \geq W_{n-1}/W_n$ for any representation i in addition to $D(n+3,n)$, one of the two sum rules must be violated. The proof does not quite work for the $\underline{8}\otimes\underline{35}$ crossing matrix; the assertion may be verified by inspection in the $\underline{8}\otimes\underline{10}$ case.[7] A similar proof applies to all the spin crossing matrices.

Another force that exists in the model is the B^{n+1} exchange force in the (n, P) state. However, the crossing-matrix factors are such that this force is strong only in the representation corresponding to B^{n-1}. We neglect this force because the (n, P) state is heavier than the $(n-2, P)$ state, in which the B^{n-1} resonance is assumed to exist.

Our assumption that the B^{n+1} are resonances in the states (n, P) is reasonable because the lighter states (m, P) (where $m < n$) are not coupled to the SU(3) representation $D(n+3,n)$. It is interesting to consider also the possibility of a resonance in the (n, P) state corresponding to the representation $D(n+1,n-2)d(n+\frac{3}{2})$, since this type of resonance could be a Regge recurrence of B^{n-1}. We have shown that the SU(3) crossing matrices favor $D(n+3,n)$ over the Regge multiplet. On the other hand, the SU(3) algebra is such that the Regge recurrence representation is coupled to lighter two-particle states also [such as the $(n-1, P)$ states]; this coupling may increase the attraction. Therefore, we can argue only that our proposed resonance chain may exist; we cannot argue that other resonances do not exist or lie at higher energies.

We now make a rough prediction of the masses of the particles in the proposed $j = \frac{5}{2}$ multiplet $B^2(35)$. The hypercharge and isotopic-spin structure of this multiplet is (2, 2), (1, $\frac{3}{2}$ and $\frac{5}{2}$), (0, 1 and 2), (−1, $\frac{1}{2}$ and $\frac{3}{2}$), (−2, 0 and 1), (−3, $\frac{1}{2}$). The Gell-Mann–Okubo mass formula for the particles in this multiplet is[8]

$$M = a_2 + b_2 Y + c_2[I(I+1) - \tfrac{1}{4}Y^2 - 4], \tag{5}$$

where a_2, b_2, and c_2 are constants. The $(-4c_2)$ term is included so that a_2 is the average mass within the multiplet. It may be shown from the P-wave, static-model dispersion relations that a resonance occurs at an energy ω that corresponds to a zero in the quantity $1-(\omega-\omega_F)FI$, where ω_F and F are the position and residue of the force pole and I is the integral

$$I = \pi^{-1}\int \frac{d\omega' q'^3 P(\omega')}{(\omega'-\omega_F)^2(\omega'-\omega)}.$$

The quantity q' is the magnitude of the meson momentum corresponding to ω', and $P(\omega')$ is a cutoff function. We make the crude approximation that the integrals I for all resonances of the model are equal to a constant, Δ^{-1}, so that the resonance energies satisfy the formula $\omega-\omega_F = \Delta/F$. In order to predict the average mass a_n in the multiplet B^n, we neglect the mass differences within the multiplets, in which case our assumption leads to the formula,

$$a_n = 2a_{n-1} - a_F + \Delta/F, \tag{6}$$

where a_F is the average mass in the multiplet that transmits the force. In our model, $a_F = a_{n-2}$ if $n > 1$. We estimate Δ/F by taking $n = 1$ in Eq. (6) and using experimental values for the average masses in the octet and decuplet. (In this case the force is transmitted by the octet B^0.) The result is $\Delta/F(1 \to 0, P) \sim 235$ MeV. In the effective-range approximation, this implies that $\Delta/F(2 \to 1, P) \sim 470$ MeV. Application of

Eq. (6) to the multiplet $B^2(35)$ then leads to the estimate for the average mass, $a_2 \sim 2090$ MeV. Since the assumptions involved are crude, this estimate may be considerably in error.

It is seen from Eq. (6) that the differences between the average masses of adjacent members of the chain of multiplets increases as n increases. For this reason we do not expect that the entire infinite chain is realized in nature.

We may predict the mass splitting parameters of $B^2(35)$ by using individual particle masses, rather than average masses, in Eq. (6). The $(Y=-3, I=\frac{1}{2})$ and $(Y=2, I=2)$ members of B^2 are regarded as $\bar{K}\Omega$ and KN^* resonances, produced, respectively, by Ξ and Σ exchange forces. We assume that the value of the Δ/F term is common to all members of a multiplet, so that this term may be eliminated by appropriate subtraction, viz.,

$$M_{(\bar{K}\Omega)}-a_2=2(M_\Omega-a_1)-(M_\Xi-a_0), \tag{7a}$$

$$M_{(KN^*)}-a_2=2(M_{N^*}-a_1)-(M_\Sigma-a_0). \tag{7b}$$

These equations, when combined with Eq. (5), lead to the predictions,

$$b_2 \sim -178 \text{ MeV}, \quad c_2 \sim 20 \text{ MeV}. \tag{8}$$

In order to check the consistency of this calculation, we apply Eq. (6) to the $(Y=1, I=\frac{5}{2})$ member of B^2, assumed to be a πN^* resonance, produced by the N exchange force. The result is $M_{(\pi N^*)}-a_2 \sim -82$ MeV, whereas the b_2 and c_2 values of Eq. (8) lead to $M_{(\pi N^*)}-a_2 \sim -86$ MeV.

We may apply this method to the mass splitting of the decuplet also, by regarding the Ω as a $\bar{K}\Xi$ bound state, produced by the Σ exchange force,[9] and by taking the parameters of the N^* midway between those of a (πN) resonance, produced by the N exchange force, and those of a $K\Sigma$ bound state, produced by the Ξ exchange force. If we set $a_1=M_{Y^*}$, the result of this estimate is $M_\Omega-M_{Y^*}=2M_\Xi-M_\Sigma-a_0 \sim 294$ MeV; $M_{N^*}-M_{Y^*}=\frac{1}{2}M_N+M_\Sigma-\frac{1}{2}M_\Xi-a_0 \sim -148$ MeV. The agreement with experiment is surprisingly good.

The predicted average mass difference a_2-a_1 is sufficiently large that one does not expect this static-model prediction to be accurate. It is not even clear that the 35-fold multiplet should exist. However, if the $B^2(35)$ does exist, the 81-fold, $j=\frac{7}{2}$ multiplet B^3 may exist also. An extension of the argument used above leads to a predicted average mass of about 3.4 MeV for $B^3(81)$. It should be emphasized that neither of these multiplets is expected to decay strongly into (B^0, P) states. $B^2(35)$ and $B^3(81)$ should decay primarily into (B^1, P) and (B^2, P) states, respectively.

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[5]This may be shown from a slight extension of the argument in R. H. Capps, to be published.

[6]R. H. Capps, Phys. Rev. 134, B460 (1964).

[7]This crossing matrix is given by Donald E. Neville, Phys. Rev. 132, 844 (1963). This was pointed out to the author by Juergen Koerner.

[8]See S. Okubo, Progr. Theoret. Phys. (Kyoto) 27, 949 (1962).

[9]The Λ exchange force is neglected because $g_{K\Lambda\Xi}^2$ is much smaller than $g_{K\Sigma\Xi}^2$, if the F/D interaction ratio is close to the value predicted by the reciprocal bootstrap model to the octet and decuplet.