hypernuclei with large nucleon number and hypernuclei which correspond to the excited states of nuclei. They not only afford an enormous number of experimental tests of the unitary symmetry model, but also constitute a chart of hypernuclear spectroscopy.[15]

It is a pleasure to thank Professor J. R. Oppenheimer for the hospitality extended at the Institute for Advanced Study.

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SU(6) AND ELECTROMAGNETIC INTERACTIONS

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1. The purpose of this note is to discuss some properties of the electromagnetic vertex of baryons under the assumption that the effective electromagnetic current associated with the strongly interacting particles transforms according to the adjoint representation of the group[1-3] SU(6). In particular we show that, in the limit where SU(6) is broken by electromagnetism only, all of the following quantities can be expressed uniquely in terms of the proton magnetic moment $\mu(p)$: (a) the magnetic moments of all baryon octet members, (b) those of the spin-$\frac{3}{2}$ decuplet, (c) all allowed transition moments between octet and decuplet. We recall[1,2] that the octet and the decuplet are united in the 56-dimensional representation of SU(6) and that $\underline{56}^*\otimes\underline{56}$ contains $\underline{35}$ only once. All our results about baryons stem from this single occurrence of $\underline{35}$.

In a pure SU(3) treatment it is customary to define the charge operator Q as follows:

$$Q=(F_3+F_8/\sqrt{3}). \tag{1}$$

The magnetic moment operator is

$$\vec{\mathrm{M}}=\mu_0 Q'\vec{\mathrm{J}}. \tag{2}$$

$\vec{\mathrm{J}}$ is the appropriate spin matrix ($\vec{\mathrm{J}}=\vec{\sigma}/2$ for spin $\frac{1}{2}$), μ_0 is a scale factor. Q' is an operator with the same SU(3) transformation properties as Q. The quantity commonly called the magnetic moment is the matrix element of M_3 between states of highest J_3. The (diagonal) magnetic moments within any given SU(3) multiplet are given by[4]

$$\mu=bQ+c[U(U+1)-\tfrac{1}{4}Q^2-\tfrac{1}{6}C_2^{(3)}], \tag{3}$$

U being the usual U spin.[5] The embedding of SU(3)⊗SU(2) in SU(6) removes the D/F arbitrariness reflected in Eq. (3) and gives the unique relations mentioned earlier. We next state our results:

(a) Baryon octet.—We find

$$\mu_8=\tfrac{2}{3}[(\tfrac{1}{2}Q+1)^2-U(U+1)]\mu(p). \tag{4}$$

Beyond the SU(3) relations, first tabulated by Coleman and Glashow,[6] Eq. (4) gives the additional SU(6) relation

$$\beta=\mu(n)/\mu(p)=-\tfrac{2}{3}, \tag{5}$$

in remarkable agreement with the experimental ratio ≈ -0.684.

(b) The spin-$\frac{3}{2}$ decuplet.—As $U=1-Q/2$, SU(3) predicts that $\mu_{10}=\mathrm{const}\times Q$. More specifically we find from SU(6) that

$$\mu_{10}=Q\mu(p). \tag{6}$$

Thus, for example, $\mu(\Omega)=-\mu(p)$.

(c) Decuplet-octet transitions.—We denote the amplitude of the M_1 transitions by $\langle n'J'M'|\mu|nJM\rangle$, where n' and n are particle labels. In this notation $\langle p\frac{1}{2}\frac{1}{2}|\mu|p\frac{1}{2}\frac{1}{2}\rangle\equiv\mu(p)$. For the decuplet-octet transitions $J=\frac{3}{2}$, $J'=\frac{1}{2}$, and it is sufficient to quote the results for $M=\frac{1}{2}$ and $M'=\frac{1}{2}$. Amplitudes for other M and M' can be obtained by elementary SU(2) rotations. In the following it is therefore understood that $J=\frac{3}{2}$, $M=\frac{1}{2}$, $J'=M'=\frac{1}{2}$, and the explicit dependence need not be exhibited.

Note that SU(3) alone gives the following relationships for transitions allowed by conservation of charge and hypercharge[7]:

$$\langle p|\mu|N_+^*\rangle=-\langle\Sigma_+|\mu|Y_+^*\rangle=\langle n|\mu|N_0^*\rangle=2\langle\Sigma_0|\mu|Y_0^*\rangle$$
$$=\tfrac{1}{2}\sqrt{3}\langle\Lambda|\mu|Y_0^*\rangle=\langle\Xi_0|\mu|\Xi_0^*\rangle, \tag{7}$$

$$\langle\Sigma_-|\mu|Y_-^*\rangle=\langle\Xi_-|\mu|\Xi_-^*\rangle=0. \tag{8}$$

SU(6) now gives the additional relation

$$\langle p|\mu|N_+^*\rangle=\tfrac{2}{3}\sqrt{2}\mu(p). \tag{9}$$

This is in qualitative agreement with the estimates of Gourdin and Salin[8] who obtain $\langle p|\mu|N_+^*\rangle\cong 1.6\times(2\sqrt{2}/3)\mu(p)$ from a study of $\gamma+p\to\pi+N$ near the 33 resonance.

2. Derivations.—For given momentum $\vec{\mathrm{q}}$, the states of the 56-dimensional representation of $\mathrm{SU}(6)_{\vec{\mathrm{q}}}$ are described by the completely symmetric tensor $B^{\alpha\beta\gamma}(\vec{\mathrm{q}})$, $\alpha,\beta,\gamma=1,2,\cdots,6$. This tensor is reducible under the group $\mathrm{SU}(3)\otimes\mathrm{SU}(2)_{\vec{\mathrm{q}}}$, the explicit reduction[9] in the rest frame ($\vec{\mathrm{q}}=0$) being

$$B^{\alpha\beta\gamma}(0)=B^{\alpha\beta\gamma}=\chi^{(ijk)}d^{(ABC)}$$
$$+\frac{1}{3\sqrt{2}}[(2\epsilon^{ij}\chi^k+\epsilon^{jk}\chi^i)\epsilon^{ABD}b_D{}^C$$
$$+(\epsilon^{ij}\chi^k+2\epsilon^{jk}\chi^i)\epsilon^{BCD}b_D{}^A], \tag{10}$$

$i,j,k=1,2$; $A,B,C,D=1,2,3$. Here ϵ^{ij} and ϵ^{ABC} are the Levi-Civita symbols in two and three dimensions, respectively. χ^i is a (normalized) Pauli spinor. The $\chi^{(ijk)}$ are the spin-$\frac{3}{2}$ wave functions.[10] $b_B{}^A$ is the usual baryon octet tensor,[11] $d^{(ABC)}$ is the SU(3)-decuplet tensor.[11]

Our assumption is that the charge operator transforms like an $(\underline{8},\underline{1})$ member of a $\underline{35}$ representation, and the magnetic moment operator transforms like an $(\underline{8},\underline{3})$ member of a $\underline{35}$ representation[12] (we do not assume that the same $\underline{35}$ representation appears in both cases). Under these assumptions, the effective, low-frequency limit of the electromagnetic vertex of the baryons may be written as[13]

$$3B_{\alpha\beta\gamma}{}^+B^{\alpha\beta\delta}[e\varphi\delta_l{}^k+\mu(p)\cdot i(\vec{\sigma}\cdot\vec{\mathrm{q}}\times\vec{\epsilon})_l{}^k]Q_D{}^C;$$
$$\gamma=(k,C),\ \ 8=(l,D), \tag{11}$$

where φ is an electrostatic potential and $\vec{\epsilon}$ a polarization vector $\perp\vec{\mathrm{q}}$. Expanding the coefficient of φ in terms of particle states we get the respective charges of the particles, while the magnetic term yields the results quoted in Eqs. (4)-(9).[14]

3. Remarks.—(a) A more general definition of Q has been proposed[15] which would lead to the addition on the right-hand side of Eq. (3) of a constant (independent of Q, U, and $C_2^{(3)}$). The inclusion of such a term would diminish the pre-

dictive power of SU(6) and would in particular render β arbitrary [see Eq. (5)].

(b) It has been noted[3] that the subgroup SU(4)(T) of SU(6) gives an arbitrary mixture of Wigner versus Majorana forces between nucleons, while this mixture is unique for SU(6). This statement has an electromagnetic analog, namely, the isoscalar vs isovector ratio is fixed in SU(6) but arbitrary in SU(4)(T), so that SU(4)(T) does not make any predictions for β. However, if one assumes that the effective electromagnetic current transforms according to the adjoint representation of SU(4)(T), one obtains[16] $\beta=-1$.

(c) It has been noted in reference 3 and independently by Sakita[17] that SU(6) relates the structure of Pauli-type vector-meson terms to that of the p-wave pseudoscalar term. Sakita has studied an assignment where the baryon octet is contained in the 20-dimensional representation of SU(6). β is unique also for this choice and we find $\beta=-2$ in this case. This may serve as a further indication that the $\underline{56}$ representation is preferable.

(d) All our results can also be obtained by the method of vector addition of magnetic moments,[18] by regarding the baryons as composite structures built up out of spin-$\frac{1}{2}$ quarks[19] with composite wave functions dictated by SU(6). This method can of course be applied to other SU(6) representations as well. In this way one easily shows that SU(6) yields a new relation for the $\underline{35}$ meson representation, namely $\mu(\rho_+)=3\langle\pi_+,0,0|\mu|\rho_+,1,0\rangle$. We hasten to add that this remark is not meant to shed light on the existence of quarks.

4. Finally, we discuss some implications of our results from the point of view of a local Lagrangian field theory. It should be stressed that the conclusions obtained so far have come from an analysis of an <u>effective</u> vertex[14] under the assumption that this vertex has prescribed SU(6) properties. Likewise the results found in reference 3 referred exclusively to an SU(6)-invariant effective strong-interaction vertex. However, in the present electromagnetic case we are in the unique position to be able to compare a specific numerical prediction of the SU(6) theory with an equally specific answer of local field theory. Loosely speaking, the situation is the following: According to Eq. (5), $\beta=-\frac{2}{3}$. This comfortable value for β is a pure number, independent of any coupling constants. In field theory we have been accustomed for many years to say, "In the limit where the strong interactions are 'turned off,' we should have $\mu(n)=0$, $\mu(p)=1$, hence $\beta=0$; or, conversely, the 'anomalous' magnetic moments of nucleons come about by 'turning on' the strong interactions." Thus we arrive at a paradox which comforts while it mocks: We cannot assume both that the SU(6) group is valid and that local field theory with minimal electromagnetic interactions applies to nucleons.

We shall next attempt to state this imcompatibility in more precise terms. Let us consider the following set of assumptions: (I) Strong and electromagnetic effects are derivable from a Lagrangian $\mathcal{L}=\mathcal{L}_0+\mathcal{L}(g)+\mathcal{L}(e)$. Here $\mathcal{L}_0$ is the free Lagrangian, $\mathcal{L}(g)$ symbolizes all strong-interaction terms, and $\mathcal{L}(e)$ stands for the electromagnetic terms. $\mathcal{L}_0$, $\mathcal{L}(g)$, and $\mathcal{L}(e)$ contain explicitly the local nucleon fields. (II) $\mathcal{L}(e)$ is minimal, that is, it contains no derivatives of the electromagnetic potentials, while also the SU(3) trace of the charge operator shall vanish ($Q=F_3+F_8/\sqrt{3}$). (III) $\mathcal{L}_0+\mathcal{L}(g)$ is invariant under a group which contains SU(6) as a subgroup. As SU(6) is a linear group, this means in particular that $\mathcal{L}_0$ and $\mathcal{L}(g)$ are separately SU(6)-invariant. Furthermore, $\mathcal{L}(e)$ shall have the definite SU(6) properties assumed above for the effective electromagnetic vertex. (IV) It is possible to calculate in such a theory the magnetic moment of the neutron, which we denote by $\mu_n(e,g)$, and likewise for other particles. Moreover, $\mu_n(e,0)$ exists and is identical with the neutron magnetic moment calculated from $\mathcal{L}=\mathcal{L}_0+\mathcal{L}(e)$; likewise for the proton. We conclude that the assumptions (I) to (IV) are incompatible.

We are now faced with two connected questions. First, one should prove this statement in a direct fashion rather than having recourse to the numerical result for β. Second, if one believes (as we do) that the results obtained with the SU(6) assumptions are not a series of numerical coincidences, one will have to revise some of the assumptions (I) to (IV) and the question is which ones. These questions will be studied further.

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[8]M. Gourdin and Ph. Salin, Nuovo Cimento 27, 193 (1963). In their notation $\langle p|\mu|N_+^*\rangle = (\frac{2}{3})^{1/2}\frac{1}{2}(C_1 + 2m_N C_2/m_\pi)$ in units of nuclear magneton.

[9]To each fixed α corresponds a fixed pair of labels (i,A). For $\alpha = 1,\cdots,6$ these respective pairs are (1,1), (1,2), (1,3), (2,1), (2,2), and (2,3). In Eq. (11) we use the correspondence $\alpha = (i,A)$; $\beta = (j,B)$, $\gamma = (k,C)$. Note that $\epsilon^{ABD}b_D{}^C + \epsilon^{BCD}b_D{}^A + \epsilon^{CAD}b_D{}^B = 0$, $\epsilon^{ij}\chi^k + \epsilon^{jk}\chi^i + \epsilon^{ki}\chi^j = 0$.

[10]$\chi^{(ijk)}$ is totally symmetric in i, j, and k. Normalizations are $\|\chi^{(111)}\| = 1$, $\|\chi^{(112)}\| = \frac{1}{3}$, etc.

[11]We use a normalization such that $b_3{}^1 = p$. Furthermore, $d^{(111)} = N_{++}^*$, $d^{(112)} = N_+^*/\sqrt{3}$, etc. For fixed α, β, γ, the norm of $B^{\alpha\beta\gamma}$ is equal to 1 if $\alpha = \beta = \gamma$; $\frac{1}{3}$ if $\alpha = \beta \neq \gamma$; $\frac{1}{6}$ if $\alpha \neq \beta \neq \gamma$.

[12]Note that in terms of SU(6) generators the charge and magnetic moment operators are given by: $Q = e \times (A_1{}^1 + A_4{}^4)$; $M_+ = \mu_0[\frac{2}{3}T_1{}^4 - \frac{1}{3}T_2{}^5 - \frac{1}{3}T_3{}^6]$; $M_- = \mu_0[\frac{2}{3}T_4{}^1 - \frac{1}{3}T_5{}^2 - \frac{1}{3}T_6{}^3]$; $M_3 = \mu_0[\frac{1}{3}(T_1{}^1 - T_4{}^4) - \frac{1}{6}(T_2{}^2 - T_5{}^5) - \frac{1}{6}(T_3{}^3 - T_6{}^6)]$, where $T_\alpha{}^\beta$ is an irreducible tensor operator transforming as the generator $A_\alpha{}^\beta$ (see Bég and Singh, reference 4).

[13]The normalization factors are so chosen that for the proton Eq. (11) reduces to $ep^+p\varphi + \mu(p)p^+\vec{\sigma}p\cdot\vec{H}$.

[14]$B_{\alpha\beta\gamma}{}^+$ means the wave function complex conjugate to $B^{\alpha\beta\gamma}$. In the present paper we do not discuss the extension to the crossed channel.

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